Serial #: ________ ID ______________ NAME ______________________________________

Show all necessary steps for full marks.

Question 1: (5 points): If $f(x)=\dfrac{1-3x}{2+7x}$. If $f^{-1}(x)$ is written in the form $\dfrac{Ax+B}{x+C}$, then $A+B+C=?$

Solution: $y=\dfrac{1-3x}{2+7x}$

$$x=\frac{1-3y}{2+7y}$$

$$1-3y=2x+7xy$$

$$-7xy-3y=-1+2x$$

$$y(-7x-3)=-1+2x$$

$$y=\frac{-1+2x}{-7x-3}$$

$$f^{-1}(x)=\frac{-1+2x}{-7x-3}$$

We have to write the function $f^{-1}(x)=\dfrac{2x-1}{-7x-3}$ in the required form $\dfrac{Ax+B}{x+C}$

Therefore, we have to divide numerator and denominator by -5

$$f^{-1}(x)=\frac{2x-1}{-7x-3}=\frac{\frac{2x-1}{-7}}{\frac{-7x-3}{-7}}=\frac{-\frac{2}{7}x+\frac{1}{7}}{x+\frac{3}{7}} \Rightarrow A=-\frac{2}{7},\ B=\frac{1}{7} \text{ and } C=\frac{3}{7}$$

$$A+B+C=-\frac{2}{7}+\frac{1}{7}+\frac{3}{7}=\frac{-2+1+3}{7}=\frac{2}{7}$$

Question 2: (5 points): Let $f(x)=2x^2+4$ for $x\le 0$. Find $f^{-1}(x)$ and state the domain and range of f and f^{-1}. Sketch the graph of f and f^{-1}

Solution: $y=2x^2+4,\ x\le 0$

$$x=2y^2+4,\ y\le 0$$

$$2y^2=x-4,\ y\le 0,\ x-4\ge 0$$

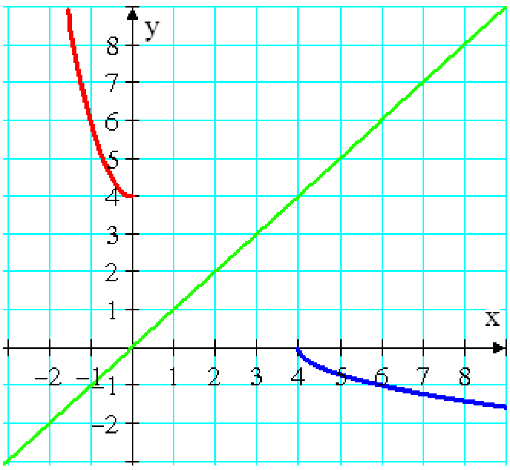
$$y^2=\frac{1}{2}x-2,\ y\le 0,\ x\ge 4$$

$$\sqrt{y^2}=\sqrt{\frac{1}{2}x-2},\ y\le 0,\ x\ge 4$$

$$|y|=\sqrt{\frac{1}{2}x-2},\ y\le 0,\ x\ge 4$$

$$y=-\sqrt{\frac{1}{2}x-2},\ y\le 0,\ x\ge 4$$

$$f^{-1}(x)=-\sqrt{\frac{1}{2}x-2},\ D_{f^{-1}}=[4,\infty),\ R_{f^{-1}}=(-\infty,0]$$



Question 3: (5 points): Consider the function $f(x) = -2^{-2x+3} + 4$

(a): Find the *y*-intercepts, if any.
(b): Find the *x*-intercepts, if any.
(c): Find the range of f in interval notation.
(d): Sketch the graph of f.
(e): Sketch the graph of $g(x) = \left|-2^{-2x+3} + 4\right|$.

Solution:

(a): Let $x = 0$, then $f(0) = -2^{-0+3} + 4 = -2^3 + 4 = -4$

The y-intercept is: $\boxed{y = -4}$, $(0, -4)$

(b): Let $f(x) = 0$, then $0 = -2^{-2x+3} + 4 \Rightarrow 2^{-2x+3} = 2^2 \Rightarrow -2x + 3 = 2 \Rightarrow x = \frac{1}{2}$

The x-intercept is: $\boxed{x = \frac{1}{2}}$, $(1/2, 0)$

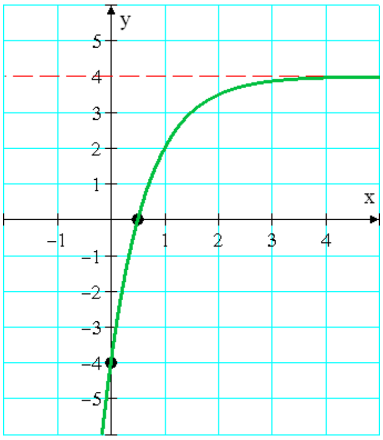
(c): $D_f = (-\infty, \infty)$

$-2^{-2x+3} < 0 \Rightarrow -2^{-2x+3} + 4 < 4 \Rightarrow y < 4 \Rightarrow R_f = (-\infty, 4)$

(d): As $x \to \infty$, $f(x) \to -0 + 4$

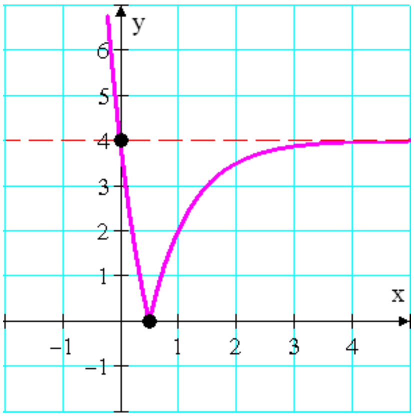
$\Rightarrow$ The line $y = 4$ is the horizontal asymptote.

x	0	1/2	1
$y = f(x)$	-4	0	2



$D_f = (-\infty, \infty)$ $R_f = (-\infty, 4]$

(e): $g(x) = \left|-2^{-2x+3} + 4\right|$



$D_g = (-\infty, \infty)$ $R_g = [0, \infty)$

Question 4: (5 points): If the function $y = 4^{x+2} - 5$ is written as $y = k\left(\frac{1}{2}\right)^{bx} + c$, then $k + b + c =$

(a) 11 (b) 7 (c) 9 (d) 13 (e) 12

Solution:

$$\begin{aligned} y &= 4^{x+2} - 5 \\ &= \left(2^2\right)^{x+2} - 5 \\ &= (2)^{4+2x} - 5 \\ &= (2)^4 (2)^{2x} - 5 \\ &= (16)\left(\frac{1}{2}\right)^{-2x} - 5 \\ &= k\left(\frac{1}{2}\right)^{bx} + c \end{aligned}$$

$$\Rightarrow \quad \boxed{k = 16}, \quad \boxed{b = -2}, \quad \boxed{c = -5} \quad \Rightarrow \quad k + b + c = 9$$