Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (6 points) (9.1 Recitation Q5):

If (x, y) is the solution of the system $\begin{cases} y = \log(x+1)+3 \\ y = \log(x+2)+2 \end{cases}$, then $27x = ?$

Solution:

$$\begin{cases} y = \log(x+1)+3 \\ y = \log(x+2)+2 \end{cases}$$

$$\log(x+1)+3 = \log(x+2)+2$$

$$\log(x+1)+1 = \log(x+2)$$

$$\log(x+1)+\log 10 = \log(x+2)$$

$$\log[10(x+1)] = \log(x+2)$$

$$10x+10 = x+2$$

$$9x = -8$$

$$x = -\frac{8}{9}$$

Check: $-\frac{8}{9}$ is from domain of the logarithmic expressions.

$$27x = 27\left(-\frac{8}{9}\right) = 3(-8) = -24$$

Question 2: (7 points) (9.5 Additional Exercise 8) (Textbook Exercise 58, page 876):

Find the value(s) of b such that the line $x+2y=b$ **touches** the circle $x^2+y^2=9$.

Solution:

$x+2y=b$ (I)

$x^2+y^2=9$ (II)

(I) $\Rightarrow$ $x = b-2y$

(II) $\Rightarrow$ $(b-2y)^2+y^2=9$

$$b^2-4by+4y^2+y^2=9$$

$$5y^2-4by+b^2-9=0$$

This is a quadratic equation in terms of y and will have a unique solution when the Discriminant is 0.

$$(-4b)^2-4(5)(b^2-9)=0$$

$$16b^2-20b^2+180=0$$

$$-4b^2=-180$$

$b^2=45$ $\Rightarrow$ $b=\pm 3\sqrt{5}$

Thus, the line $x+2y=b$ will touch the circle $x^2+y^2=9$ in only one point if

$b=3\sqrt{5}$ or $b=-3\sqrt{5}$.

Question 3: (7 points) (9.7 Additional Exercise 14): If $A=\begin{bmatrix}3 & 2 & 0\\ 3 & 5 & 1\end{bmatrix}$, $B=\begin{bmatrix}5 & 0\\ -3 & 1\\ 0 & -1\end{bmatrix}$ and

$C=\begin{bmatrix}\frac{3}{2} & 1\\ 0 & \frac{3}{2}\end{bmatrix}$ then find the matrix $D=AB-4C^2$.

Solution: $D=AB-4C^2=AB-(2C)^2=AB-(2C)(2C)$

$$=\begin{bmatrix}3 & 2 & 0\\ 3 & 5 & 1\end{bmatrix}\begin{bmatrix}5 & 0\\ -3 & 1\\ 0 & -1\end{bmatrix}-\begin{bmatrix}3 & 2\\ 0 & 3\end{bmatrix}\begin{bmatrix}3 & 2\\ 0 & 3\end{bmatrix}=\begin{bmatrix}9 & 2\\ 0 & 4\end{bmatrix}-\begin{bmatrix}9 & 12\\ 0 & 9\end{bmatrix}=\begin{bmatrix}0 & -10\\ 0 & -5\end{bmatrix}$$