

King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 151

Recitation (7.4)

Question 1:

If $\csc x = -\frac{5}{4}$, where $\frac{3\pi}{2} < x < 2\pi$, then find $\tan 2x$, $\cos \frac{x}{2}$.

Answer: (a): $\tan 2x = \frac{24}{7}$ **(b):** $\cos \frac{x}{2} = -\frac{2\sqrt{5}}{5}$

Question 2 $\cos 2\alpha = \frac{3}{4}$ and α terminates in quadrant III, then find $\sin \alpha + \cos \alpha$

Answer: $\sin \alpha + \cos \alpha = \frac{-\sqrt{2} - \sqrt{14}}{4}$

Question 3: Find the exact value of: $\left(\sin \frac{3\pi}{8} + \cos \frac{3\pi}{8} \right)^2$

Answer: $\frac{2 + \sqrt{2}}{2}$

Question 4 Verify the following identity: $\sin^2 \frac{x}{2} (1 + \sec x)^2 = \cos^2 \frac{x}{2} \tan^2 x$

Solution:

$$\begin{aligned} LHS &= \sin^2 \frac{x}{2} (1 + \sec x)^2 = \frac{1 - \cos x}{2} \left(1 + \frac{1}{\cos x}\right)^2 = \frac{1 - \cos x}{2} \left(\frac{1 + \cos x}{\cos x}\right)^2 = \frac{1 - \cos x}{2} \frac{(1 + \cos x)^2}{\cos^2 x} \\ &= \frac{1 + \cos x}{2} \frac{(1 + \cos x)(1 - \cos x)}{\cos^2 x} = \cos^2 \frac{x}{2} \cdot \frac{1 - \cos^2 x}{\cos^2 x} = \cos^2 \frac{x}{2} \cdot \frac{\sin^2 x}{\cos^2 x} = \cos^2 \frac{x}{2} \tan^2 x = RHS \end{aligned}$$

Another Method:

$$\begin{aligned} RHS &= \cos^2 \frac{x}{2} \tan^2 x = \frac{1 + \cos x}{2} \cdot (\sec^2 x - 1) = \frac{1 + \cos x}{2} \cdot (\sec x - 1)(\sec x + 1) \\ &= \frac{1 + \cos x}{2} \cdot \left(\frac{1}{\cos x} - 1\right)(\sec x + 1) = \frac{1 + \cos x}{2} \cdot \left(\frac{1 - \cos x}{\cos x}\right)(\sec x + 1) = \frac{1 - \cos x}{2} \cdot \frac{1 + \cos x}{\cos x} (\sec x + 1) \\ &= \sin^2 \frac{x}{2} \cdot \left(\frac{1}{\cos x} + 1\right)(\sec x + 1) = \sin^2 \frac{x}{2} \cdot (\sec x + 1)(\sec x + 1) = \sin^2 \frac{x}{2} \cdot (\sec x + 1)^2 = LHS \end{aligned}$$

Question 5: If $\csc \theta = -\frac{5}{4}$, where $\frac{3\pi}{2} < \theta < 2\pi$, then $\sec \frac{\theta}{2} = ?$

Answer: $\sec \frac{\theta}{2} = -\frac{\sqrt{5}}{2}$