King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test I
Textbook Sections: 4.1 to 6.2
Term 132
Time Allowed: 90 Minutes

Time: 6:00 pm – 7:30 pm

Student's Name: …………… ………………………………………………
ID #:……………………….   **Section:** …….   **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	6	
8	4	
9	4	
10	4	
11	4	
12	4	
Total	**50**	____ / 50
		____ / 100

Q1. (4 points): (4.1Textbook Exercise 74): Find the inverse of $f(x)=\dfrac{-3x+12}{x-6}, \quad x \neq 6$

Solution:

Step 1: Replace $f(x)$ with y and interchange x and y.

$$y=\frac{-3x+12}{x-6} \Rightarrow x=\frac{-3y+12}{y-6}$$

Step 2: Solve for y.

$$x=\frac{-3y+12}{y-6} \Rightarrow x(y-6)=-3y+12 \Rightarrow$$

$$xy-6x=-3y+12 \Rightarrow xy+3y=6x+12 \Rightarrow$$

$$y(x+3)=6x+12 \Rightarrow y=\frac{6x+12}{x+3}$$

Step 3: Replace y with $f^{-1}(x)$.

$$f^{-1}(x)=\frac{6x+12}{x+3},\ x \neq -3.$$

Q2. (4 points): (4.2 Textbook Example 6): Solve $x^{4/3}=81$.

Solution:

EXAMPLE 6 Solving an Equation with a Fractional Exponent

Solve $x^{4/3}=81$.

SOLUTION Notice that the variable is in the base rather than in the exponent.

$$x^{4/3}=81$$

$$(\sqrt[3]{x})^4=81 \quad \text{Radical notation for } a^{m/n} \text{ (Section R.7)}$$

$$\sqrt[3]{x}=\pm 3 \quad \text{Take fourth roots on each side. Remember to use } \pm. \text{ (Section 1.6)}$$

$$x=\pm 27 \quad \text{Cube each side.}$$

Check *both* solutions in the original equation. Both check, so the solution set is $\{\pm 27\}$.

Alternative Method There may be more than one way to solve an exponential equation, as shown here.

$$x^{4/3}=81$$

$$(x^{4/3})^3=81^3 \quad \text{Cube each side.}$$

$$x^4=(3^4)^3 \quad \text{Write 81 as } 3^4.$$

$$x^4=3^{12} \quad (a^m)^n=a^{mn}$$

$$x=\pm\sqrt[4]{3^{12}} \quad \text{Take fourth roots on each side.}$$

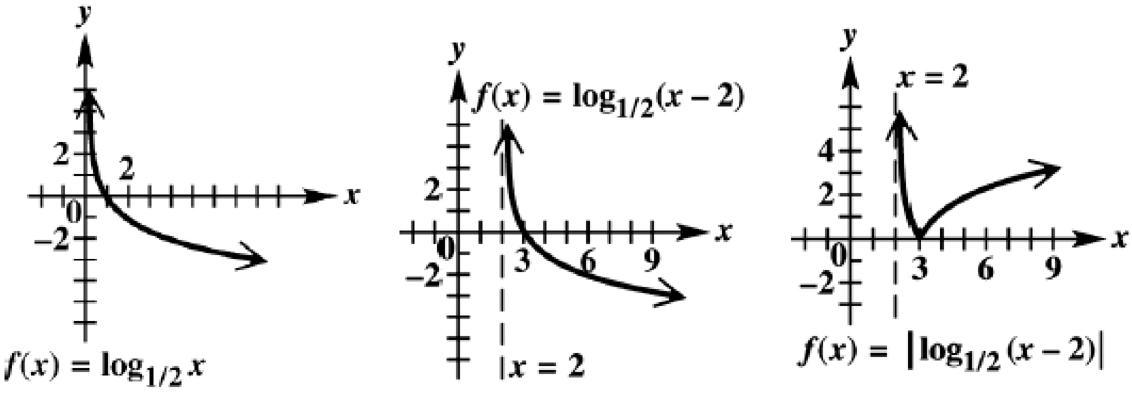
$$x=\pm 3^3 \quad \text{Simplify the radical.}$$

$$x=\pm 27 \quad \text{Apply the exponent.}$$

The same solution set, $\{\pm 27\}$, results.

Q3. (4 points): (4.3 Textbook Exercise 44): Graph $f(x) = \left|\log_{\frac{1}{2}}(x-2)\right|$.

Solution:



Q4. (4 points): (4.4 Recitation Question 2): If $\log x = a$, $\log y = b$ then write the expression $\log_x x^3\sqrt{10y}$ in terms of a and b

. **Solution:** Note that we must have: $x > 0$ and $x \neq 1$

$$\log_x x^3\sqrt{10y} = \frac{\log x^3\sqrt{10y}}{\log x} = \frac{\log x^3 + \log\sqrt{10y}}{a} = \frac{3a + \log(10y)^{1/2}}{a}$$

$$= \frac{3a + \frac{1}{2}\log(10y)}{a} = \frac{6a + \log(10y)}{2a} = \frac{6a + \log 10 + \log y}{2a}$$

$$= \frac{6a + 1 + b}{2a}$$

Answer: $\frac{6a+1+b}{2a}$

Q5. (4 points): (4.5 Textbook Example 2): Solve $3^{2x-1} = 0.4^{x+2}$

SOLUTION

$3^{2x-1} = 0.4^{x+2}$	
$\ln 3^{2x-1} = \ln 0.4^{x+2}$	Take the natural logarithm on each side.
$(2x-1)\ln 3 = (x+2)\ln 0.4$	Power property
$2x\ln 3 - \ln 3 = x\ln 0.4 + 2\ln 0.4$	Distributive property (Section R.2)
$2x\ln 3 - x\ln 0.4 = 2\ln 0.4 + \ln 3$	Write the terms with x on one side.
$x(2\ln 3 - \ln 0.4) = 2\ln 0.4 + \ln 3$	Factor out x. (Section R.4)
$x = \frac{2\ln 0.4 + \ln 3}{2\ln 3 - \ln 0.4}$	Divide by $2\ln 3 - \ln 0.4$.
$x = \frac{\ln 0.4^2 + \ln 3}{\ln 3^2 - \ln 0.4}$	Power property
$x = \frac{\ln 0.16 + \ln 3}{\ln 9 - \ln 0.4}$	Apply the exponents.
This is *exact.* $x = \frac{\ln 0.48}{\ln 22.5}$	Product and quotient properties (Section 4.3)

Q6. (4 points): (5.1 Textbook Example 4b): Convert $34.817°$ to degrees, minutes, and second.

Solution:

$34.817° = 34° + 0.817°$	Write as a sum.
$= 34° + 0.817(60')$	$1° = 60'$
$= 34° + 49.02'$	Multiply.
$= 34° + 49' + 0.02'$	Write as a sum.
$= 34° + 49' + 0.02(60'')$	$1' = 60''$
$= 34° + 49' + 1.2''$	Write as a sum.
$\approx 34° 49' 01''$	Approximate to the nearest second.

Q7. (6 points): (5.1 Textbook Example 5): Find the angle of least positive measure that are coterminal with each angle.

(a): 908° **(b):** –75° **(c):** –800°

Solution:

(a) Subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°. Since

$$908° - 2 \cdot 360° = 188°,$$

an angle of 188° is coterminal with an angle of 908°. See **Figure 11.**

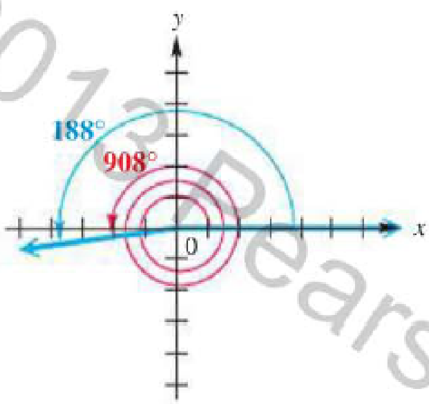


Figure 11

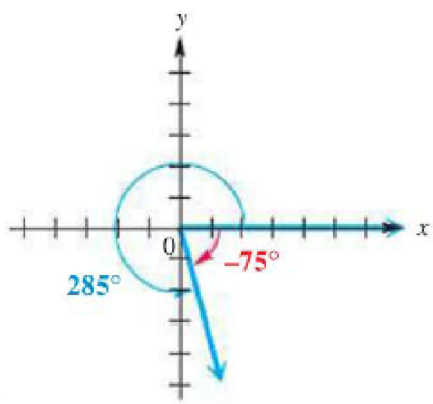


Figure 12

(b) See **Figure 12.** Use a rotation of

$$360° + (-75°) = 285°.$$

(c) The least integer multiple of 360° greater than 800° is

$$360° \cdot 3 = 1080°.$$

Add 1080° to −800° to obtain

$$1080° + (-800°) = 280°.$$

Q8. (4 points): (5.2 Textbook Example 10):

Find $\sin\theta$ and $\tan\theta$, given $\cos\theta = -\frac{\sqrt{3}}{4}$ and $\sin\theta > 0$.

Solution:

$$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1 \qquad \text{Replace } \cos\theta \text{ with } -\tfrac{\sqrt{3}}{4}.$$

$$\sin^2\theta + \frac{3}{16} = 1 \qquad \text{Square } -\tfrac{\sqrt{3}}{4}.$$

$$\sin^2\theta = \frac{13}{16} \qquad \text{Subtract } \tfrac{3}{16}.$$

$$\sin\theta = \pm\frac{\sqrt{13}}{4} \qquad \text{Take square roots. (Section 1.4)}$$

Choose the correct sign here.

$$\sin\theta = \frac{\sqrt{13}}{4} \qquad \text{Choose the positive square root since } \sin\theta \text{ is positive.}$$

To find $\tan\theta$, use the quotient identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

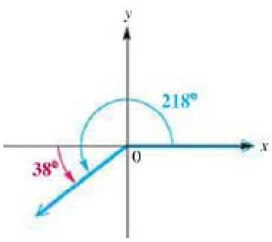
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}} = \frac{\sqrt{13}}{4}\left(-\frac{4}{\sqrt{3}}\right) = -\frac{\sqrt{13}}{\sqrt{3}}$$

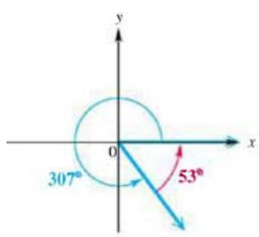
$$= -\frac{\sqrt{13}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{39}}{3} \qquad \text{Rationalize the denominator.}$$

Q9. (4 points): (5.3 Textbook Example 4): Find the reference angle for each angle:

(a): 218° **(b):** 1387°

Solution:





(a) As shown in **Figure 34(a),** the positive acute angle made by the terminal side of this angle and the x-axis is

$$218° - 180° = 38°.$$

For $\theta = 218°$, the reference angle $\theta' = 38°$.

(b) First find a coterminal angle between 0° and 360°. Divide 1387° by 360° to get a quotient of about 3.9. Begin by subtracting 360° three times (because of the whole number 3 in 3.9).

$$1387° - 3 \cdot 360° = 1387° - 1080° \qquad \text{Multiply. (Section 5.1)}$$

$$= 307° \qquad \text{Subtract.}$$

The reference angle for 307° (and thus for 1387°) is

$$360° - 307° = 53°.$$

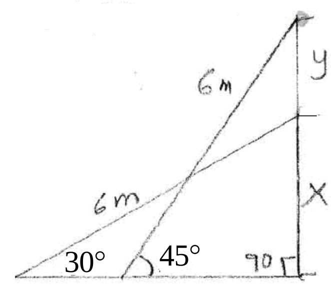
Q10. (4 points): (5.4 Recitation Question 3):

A ladder of 6 meters length is placed against a wall forms an angle of 30° with the ground. If the foot of the ladder is moved towards the wall, the angle changed to 45°. Find the exact distance moved by the top of the ladder on the wall.

Solution:

$\sin 30^\circ = \dfrac{x}{6} \Rightarrow \boxed{x = 3}$

$\sin 45^\circ = \dfrac{x+y}{6} \Rightarrow x + y = 3\sqrt{2} \Rightarrow y = 3\sqrt{2} - 3 = 3\left(\sqrt{2} - 1\right)$



Q11. (4 points): (6.1 Recitation Question 4):
Find the length s of the arc that subtends the central angle $\theta = 35^\circ\,30'$ in a circle of diameter 720 centimeters.

Solution:

$$\theta = 35^\circ\,30' = 35^\circ + (30') = 35^\circ + \left(\frac{30}{60}\right)^\circ = 35^\circ + \left(\frac{1}{2}\right)^\circ = \frac{71^\circ}{2} = \frac{71^\circ}{2}\cdot\frac{\pi}{180^\circ} = \frac{71\pi}{360} \text{ radinas}$$

$$s = r\,\theta = \frac{720}{2}\cdot\frac{71\pi}{360} = 71\pi \text{ centimeters}$$

Q12. (4 points): (6.2 Textbook Exercises 14 and 19): Find the exact value of

$\tan\left(-\dfrac{17\pi}{3}\right) + \sec\dfrac{23\pi}{6}$: (Show your work)

Solution: First, we find the value of $\tan\left(-\dfrac{17\pi}{3}\right)$:

14. $-\dfrac{17\pi}{3}$ is coterminal with

$-\dfrac{17\pi}{3} + 6\pi = -\dfrac{17\pi}{3} + \dfrac{18\pi}{3} = \dfrac{\pi}{3}$. Since $\dfrac{\pi}{3}$ is in quadrant I, $\dfrac{\pi}{3}$ is the reference angle. In quadrant I, the tangent is positive. Thus,

$\tan\left(-\dfrac{17\pi}{3}\right) = \tan\dfrac{\pi}{3} = \sqrt{3}$. Converting $\dfrac{\pi}{3}$ to degrees, we have $\dfrac{\pi}{3} = \dfrac{1}{3}(180^\circ) = 60^\circ$. Thus,

$\tan\dfrac{\pi}{3} = \tan 60^\circ = \sqrt{3}$.

Now, we find the value of $\sec\dfrac{23\pi}{6}$**:**

19. $\frac{23\pi}{6}$ is coterminal with

$\frac{23\pi}{6} - 2\pi = \frac{23\pi}{6} - \frac{12\pi}{6} = \frac{11\pi}{6}$. Since $\frac{11\pi}{6}$ is in quadrant IV, the reference angle is

$2\pi - \frac{11\pi}{6} = \frac{12\pi}{6} - \frac{11\pi}{6} = \frac{\pi}{6}$. In quadrant IV, the secant is positive. Thus,

$$\sec\frac{23\pi}{6} = \sec\frac{11\pi}{6} = \sec\frac{\pi}{6} = \frac{2\sqrt{3}}{3}.$$

Converting $\frac{11\pi}{6}$ to degrees, we have

$\frac{11\pi}{6} = \frac{11}{6}(180°) = 330°$. The reference angle is $360° - 330° = 30°$. Thus,

$$\sec\frac{23\pi}{6} = \sec\frac{11\pi}{6} = \sec 330° = \sec 30° = \frac{2\sqrt{3}}{3}.$$

$$\tan\left(-\frac{17\pi}{3}\right) + \sec\frac{23\pi}{6} = \sqrt{3} + \frac{2\sqrt{3}}{3} = \frac{5\sqrt{3}}{3}$$