Serial # ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Q1. (5 points)(4.3 Exercise 54): Graph the function $f(x) = \log_{1/3}(3-x)$ **and** give the domain and the range.

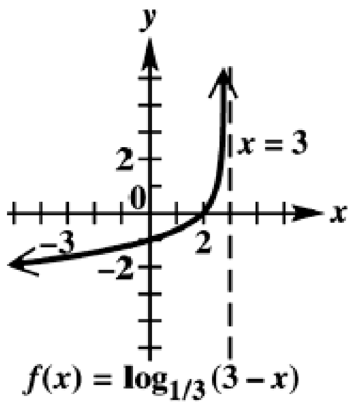
Solution:

54. $f(x) = \log_{1/3}(3-x)$

Since $f(x) = y = \log_{1/3}(3-x)$, we can write the exponential form as

$3 - x = \left(\frac{1}{3}\right)^y \Rightarrow x = 3 - \left(\frac{1}{3}\right)^y$ to find ordered pairs that satisfy the equation. It is easier to choose values for y and find the corresponding values of x. Make a table of values.

x	$y = \log_{1/3}(3-x)$
-6	-2
0	-1
2	0
$\frac{8}{3} \approx 2.7$	1
$\frac{26}{9} \approx 2.9$	2



The graph has the line $x = 3$ as a vertical asymptote.

$D_f = (-\infty, 3)$ and $R_f = (-\infty, \infty)$

Q2. (5 points) (4.3 Recitation Q2): Expand the logarithm: $\log_2\left(\sqrt[3]{\frac{8x \cdot \sqrt{z}}{y^2+4}}\right)$

Solution: Assume all variables represent real numbers.

$$\log_2\left(\sqrt[3]{\frac{8x \cdot \sqrt{z}}{y^2+4}}\right) = \frac{1}{3}\left[\log_2 8 + \log_2 x + \log_2 z^{1/2} - \log_2\left(y^2+4\right)\right]$$

$$= \frac{1}{3}\left[\log_2 2^3 + \log_2 x + \frac{1}{2}\log_2 z - \log_2\left(y^2+4\right)\right]$$

$$= \frac{1}{3}\left[3 + \log_2 x + \frac{1}{2}\log_2 z - \log_2\left(y^2+4\right)\right]$$

$$= 1 + \frac{1}{3}\log_2 x + \frac{1}{6}\log_2 z - \frac{1}{3}\log_2\left(y^2+4\right)$$

Q3. (4 points) (4.3 Exercise 96): Evaluate each expression:

(a): $100^{\log 3}$ **(b):** $\log(0.01)^3$ **(c):** $\log(0.0001)^5$ **(d):** $1000^{\log 5}$

Solution:

96. **(a)** $100^{\log 3} = 10^{2\log 3} = 10^{\log 3^2} = 10^{\log 9} = 9$

(b) $\log_{10} 0.01^3 = \log_{10}\left(10^{-2}\right)^3$
$= \log_{10} 10^{-6} = -6$

(c) $\log_{10} 0.0001^5 = \log_{10}\left(10^{-4}\right)^5$
$= \log_{10} 10^{-20} = -20$

(d) $1000^{\log_{10} 5} = 10^{3\log_{10} 5} = 10^{\log_{10} 5^3}$
$= 10^{\log_{10} 125} = 125$

Q4. (6 points)(Additional Exercise 13): Assume all variables represent positive real numbers, write the logarithmic expression: $2 - \log_3 x^2 - 8\log_9 y + \log_{\sqrt{3}} xy$ as a single logarithm with a base of 3

Solution:

$$\begin{aligned}
2 - \log_3 x^2 - 8\log_9 y + \log_{\sqrt{3}} xy &= \log_3 3^2 - \log_3 x^2 - 8\frac{\log_3 y}{\log_3 9} + \frac{\log_3 xy}{\log_3 \sqrt{3}} \\
&= \log_3 9 - \log_3 x^2 - 8\frac{\log_3 y}{2} + \frac{\log_3 xy}{\frac{1}{2}} \\
&= \log_3 9 - \log_3 x^2 - 4\log_3 y + 2\log_3 xy \\
&= \log_3 9 - \log_3 x^2 - \log_3 y^4 + \log_3 (xy)^2 \\
&= \log_3 9 + \log_3 (xy)^2 - \left(\log_3 x^2 + \log_3 y^4\right) \\
&= \log_3 9(xy)^2 - \log_3 x^2 y^4 \\
&= \log_3 \frac{9(xy)^2}{x^2 y^4} \\
&= \log_3 \frac{9}{y^2}
\end{aligned}$$