King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test II
Textbook Sections: 6.3 to 8.3
Term 132
Time Allowed: 90 Minutes

Time: 8:30 pm – 10:00 pm

Student's Name: …………… ……………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

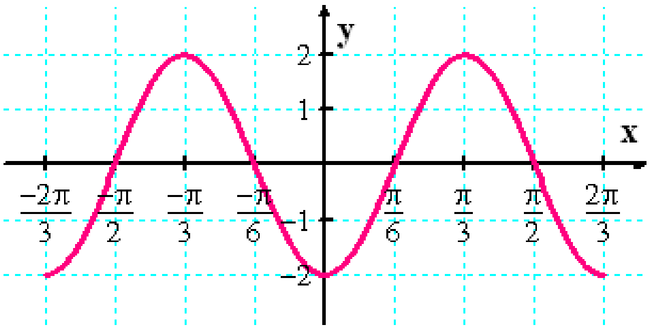
Question	**Points**	Student's Score
1	4	
2	4	
3	4	
4	5	
5	4	
6	5	
7	4	
8	4	
9	4	
10	4	
11	6	
12	6	
13	6	
Total	**60**	____ 60
		____ 100

Q1. (4 points): (6.3Textbook Exercise 31): Graph the function $y = -2\cos 3x$ over **two** periods.

Solution:

$0 \le 3x \le 2\pi$

$0 \le x \le \dfrac{2\pi}{3}$

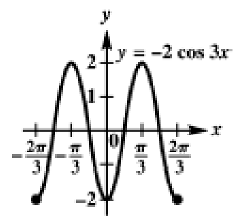


31. $y = -2\cos 3x$

Period: $\frac{2\pi}{3}$ and amplitude: $|-2| = 2$

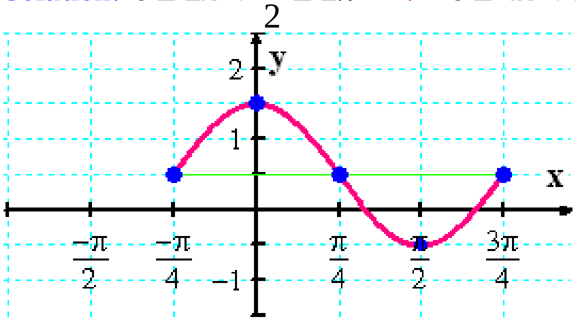
Divide the interval $\left[0, \frac{2\pi}{3}\right]$ into four equal parts to get the x-values that will yield minimum and maximum points and x-intercepts. Then make a table. Repeat this cycle for the interval $\left[-\frac{2\pi}{3}, 0\right]$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$3x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 3x$	1	0	-1	0	1
$-2\cos 3x$	-2	0	2	0	-2



Q2. (4 points): (6.4Textbook Exercise 31): Graph $y = \dfrac{1}{2} + \sin\left(2x + \dfrac{\pi}{2}\right)$ over one period

Solution: $0 \le 2x + \dfrac{\pi}{2} \le 2\pi \Rightarrow 0 \le 4x + \pi \le 4\pi \Rightarrow -\pi \le 4x \le 3\pi \Rightarrow -\dfrac{\pi}{4} \le x \le \dfrac{3\pi}{4}$



55. $y = \frac{1}{2} + \sin\left[2\left(x + \frac{\pi}{4}\right)\right]$

Step 1: Find the interval whose length is $\frac{2\pi}{b}$.

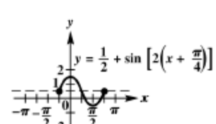
$0 \le 2\left(x + \frac{\pi}{4}\right) \le 2\pi \Rightarrow 0 \le x + \frac{\pi}{4} \le \frac{2\pi}{2} \Rightarrow$

$0 \le x + \frac{\pi}{4} \le \pi \Rightarrow -\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

Step 2: Divide the period into four equal parts to get the following x-values: $-\frac{\pi}{4}$, 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$

Step 3: Evaluate the function for each of the five x-values.

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$2\left(x + \frac{\pi}{4}\right)$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\left[2\left(x + \frac{\pi}{4}\right)\right]$	0	1	0	-1	0
$\frac{1}{2} + \sin\left[2\left(x + \frac{\pi}{4}\right)\right]$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$



The amplitude is $|1|$, which is 1. The period is $\frac{2\pi}{2}$, which is π. The vertical translation is $\frac{1}{2}$ unit up. The phase shift is $\frac{\pi}{4}$ units to the left.

Q3. (4 points): Find the range of $y = 1 - 3\sec\left(\frac{\pi}{2}x - 1\right)$

Solution: $y = d + a\sec(bx + c) \Rightarrow Range = (-\infty, -|a| + d] \cup [|a| + d, \infty)$

$Range = (-\infty, -|-3| + 1] \cup [|-3| + 1, \infty)$

$Range = (-\infty, -2] \cup [4, \infty)$

Q4. (5 points): Given $y = 3\tan\left(2x + \frac{\pi}{2}\right)$, where $-\frac{5\pi}{4} \le x \le \frac{3\pi}{4}$

Answer **True** or **False**.

(a) The graph is decreasing on $\left(0, \frac{\pi}{2}\right)$.

(b) The graph has three vertical asymptotes

(c) The graph has one y-intercept.

(d) The graph has five vertical asymptotes.

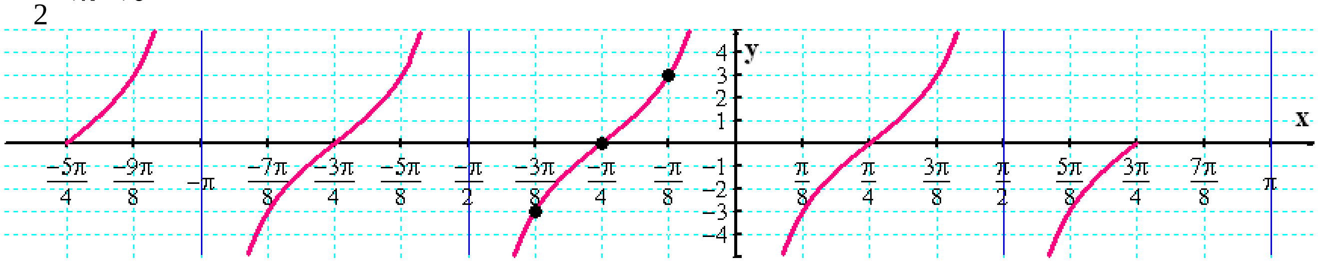
(e) The graph has 5 x-intercepts.

Solution:

$-\frac{\pi}{2} < 2x + \frac{\pi}{2} < \frac{\pi}{2}$

$-\pi < 2x < 0$

$-\frac{\pi}{2} < x < 0$



(a): The graph is decreasing on $\left(0, \frac{\pi}{2}\right)$. **FALSE**.

(b): The graph has three vertical asymptotes. **FALSE**.

(c): The graph has one y-intercept. . **FALSE**.

(d): The graph has five vertical asymptotes. . **FALSE**.

(e): The graph has 5 x-intercepts. **TRUE**.

Q5. (4 points):(Recitation 6.5and6.6 Q#2) If $x = a$, $x = b$ and $x = c$ are the vertical asymptotes of $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right)$, in the interval $[0, 3\pi]$ then $a + b + c = ?$

Solution: $y = 1 - \frac{1}{2}\csc\left(x - \frac{3\pi}{4}\right) = 1 - \frac{1}{2}\frac{1}{\sin\left(x - \frac{3\pi}{4}\right)}$

Vertical asymptotes are at the values of x when: $\sin\left(x - \frac{3\pi}{4}\right) = 0$

$\Rightarrow \quad x - \frac{3\pi}{4} = n\pi$ OR $x = \frac{3\pi}{4} + n\pi = \frac{3 + 4n}{4}\pi$

$x = \frac{3+4n}{4}\pi$, where n is an integer.

$n = 0 \Rightarrow \boxed{x = \frac{3\pi}{4}}$

$n = 1 \Rightarrow \boxed{x = \frac{7\pi}{4}}$

$n = 2 \Rightarrow \boxed{x = \frac{11\pi}{4}}$

$$a + b + c = \frac{3\pi}{4} + \frac{7\pi}{4} + \frac{11\pi}{4} = \frac{21\pi}{4}$$

Q6. (5 points): (7.1 Exercise 34): Given $\csc\theta = -\frac{5}{2}$, θ is in quadrant III. Find the remaining five trigonometric functions of θ.

Solution:

34. $\csc\theta = -\frac{5}{2}, \theta$ in quadrant III

Since θ is in quadrant III, the tangent, and cotangent function values are positive. The sine, cosine, cosecant, and secant function values are negative.

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{5}{2}} = -\frac{2}{5}$$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(-\frac{2}{5}\right)^2 = 1 - \frac{4}{25} = \frac{21}{25} \Rightarrow$$

$$\cos\theta = -\frac{\sqrt{21}}{5}, \text{ since } \cos\theta < 0$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2}{5}}{-\frac{\sqrt{21}}{5}} = \frac{2}{\sqrt{21}}$$

$$= \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{2}{\sqrt{21}}} = \frac{\sqrt{21}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{\sqrt{21}}{5}} = -\frac{5}{\sqrt{21}}$$

$$= -\frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

Q7. (4 points): (7.2 Recitation 6.5and6.6 Q#1): Simplify the following expression

$$\frac{(\sec\theta - \tan\theta)^2 + 1}{\sec\theta\csc\theta - \tan\theta\csc\theta} = ?$$

Solution:

$$\frac{(\sec\theta-\tan\theta)^2+1}{\sec\theta\csc\theta-\tan\theta\csc\theta}=\frac{\sec^2\theta-2\sec\theta\tan\theta+\tan^2\theta+1}{\csc\theta(\sec\theta-\tan\theta)}=\frac{\sec^2\theta-2\sec\theta\tan\theta+\sec^2\theta}{\csc\theta(\sec\theta-\tan\theta)}$$

$$=\frac{2\sec^2\theta-2\sec\theta\tan\theta}{\csc\theta(\sec\theta-\tan\theta)}=\frac{2\sec\theta(\sec\theta-\tan\theta)}{\csc\theta(\sec\theta-\tan\theta)}=2\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}=2\frac{\sin\theta}{\cos\theta}=2\tan\theta$$

Q8. (4 points): (7.3 Recitation 6.5and6.6 Q#1): $\dfrac{1-\tan\frac{13\pi}{9}\tan\frac{2\pi}{9}}{\tan\frac{13\pi}{9}+\tan\frac{2\pi}{9}}=?$

Solution:

$$\frac{\tan\frac{13\pi}{9}+\tan\frac{2\pi}{9}}{1-\tan\frac{13\pi}{9}\tan\frac{2\pi}{9}}=\frac{1}{\tan\left(\frac{13\pi}{9}+\frac{2\pi}{9}\right)}=\frac{1}{\tan\frac{15\pi}{9}}=\frac{1}{\tan\frac{5\pi}{3}}=\frac{1}{-\tan\frac{\pi}{3}}=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

Q9. (4 points): (7.4 Exercise 63): Find $\tan\frac{\theta}{2}$, given $\tan\theta=\frac{\sqrt{7}}{3}$, with $180°<\theta<270°$.

Solution:

63. Find $\tan\frac{\theta}{2}$, given $\tan\theta=\frac{\sqrt{7}}{3}$, with $180°<\theta<270°$.

$\sec^2\theta=\tan^2\theta+1\Rightarrow$

$\sec^2\theta=\left(\frac{\sqrt{7}}{3}\right)^2+1=\frac{7}{9}+1=\frac{16}{9}$

Since θ is in quadrant III, $\sec\theta<0$ and $\sin\theta<0$.

$\sec\theta=-\sqrt{\frac{16}{9}}=-\frac{4}{3}$ and

$\cos\theta=\frac{1}{\sec\theta}=\frac{1}{-\frac{4}{3}}=-\frac{3}{4}$

$\sin\theta=-\sqrt{1-\cos^2\theta}$

$=-\sqrt{1-\left(-\frac{3}{4}\right)^2}=-\sqrt{1-\frac{9}{16}}=-\frac{\sqrt{7}}{4}$

$\tan\frac{\theta}{2}=\frac{\sin\theta}{1+\cos\theta}=\frac{-\frac{\sqrt{7}}{4}}{1+\left(-\frac{3}{4}\right)}=\frac{-\sqrt{7}}{4-3}=-\sqrt{7}$

Q10. (4 points): (7.5 Exercise 13-36): Find the exact value of each real number y if it exists.

(a): $y=\arctan\left(-\sqrt{3}\right)$

(b): $y=\csc^{-1}(-\sqrt{2})$

(c): $y=\sec^{-1}(-\sqrt{2})$

(d): $y=\cot^{-1}(-1)$

Solution:

(a): $y=\arctan\left(-\sqrt{3}\right)$

Check that $-\sqrt{3} \in D_{\text{arctan}} = (-\infty, \infty)$ and $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) =$ range of arctan

$y = \arctan\left(-\sqrt{3}\right) \Rightarrow \tan y = -\sqrt{3} \Rightarrow \boxed{y = -\frac{\pi}{3}}$

(b): $y = \csc^{-1}(-\sqrt{2})$

Check that $-\sqrt{2} \in D_{\csc^{-1}} = (-\infty, -1] \cup [1, \infty)$ and $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right] =$ range of arccsc

$\csc y = -\sqrt{2} \Rightarrow \boxed{y = -\frac{\pi}{4}}$ because we know that $\csc\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(c): $y = \sec^{-1}(-\sqrt{2})$

Check that $-\sqrt{2} \in D_{\sec^{-1}} = (-\infty, -1] \cup [1, \infty)$ and $y \in \left[o, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

$\Rightarrow \sec y = -\sqrt{2} \Rightarrow \boxed{y = \frac{3\pi}{4}}$ because $\sec(135°) = -\sqrt{2}$ and

$\frac{3\pi}{4} = y \in \left[o, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

(d): $y = \cot^{-1}(-1)$

Check that $-1 \in (-\infty, \infty) =$ domain of $\cot^{-1}$ and $y \in (0, \pi) =$ Range of $\cot^{-1}$

$y = \cot^{-1}(-1) \Rightarrow \cot y = \cot\left(\cot^{-1}(-1)\right) = -1$

$\Rightarrow \cot y = -1 \Rightarrow \boxed{y = \frac{3\pi}{4}}$ because we know that $\cot\left(\frac{3\pi}{4}\right) = -1$ and

$y = \frac{3\pi}{4} \in R_{\cot^{-1}} =$ range of $\cot^{-1} = (0, \pi)$

Q11. (6 points): (7.6 Exercise 81): Solve $\cos 2x + \cos x = 0$ over the interval $[0, 2\pi)$.

Solution:

81. $\cos 2x + \cos x = 0$

We choose an identity for $\cos 2x$ that involves only the cosine function.

$$\cos 2x + \cos x = 0$$
$$\left(2\cos^2 x - 1\right) + \cos x = 0$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow$$
$$\cos x = \frac{1}{2} \text{ or } \cos x = -1$$

$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\cos x = -1 \Rightarrow x = \pi$

Solution set: $\left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

Q12. (6 points): Solve the equation $\cos^{-1}\frac{x}{2}+\sin^{-1}\left(\frac{-3}{5}\right)-\frac{\pi}{3}=0$

Solution

$$\cos^{-1}\frac{x}{2}=\frac{\pi}{3}-\sin^{-1}\left(\frac{-3}{5}\right)$$

$$\cos\left(\cos^{-1}\frac{x}{2}\right)=\cos\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-3}{5}\right)\right)$$

$$\frac{x}{2}=\cos\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-3}{5}\right)\right)$$

Let $\theta=\sin^{-1}\left(\frac{-3}{5}\right)$ where $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$.

$\sin\theta=\frac{-3}{5},\ -\frac{\pi}{2}\le\theta\le\frac{\pi}{2} \Rightarrow$ θ is in Quadrant IV

$$y=-3,\ r=5,\ x=+\sqrt{r^2-y^2}=\sqrt{25-9}=\sqrt{16}=4$$

$$\frac{x}{2}=\cos\left(\frac{\pi}{3}-\theta\right)$$

$$\frac{x}{2}=\cos\frac{\pi}{3}\cos\theta+\sin\frac{\pi}{3}\sin\theta$$

$$\frac{x}{2}=\frac{1}{2}\left(\frac{4}{5}\right)+\frac{\sqrt{3}}{2}\left(\frac{-3}{5}\right)$$

$$x=\frac{4-3\sqrt{3}}{5}$$

$$SS=\left\{\frac{4-3\sqrt{3}}{5}\right\}$$

Q13. (6 points): Given the vectors $\vec{u}=\left\langle\frac{2\sqrt{3}}{3},\frac{2}{3}\right\rangle$ and $\vec{v}=\left\langle-\frac{1}{5},\frac{\sqrt{3}}{5}\right\rangle$

(a): Find a unit vector in the direction of $\vec{u}$.

(b): Find $\vec{u}\cdot\vec{v}$

(c): Find the angle between $\vec{u}$ and $\vec{v}$.

Solution

$$\|\vec{u}\|=\left\|\left\langle\frac{2\sqrt{3}}{3},\frac{2}{3}\right\rangle\right\|=\left|\frac{2}{3}\right|\left\|\left\langle\sqrt{3},1\right\rangle\right\|=\frac{2}{3}\sqrt{3+1}=\frac{4}{3}$$

(a): A unit vector in the direction of $\vec{u}$ is equal to $\frac{1}{\|\vec{u}\|}\vec{u}=\frac{3}{4}\left\langle\frac{2\sqrt{3}}{3},\frac{2}{3}\right\rangle=\left\langle\frac{\sqrt{3}}{2},\frac{1}{2}\right\rangle$

(b): $\vec{u}\cdot\vec{v}=\left\langle\frac{2\sqrt{3}}{3},\frac{2}{3}\right\rangle\cdot\left\langle-\frac{1}{5},\frac{\sqrt{3}}{5}\right\rangle=-\frac{2\sqrt{3}}{15}+\frac{2\sqrt{3}}{15}=0$

(c): $\cos\alpha=\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\cdot\|\vec{v}\|}=0 \Rightarrow \alpha=\frac{\pi}{2}$