

Show all necessary steps for full marks.

Question 1: (5 points): (R.5 Exercise 38): Simplify $\frac{ac + ad + bc + bd}{a^2 - b^2} \cdot \frac{a^3 - b^3}{2a^2 + 2ab + 2b^2}$

Solution:

$$\begin{aligned}
 38. \quad & \frac{ac + ad + bc + bd}{a^2 - b^2} \cdot \frac{a^3 - b^3}{2a^2 + 2ab + 2b^2} \\
 &= \frac{a(c + d) + b(c + d)}{(a + b)(a - b)} \cdot \frac{(a - b)(a^2 + ab + b^2)}{2(a^2 + ab + b^2)} \\
 &= \frac{(c + d)(a + b)}{(a + b)(a - b)} \cdot \frac{(a - b)(a^2 + ab + b^2)}{2(a^2 + ab + b^2)} = \frac{c + d}{2}
 \end{aligned}$$

Question 2: (5 points): (R.5 Chapter R Test Question 21, Page 78):

Perform the indicated operations: $\frac{5x^2 - 9x - 2}{30x^3 + 6x^2} \div \frac{x^4 - 3x^2 - 4}{2x^8 + 6x^7 + 4x^6}$

Solution:

$$\begin{aligned}
 21. \quad & \frac{5x^2 - 9x - 2}{30x^3 + 6x^2} \div \frac{x^4 - 3x^2 - 4}{2x^8 + 6x^7 + 4x^6} \\
 &= \frac{5x^2 - 9x - 2}{30x^3 + 6x^2} \cdot \frac{2x^8 + 6x^7 + 4x^6}{x^4 - 3x^2 - 4} \\
 &= \frac{(5x + 1)(x - 2)}{6x^2(5x + 1)} \cdot \frac{2x^6(x^2 + 3x + 2)}{(x^2 - 4)(x^2 + 1)} \\
 &= \frac{(5x + 1)(x - 2)}{6x^2(5x + 1)} \cdot \frac{2x^6(x + 2)(x + 1)}{(x + 2)(x - 2)(x^2 + 1)} \\
 &= \frac{2x^6(x + 1)}{6x^2(x^2 + 1)} = \frac{x^4(x + 1)}{3(x^2 + 1)}
 \end{aligned}$$

Question 3: (5 points): (R.6 Exercise 69, page 57): Simplify $\left(\frac{16m^3}{n}\right)^{1/4} \left(\frac{9n^{-1}}{m^2}\right)^{1/2}$

Solution:

$$\begin{aligned}
 69. \quad & \left(\frac{16m^3}{n}\right)^{1/4} \left(\frac{9n^{-1}}{m^2}\right)^{1/2} = \frac{16^{1/4} m^{3/4}}{n^{1/4}} \cdot \frac{9^{1/2} n^{-1/2}}{m} \\
 &= \frac{2m^{3/4}}{n^{1/4}} \cdot \frac{3n^{-1/2}}{m} \\
 &= (2 \cdot 3) \frac{m^{3/4}}{m} \cdot \frac{n^{-1/2}}{n^{1/4}} \\
 &= 6m^{(3/4)-1} n^{(-1/2)-(1/4)} \\
 &= 6m^{(3/4)-(4/4)} n^{(-2/4)-(1/4)} \\
 &= 6m^{-1/4} n^{-3/4} = \frac{6}{m^{1/4} n^{3/4}}
 \end{aligned}$$

Question 4: (5 points): (R.6 Exercise 94, page 58):

Factor $(3r + 1)^{-2/3} + (3r + 1)^{1/3} + (3r + 1)^{4/3}$, using the common factor $(3r + 1)^{-2/3}$. Assume all variables represent positive real numbers.

Solution:

94. Factor $(3r + 1)^{-2/3} + (3r + 1)^{1/3} + (3r + 1)^{4/3}$,

using the common factor $(3r + 1)^{-2/3}$.

$$\begin{aligned} & (3r + 1)^{-2/3} + (3r + 1)^{1/3} + (3r + 1)^{4/3} \\ &= (3r + 1)^{-2/3} \cdot [1 + (3r + 1)^{3/3} + (3r + 1)^{6/3}] \\ &= (3r + 1)^{-2/3} \cdot [1 + (3r + 1) + (3r + 1)^2] \\ &= (3r + 1)^{-2/3} \cdot (1 + 3r + 1 + 9r^2 + 6r + 1) \\ &= (3r + 1)^{-2/3} (9r^2 + 9r + 3) \\ &= 3(3r + 1)^{-2/3} (3r^2 + 3r + 1) \end{aligned}$$