King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 142

Recitation (7.4 Reduction Identity)

$a\sin x + b\cos x = k\sin(x+\alpha)$

where $k = \sqrt{a^2+b^2}$ and α is determined by: $\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$ and $\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$

Or $\tan\alpha = \dfrac{b}{a}$ where α can be detrmined from the quadrant that contains the point (a,b)

Question 1:

Given the function $f(x) = 2\sin\frac{x}{3} - 2\sqrt{3}\cos\frac{x}{3}$

a) Rewrite $f(x)$ in the form $f(x) = k\sin(bx+\alpha)$

b) Find the amplitude, the phase shift, the period, and the range for the graph of $f(x)$.

Answer:

(a): $f(x) = 4\sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ OR $f(x) = 4\sin\left(\frac{x}{3} + \frac{5\pi}{3}\right)$

(b):

Amplitude = 4

Phase shift = π units to the right. OR Phase shift = -5π $|-5\pi|$ units to the left.

Period = 6π

Range = $[-4,4]$

Question 2: If $\sin 20^\circ - \sqrt{3}\cos 20^\circ = k\sin\theta,\ 0^\circ < \theta < 90^\circ$. Then k and θ are equal to

a) $-2,\ 40^\circ$

b) $2,\ 20^\circ$

c) $1-\sqrt{3},\ 20^\circ$

d) $-2,\ 20^\circ$

e) $-2,\ 30^\circ$

Answer: $-2,\ 40^\circ$