King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 142

Recitation (7.4)

Question 1: If $\csc x = -\dfrac{5}{4}$, where $\dfrac{3\pi}{2} < x < 2\pi$, then find $\tan 2x$, $\cos\dfrac{x}{2}$.

Answer: $\tan 2x = \dfrac{24}{7}$ $\qquad \cos\dfrac{x}{2} = -\dfrac{2\sqrt{5}}{5}$

Question2 $\cos 2\alpha = \dfrac{3}{4}$ and α terminates in quadrant III, then find $\sin\alpha + \cos\alpha$

Answer: $\sin\alpha + \cos\alpha = \dfrac{-\sqrt{2}-\sqrt{14}}{4}$

Question3: Find the exact value of: $\left(\sin\dfrac{3\pi}{8} + \cos\dfrac{3\pi}{8}\right)^2$

Answer: $\left(\sin\dfrac{3\pi}{8} + \cos\dfrac{3\pi}{8}\right)^2 = \dfrac{2+\sqrt{2}}{2}$

Question 4 Verify the following identity: $\sin^2\dfrac{x}{2}(1+\sec x)^2 = \cos^2\dfrac{x}{2}\tan^2 x$

Solution:

$$LHS = \sin^2\frac{x}{2}(1+\sec x)^2 = \frac{1-\cos x}{2}\left(1+\frac{1}{\cos x}\right)^2 = \frac{1-\cos x}{2}\left(\frac{1+\cos x}{\cos x}\right)^2 = \frac{1-\cos x}{2}\frac{(1+\cos x)^2}{\cos^2 x}$$

$$= \frac{1+\cos x}{2}\frac{(1+\cos x)(1-\cos x)}{\cos^2 x} = \cos^2\frac{x}{2}\cdot\frac{1-\cos^2 x}{\cos^2 x} = \cos^2\frac{x}{2}\cdot\frac{\sin^2 x}{\cos^2 x} = \cos^2\frac{x}{2}\tan^2 x = RHS$$

Another Method:

$$RHS = \cos^2\frac{x}{2}\tan^2 x = \frac{1+\cos x}{2}\cdot\left(\sec^2 x - 1\right) = \frac{1+\cos x}{2}\cdot(\sec x - 1)(\sec x + 1)$$

$$= \frac{1+\cos x}{2}\cdot\left(\frac{1}{\cos x}-1\right)(\sec x + 1) = \frac{1+\cos x}{2}\cdot\left(\frac{1-\cos x}{\cos x}\right)(\sec x + 1) = \frac{1-\cos x}{2}\cdot\frac{1+\cos x}{\cos x}(\sec x + 1)$$

$$= \sin^2\frac{x}{2}\cdot\left(\frac{1}{\cos x}+1\right)(\sec x + 1) = \sin^2\frac{x}{2}\cdot(\sec x + 1)(\sec x + 1) = \sin^2\frac{x}{2}\cdot(\sec x + 1)^2 = LHS$$

Question5 $\tan 427.5^\circ =$

A) $\sqrt{2}+1$

B) $\dfrac{1-\sqrt{3}}{2}$

C) $\sqrt{2}-1$

D) $\sqrt{3}+2$

E) $\dfrac{2-\sqrt{2}}{2}$

Answer: (A) $\sqrt{2}+1$