SN ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: **(7 points) (9.2 Textbook Exercise 26):** Use Gauss-Jordan Method to find the solution set of the following system of equations.

$$\begin{aligned} -x + 2y + 6z &= 2 \\ 3x + 2y + 6z &= 6 \\ x + 4y - 3z &= 1 \end{aligned}$$

Solution:

26. $\begin{aligned} -x+2y+6z&=2 \\ 3x+2y+6z&=6 \\ x+4y-3z&=1 \end{aligned}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} -1 & 2 & 6 & 2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array}\right] -1R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 8 & 24 & 12 \\ 1 & 4 & -3 & 1 \end{array}\right] -3R1+R2 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 8 & 24 & 12 \\ 0 & 6 & 3 & 3 \end{array}\right] -1R1+R3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 6 & 3 & 3 \end{array}\right] \tfrac{1}{8}R2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 6 & 3 & 3 \end{array}\right] 2R2+R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & -15 & -6 \end{array}\right] -6R2+R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{2}{5} \end{array}\right] -\tfrac{1}{15}R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{3}{10} \\ 0 & 0 & 1 & \frac{2}{5} \end{array}\right] -3R3+R2$$

Solution set: $\left\{\left(1, \frac{3}{10}, \frac{2}{5}\right)\right\}$

Question 2: **(6 points) (9.3Textbook Exercise 40 and 45):** Find the solution set of the following equations

(a): $\begin{vmatrix} 2x & x \\ 11 & x \end{vmatrix} = 6$ **(b):** $\begin{vmatrix} x & 0 & -1 \\ 2 & -3 & x \\ x & 0 & 7 \end{vmatrix} = 12$

Solution (a):

40. To solve the equation $\begin{vmatrix} 2x & x \\ 11 & x \end{vmatrix} = 6$, we need to solve $2x \cdot x - 11x = 6$.

$2x \cdot x - 11x = 6 \Rightarrow 2x^2 - 11x = 6 \Rightarrow$
$2x^2 - 11x - 6 = 0 \Rightarrow (2x+1)(x-6) = 0$
$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$ or $x - 6 = 0 \Rightarrow x = 6$

$$SS = \left\{-\frac{1}{2}, 6\right\}$$

Solution (b):

45. To solve the equation $\begin{vmatrix} x & 0 & -1 \\ 2 & -3 & x \\ x & 0 & 7 \end{vmatrix} = 12$, expand by the second column. We will need to find $-a_{12} \cdot M_{12} + a_{22} \cdot M_{22} - a_{32} \cdot M_{32}$. However, we do not need to calculate M_{12} or M_{32} since $a_{12} = 0$ and $a_{32} = 0$.

$M_{22} = \begin{vmatrix} x & -1 \\ x & 7 \end{vmatrix} = 7x + x = 8x$

$-a_{12} \cdot M_{12} + a_{22} \cdot M_{22} - a_{32} \cdot M_{32}$.
$= 0 \cdot M_{12} + (-3)8x + 0 \cdot M_{32} = -24x$

Set this equal to 12 and solve to get
$-24x = 12 \Rightarrow x = -\frac{1}{2}$

$$SS = \left\{-\frac{1}{2}\right\}$$

Question 3: **(7 points):** Find the inverse of $\begin{bmatrix} 1 & -3 & 2 \\ 3 & -8 & 7 \\ 2 & -3 & 6 \end{bmatrix}$ if it exists.

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 3 & -8 & 7 & 0 & 1 & 0 \\ 2 & -3 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1+R_3]{-3R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 3 & 2 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-3R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 0 & -1 & 7 & -3 & 1 \end{array}\right]$$

$$\xrightarrow{-1R_3} \left[\begin{array}{ccc|ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & -7 & 3 & -1 \end{array}\right] \xrightarrow{3R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & -8 & 3 & 0 \\ 0 & 1 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & -7 & 3 & -1 \end{array}\right] \xrightarrow[-R_3+R_2]{-5R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 27 & -12 & 5 \\ 0 & 1 & 0 & 4 & -2 & 1 \\ 0 & 0 & 1 & -7 & 3 & -1 \end{array}\right]$$

The inverse matrix is $\begin{bmatrix} 27 & -12 & 5 \\ 4 & -2 & 1 \\ -7 & 3 & -1 \end{bmatrix}$.