King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001- Class Test II
Textbook Sections: 1.1 to 2.5
Term 131
November 24, 2013
Instructor: Sayed Omar

Student's Name: ....**KEY**..
ID #:........................... **Section:** .... **Serial Number:** ....

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Pagers, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	3	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	3	
9	4	
10	4	
11	4	
12	4	
13	4	
Total	**50**	

Q1. **(1.1 Example 2) (3 points):** Solve $\frac{2t+4}{3}+\frac{1}{2}t=\frac{1}{4}t-\frac{7}{3}$

Solution:

$$\frac{2t+4}{3}+\frac{1}{2}t=\frac{1}{4}t-\frac{7}{3}$$

Multiplying both sides by 12: $(12)\frac{2t+4}{3}+(12)\frac{1}{2}t=(12)\frac{1}{4}t-(12)\frac{7}{3}$

$$(4)(2t+4)+6t=3t-(4)7$$
$$8t+16+6t=3t-28$$
$$14t-3t=-16-28$$
$$11t=-44$$
$$t=-4$$
$$SS=\{-4\}$$

Q2. **(4 points):** (Recitation 1.1 Q#2)

Solve the following equations for the indicated variable:

(a) $z=y\left(1+\frac{m}{x}\right)$ for x

(b) $y=\frac{a+x}{3-ax}$ for x

Solution:

a)

$$z=y\left(1+\frac{m}{x}\right)$$
$$z=y+\frac{ym}{x}$$
$$\frac{ym}{x}=z-y$$
$$\frac{x}{ym}=\frac{1}{z-y}$$
$$x=\frac{ym}{z-y}$$

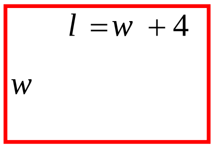
b)

$$\frac{y}{1}=\frac{a+x}{3-ax}$$
$$a+x=3y-xay$$
$$xay+x=3y-a$$
$$x(ay+1)=3y-a$$
$$x=\frac{3y-a}{ay+1}$$

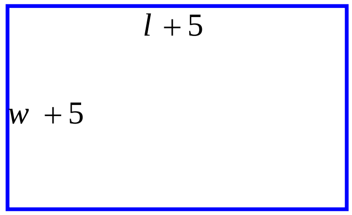
Q3. (4 points):

The length of a rectangle is 4 centimeters more than the width. If the length and width are increased by 5 cm, the perimeter of the new rectangle will be 2 cm less than 7 times the width of the original rectangle. Find the dimensions of the original rectangle.

Solution:



Original rectangle



Side is increased by 5 cm

The perimeter of the new triangle is

$$P_{new} = 7w - 2$$
$$2(l+5) + 2(w+5) = 7w - 2$$
$$2(w+4+5) + 2(w+5) = 7w - 2$$
$$2w + 18 + 2w + 10 = 7w - 2$$
$$30 = 3w$$
$$w = 10 \text{ cm}$$
$$l = 10 + 4 = 14 \text{ cm}$$

State the answer: The width and length of the original rectangle are 10 cm and 14 cm. The dimension of the rectangle is 10 cm by 14 cm.

Q4. (4 points): If $z = \dfrac{3i}{4-i} + i^{23}$ find the conjugate of z.

Solution:

$$z = \frac{3i}{4-i} + i^{23} = \frac{3i}{4-i} \cdot \frac{4+i}{4+i} + i^{22} \cdot i = \frac{12i + 3i^2}{4^2 - i^2} + \left(i^2\right)^{11} \cdot i = \frac{-3+12i}{17} - i$$

$$= \frac{-3+12i-17i}{17} = -\frac{3}{17} - \frac{5}{17}i$$

$$\bar{z} = -\frac{3}{17} + \frac{5}{17}i$$

Answer: $\bar{z} = -\dfrac{3}{17} + \dfrac{5}{17}i$

Q5. (1.4 Recitation Question 3) (4 points):

If the equation $2x^2 - \dfrac{5}{2}x = 3 - x$ is written by completing the square as $(x-a)^2 = b$ find a and b.

Solution:

$$4x^2 - 5x = 6 - 2x$$
$$4x^2 - 3x = 6$$

$$x^2 - \frac{3}{4}x = \frac{3}{2}$$

$$x^2 - \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{3}{2} + \frac{9}{64}$$

$$\left(x - \frac{3}{8}\right)^2 = \frac{105}{64} \Rightarrow \boxed{a = \frac{3}{8}}, \boxed{b = \frac{105}{64}}$$

Q6. (4 points): (1.6 Exercise 16): Solve $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2+x}$.

Solution:

16. $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x^2+x}$ or $\frac{4x+3}{x+1} + \frac{2}{x} = \frac{1}{x(x+1)}$

Multiply each term in the equation by the least common denominator, $x(x+1)$, assuming $x \neq 0, -1$.

$$x(x+1)\left[\frac{4x+3}{x+1} + \frac{2}{x}\right] = x(x+1)\left(\frac{1}{x(x+1)}\right)$$

$$x(4x+3) + 2(x+1) = 1$$

$$4x^2 + 3x + 2x + 2 = 1$$

$$4x^2 + 5x + 2 = 1 \Rightarrow 4x^2 + 5x + 1 = 0$$

$$(4x+1)(x+1) = 0$$

$4x + 1 = 0 \Rightarrow x = -\frac{1}{4}$ or $x + 1 = 0 \Rightarrow x = -1$

Because of the restriction $x \neq -1$, the only valid solution is $-\frac{1}{4}$. The solution set is $\left\{-\frac{1}{4}\right\}$.

Q7. (4 points): Solve $(-2x-1)(-3x-1)(2x+2) \leq 0$

Solution:

	$-\infty$		-1		$-\frac{1}{2}$		$-\frac{1}{3}$	$+\infty$
$-2x-1$		+	+	+	**0**	−		−
$-3x-1$		+	+	+	+	+	**0**	−
$2x+2$		−	0	+	+	+	+	+
$(-2x-1)(-3x-1)(2x+2)$		−	0	+	0	−	**0**	+

$$SS = (-\infty, -1] \cup \left[-\frac{1}{2}, -\frac{1}{3}\right]$$

Q8. (3 points): Solve each of the following equation. $|2x-3|=5x+4$

Solution:

$$|2x-3|=5x+4$$

$$2x-3=5x+4 \quad \text{or} \quad 2x-3=-(5x+4)$$

$$-7=3x \quad \text{or} \quad 2x-3=-5x-4$$

$$7x=-1$$

$$x=-\frac{7}{3} \quad \text{or} \quad x=-\frac{1}{7}$$

Check: $x=-\frac{7}{3}$, $|2x-3|=5x+4$

$$\left|\frac{-14}{3}-3\right|=\frac{-35}{3}+4$$

$$\frac{23}{3}=-\frac{23}{3}$$

$x=-\frac{7}{3}$ is rejected

Check: $x=-\frac{1}{7}$, $|2x-3|=5x+4$

$$\left|\frac{-2}{7}-3\right|=\frac{-5}{7}+4$$

$$\frac{23}{7}=\frac{23}{7}$$

$x=-\frac{1}{7}$ is accepted.

$$SS=\left\{-\frac{1}{7}\right\}$$

Q9. (4 points):

If A is the solution set of the inequality $|2x-9|-7<0$ and B is the solution set of the inequality $|x-10|-4\geq 0$, then $A\cap B=?$

Solution:

$$|2x-9|<7$$

$$-7<2x-9<7$$

$$2<2x<16$$

$$1<x<8$$

$$|x-10|\geq 4$$

$$x-10\leq -4 \quad \text{or} \quad x-10\geq 4$$

$$x\leq 6 \quad \text{or} \quad x\geq 14$$

$$x\in B=(-\infty,6]\cup[14,\infty)$$

A :

B :

$$A\cap B=(1,6]$$

Q10. (4 points): **If the distance between the points $A(x+4, 2x)$ and $B(x, -1)$ is 5, then find the value of $3x-1$ when $x>0$**

Solution:

$$5=\sqrt{[x-(x+4)]^2+(-1-2x)^2}$$

$$5=\sqrt{16+1+4x+4x^2}$$

$$25=17+4x+4x^2$$

$$4x^2+4x-8=0$$

$$x^2+x-2=0$$

$$(x+2)(x-1)=0$$

$$x=-2\ ,\ \boxed{x=1}$$

$$3x-1=3(1)-1=2$$

Q11. (4 points) (2.2 Example 4): Show that $2x^2+2y^2-6x+10y=1$ has a circle as its graph. Find the center and radius.

SOLUTION To complete the square, the coefficients of the x^2- and y^2-terms must be 1. In this case they are both 2, so begin by dividing each side by 2.

$$2x^2+2y^2-6x+10y=1$$

$$x^2+y^2-3x+5y=\frac{1}{2} \qquad \text{Divide by 2.}$$

$$(x^2-3x\qquad)+(y^2+5y\qquad)=\frac{1}{2} \qquad \text{Rearrange and regroup terms in anticipation of completing the square.}$$

$$\left(x^2-3x+\frac{9}{4}\right)+\left(y^2+5y+\frac{25}{4}\right)=\frac{1}{2}+\frac{9}{4}+\frac{25}{4} \qquad \text{Complete the square for } \textit{both } x \text{ and } y;\ \left[\tfrac{1}{2}(-3)\right]^2=\tfrac{9}{4} \text{ and } \left[\tfrac{1}{2}(5)\right]^2=\tfrac{25}{4}$$

$$\left(x-\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=9 \qquad \text{Factor and add.}$$

$$\left(x-\frac{3}{2}\right)^2+\left(y-\left(-\frac{5}{2}\right)\right)^2=3^2 \qquad \text{Center-radius form}$$

The equation has a circle with center at $\left(\frac{3}{2}, -\frac{5}{2}\right)$ and radius 3 as its graph.

Q12. (4 points): Sketch the graph and determine the domain and the range of the following functions.

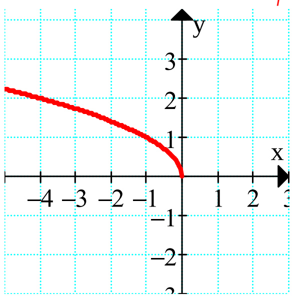
(a): $f(x)=\sqrt{-x}$

(b): $g(x)=|-x+3|$

(c): $h(x)=2-\sqrt{4-(x-3)^2}$

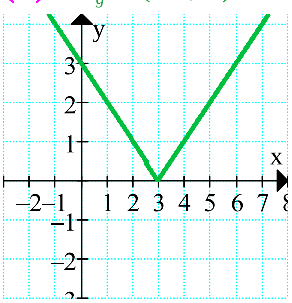
Solution:

(a): $-x\geq 0 \Rightarrow x\leq 0 \Rightarrow D_f=(-\infty,0]$



$R_f=[0,\infty)$. The function is decreasing on $(-\infty,0]$.

(b): $D_g=(-\infty,\infty)$



$R_g=[0,\infty)$. The function g is decreasing on $(-\infty,3]$ and increasing on $[3,\infty)$.

(c): $h(x)=2-\sqrt{4-(x-3)^2}$

$$y=2-\sqrt{4-(x-3)^2}$$

$$y-2=-\sqrt{4-(x-3)^2}\ ,\ y-2\leq 0$$

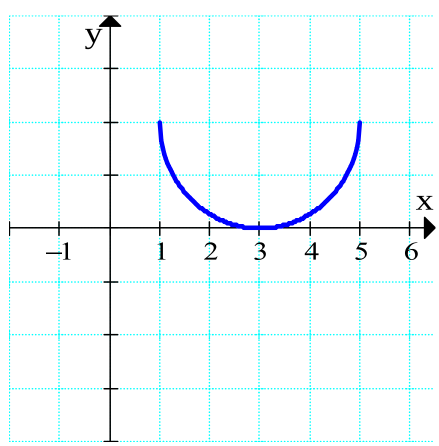
$$(y-2)^2=4-(x-3)^2\ ,\ y\leq 2$$

$$(y-2)^2+(x-3)^2=4$$

$D_h=[1,5]$, $R_h=[0,2]$

The function h is decreasing on $[1,3]$.

The function h is increasing on $[3,5]$.



Q13. **(4 points) (2.4 and 2.5 Recitation Q#2)**

(a): Find the equation of a line with x-intercept $\frac{4}{5}$ and perpendicular to the line $2y = -\frac{2}{3}x + 3$

(b): Find the x-intercept and y-intercept of the line passing through the points $(-2, 2)$ and $(1, -3)$

Solution:

(a): $2y = -\frac{2}{3}x + 3 \Rightarrow y = -\frac{1}{3}x + \frac{2}{3} \Rightarrow m_1 = -\frac{1}{3} \Rightarrow \boxed{m_2 = 3}$, $\left(\frac{4}{5}, 0\right)$

$$y - y_1 = m_2(x - x_1)$$

$$y - 0 = 3\left(x - \frac{4}{5}\right)$$

$$y = 3x - \frac{12}{5}$$

(b): $m = \frac{-3-2}{1+2} = \frac{-5}{3}$, $(-2, 2)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-5}{3}(x + 2)$$

$$3y - 6 = -5(x + 2)$$

$$3y - 6 = -5x - 10$$

$$5x + 3y = -4$$

To find x – intercept, put $y = 0$ and solve for x : $\boxed{x = -\frac{4}{5}}$ or $\left(-\frac{4}{5}, 0\right)$

To find y – intercept, put $x = 0$ and solve for y : $\boxed{y = -\frac{4}{3}}$ or $\left(0, -\frac{4}{3}\right)$