SN ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Q1. (5 points) (2.8 Exercise 40): Given $f(x)=\dfrac{1}{x}$. Find $\dfrac{f(x+h)-f(x)}{h}=?$

Solution:

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\frac{\frac{-h}{x(x+h)}}{\frac{h}{1}}=\frac{-h}{x(x+h)}\cdot\frac{1}{h}=-\frac{1}{x(x+h)}$$

Q2. (5 points) If $f(x)=\dfrac{x-1}{3-x}$ and $g(x)=\sqrt{x+2}$, find domain of $f\circ g$.

Solution: $(f\circ g)(x)=f[g(x)]=f[\sqrt{x+2}]=\dfrac{\sqrt{x+2}-1}{3-\sqrt{x+2}}$

Let D be the domain of the above expression.

Then $D=\{x \mid x+2\ge 0 \text{ and } 3+\sqrt{x+2}\ne 0\}=[-2,7)\cup(7,\infty)$

$D_{f\circ g}=D_g\cap D=[-2,\infty)\cap([-2,7)\cup(7,\infty))=[-2,7)\cup(7,\infty)$

Q3. (5 points) (3.1 Textbook Example 3): Graph $f(x)=-3x^2-2x+1$ by completing the square and locating the vertex. Identify the intercepts of the graph.

SOLUTION To complete the square, the coefficient of x^2 must be 1.

$$f(x)=-3\left(x^2+\frac{2}{3}x\qquad\right)+1 \quad \text{Factor } -3 \text{ from the first two terms.}$$

$$f(x)=-3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)+1 \quad \left[\tfrac{1}{2}\left(\tfrac{2}{3}\right)\right]^2=\left(\tfrac{1}{3}\right)^2=\tfrac{1}{9}\text{, so add and subtract } \tfrac{1}{9}.$$

$$f(x)=-3\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)-3\left(-\frac{1}{9}\right)+1 \quad \text{Distributive property (Section R.2)}$$

Be careful here.

$$f(x)=-3\left(x+\frac{1}{3}\right)^2+\frac{4}{3} \quad \text{Factor and simplify.}$$

The vertex is $\left(-\frac{1}{3},\frac{4}{3}\right)$. The intercepts are good additional points to find. The y-intercept is found by evaluating $f(0)$.

The x-intercepts are found by setting $f(x)$ equal to 0 and solving for x.

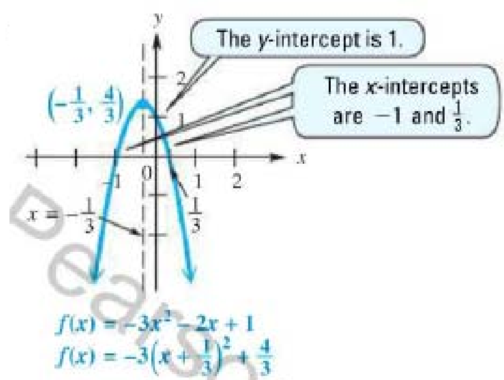
$0=-3x^2-2x+1$ Set $f(x)=0$.

$0=3x^2+2x-1$ Multiply by -1.

$0=(3x-1)(x+1)$ Factor.

$x=\frac{1}{3}$ or $x=-1$ Zero-factor property (Section 1.4)

Therefore, the x-intercepts are $\frac{1}{3}$ and -1.



Q4. (5 points) Find the quadratic function of x whose graph has a minimum at $(2,1)$ and passes through $(0,4)$

Solution: Vertex $v=(h,k)=(2,1)$

$f(x)=a(x-h)^2+k$

$$y=a(x-2)^2+1 \overset{(0,4)\text{ is a point}}{\Rightarrow} 4=a(0-2)^2+1 \Rightarrow 3=4a \Rightarrow \boxed{a=\frac{3}{4}}$$

$$f(x)=\frac{3}{4}(x-2)^2+1=\frac{3}{4}\left(x^2-4x+4\right)+1=3x^2-3x+4$$

Another Method:

Let $f(x)=ax^2+bx+c.$ We know

$f(2)=a(2)^2+b(2)+c=1 \qquad (1)$

$f(0)=a(0)^2+b(0)+c=4$

This implies $c=4$ and from Equation (1) we have

$4a+2b+4=1$ or $4a+2b=-2 \qquad (2)$

The x-value of the vertex is 2, and by the vertex formula we have $2=\frac{-b}{2a}$, which implies $b=-4a$.

Substituting $-4a$ for b in Equation (2) gives us

$$4a+2(-4a)=-3$$
$$4a-8a=-3$$
$$-4a=-3$$
$$a=\frac{3}{4}$$

Substituting $\frac{3}{4}$ for a in Equation (2) gives us

$$4\left(\frac{3}{4}\right)+2b=-3$$
$$3+2b=-3$$
$$2b=-6$$
$$b=-3$$

Thus the desired quadratic function is

$$f(x)=\frac{3}{4}x^2-3x+4.$$