Chapter 3: Polynomial and Rational Functions

Section 3.1 The Remainder Theorem and the Factor Theorem

Division of Polynomials

Use long division to divide.

1. $\left(-6x^2+4x^3-14-6x\right)\div(2x-5)$

(a) $2x^2+2x+3+\dfrac{1}{2x-5}$ (b) $2x^2+2x+2-\dfrac{4}{2x-5}$

(c) $2x^2-2x-3+\dfrac{1}{2x-5}$ (d) $2x^2-2x-2-\dfrac{4}{2x-5}$

2. $\left(-3x^3+2x+3\right)\div(x+3)$

(a) $-3x^2+11x-30+\dfrac{90}{x+3}$ (b) $-3x^2+9x+29-\dfrac{82}{x+3}$

(c) $-3x^2+9x-25+\dfrac{78}{x+3}$ (d) $-3x^2+11x+33-\dfrac{96}{x+3}$

3. $-3x^4-3x^3-8x^2-9x$ by x^2+3

(a) $-3x^2+3x-2+\dfrac{1}{x^2+3}$ (b) $-3x^2-3x-1+\dfrac{3}{x^2+3}$

(c) $-3x^2+3x-\dfrac{1}{x^2+3}$ (d) $-3x^2-3x+1-\dfrac{3}{x^2+3}$

4. $\dfrac{3x^3+5x^2-4x-7}{-x^2-3x-1}$

(a) $-3x^2+4x+\dfrac{5x-3}{-x^2-3x-1}$ (b) $4x-3+\dfrac{5x-3}{-x^2-3x-1}$

(c) $-3x+4+\dfrac{5x-3}{-x^2-3x-1}$ (d) $4x^2-3x+\dfrac{5x-3}{-x^2-3x-1}$

The Remainder Theorem

5. Use synthetic division to find $P\left(-\dfrac{1}{6}\right)$ if $P(x)=2x^4-2x^2-7$.

(a) $-\dfrac{67}{36}$ (b) $\dfrac{-571}{1296}$ (c) $\dfrac{-131}{36}$ (d) $\dfrac{-4571}{648}$

Use synthetic division to find the function value.

6. $g(x)=x^6+4x^5+9x^3-6x^2+24,\ g(-5)$ (a) 1818 (b) 1850 (c) 1930 (d) 1874

7. $f(x)=-8x^4+6x^2-6,\ f(2)$ (a) –106 (b) –109 (c) –105 (d) –110

8. For $(x^3+4x^2+kx-1)\div(x-5)$, find the value of k if the remainder is 214.

(a) –2 (b) 4 (c) –1 (d) 1

The Factor Theorem

9. Use synthetic division to determine which of the following polynomials is *not* a factor of x^3+2x^2-5x-6.

(a) $x+2$ (b) $x+1$ (c) $x+3$ (d) $x-2$

10. Use synthetic division to determine which of the following is *not* a zero of the polynomial equation.
$3x^4-23x^3+25x^2+71x+20=0$

(a) 4 (b) –1 (c) –4 (d) 5

11. Use the Factor Theorem to determine which of the following is *not* a factor of $f(x)=3x^4-5x^3-59x^2+41x+20$.

(a) $x-5$ (b) $3x+1$ (c) $x+5$ (d) $x+4$

12. Use the Factor Theorem to determine how many of the following polynomials are factors of
$3x^4-11x^3-55x^2+163x+60$.
$x-5,\ x+3,\ x-3,\ 3x-2,\ x-4,\ x+4,\ x+5$

(a) 4 (b) 1 (c) 2 (d) 3

Reduced Polynomials

Use synthetic division to complete the indicated factorization.

13. $x^3-21x^2+400=(x-20)(\quad)$ (a) x^2+x+20 (b) x^2-x-20 (c) x^2-19 (d) x^2-x-19

14. $x^4-x^3-22x^2+16x+96=(x-4)(\quad)$

(a) $x^3+3x^2-10x-24$ (b) $x^3+3x^2-11x-24$ (c) $x^3+4x^2-11x-24$ (d) $x^3-3x^2+10x-24$

15. $x^4+3x^3-91x^2-123x+1890=(x-5)(\quad)$

(a) $x^3+8x^2+51x-378$ (b) $x^3-2x^2-51x-378$ (c) $x^3-2x^2+51x-378$ (d) $x^3+8x^2-51x-378$

16. $x^4+4x^3-7x^2-22x+24=(x+4)(x+3)(\quad)$

(a) x^2-3x+2 (b) x^2-3x+1 (c) x^2+3x-2 (d) x^2-2x+1

Section 3.2 Polynomial Functions of Higher Degree

Far-Left and Far-Right Behavior

17. Determine the right-hand and left-hand behavior of the graph of the function.

$$f(x)=\frac{2x^2-3+9x^4}{7}$$

(a) Rises to the left
Rises to the right

(b) Rises to the left
Falls to the right

(c) Falls to the left
Falls to the right

(d) Falls to the left
Rises to the right

(e) None of these

Examine the leading term and determine the far-left and far-right behavior of the graph of the polynomial function.

18. $N(x) = 2 + 9x^2 - 7x^3$

(a) $a_n = 2$ and $n = 3$
The graph of N goes up to the far left and up to the far right.

(b) $a_n = -7$ and $n = 3$
The graph of N goes down to the far left and up to the far right.

(c) $a_n = -7$ and $n = 3$
The graph of N goes up to the far left and down to the far right.

(d) $a_n = 9$ and $n = 3$
The graph of N goes down to the far left and down to the far right.

19. $P(x) = -\frac{1}{7}(x-6)^4$

(a) $a_n = \frac{3}{2}$ and $n = 5$
The graph of P goes up to the far left and up to the far right.

(b) $a_n = -\frac{1}{7}$ and $n = 4$
The graph of P goes down to the far left and down to the far right.

(c) $a_n = -\frac{1}{7}$ and $n = 5$
The graph of P goes down to the far left and up to the far right.

(d) $a_n = \frac{3}{2}$ and $n = 4$
The graph of P goes up to the far left and down to the far right.

20. $S(x) = -4x^6 + 7x^5 + 11$

(a) The graph of S goes up to the far left and down to the far right.

(b) The graph of S goes down to the far left and down to the far right.

(c) The graph of S goes down to the far left and up to the far right.

(d) The graph of S goes up to the far left and up to the far right.

Maximum and Minimum Values

Find all relative extrema of the function.

21. $f(x) = x^4 - 2x^3$

(a) relative maximum: $\left(\frac{3}{2}, -\frac{27}{16}\right)$
relative minimum: none

(b) relative maximum: $\left(-\frac{3}{2}, \frac{27}{16}\right)$
relative minimum: $(0, 0)$

(c) relative maximum: none
relative minimum: $\left(\frac{3}{2}, -\frac{27}{16}\right)$

(d) The function has no relative extrema.

22. $f(x) = 64x + \frac{16}{x}$

(a) Relative minimum: $\left(-\frac{1}{2}, -64\right)$
Relative maximum: $\left(\frac{1}{2}, 64\right)$

(b) Relative maximum: $\left(-\frac{1}{2}, -64\right)$
Relative minimum: $\left(\frac{1}{2}, 64\right)$

(c) Relative minimum: $(-1, -80)$
Relative maximum: $(1, 80)$

(d) Relative maximum: $(-1, -80)$
Relative minimum: $(1, 80)$

23. $f(x) = \frac{1}{x^2 + 6x + 13}$

(a) relative minimum: $\left(-3, \frac{1}{4}\right)$

(b) relative maximum: $\left(3, \frac{1}{4}\right)$

(c) relative minimum: $\left(-3, -\frac{1}{4}\right)$

(d) relative maximum: $\left(-3, \frac{1}{4}\right)$

24. A drug that stimulates reproduction is introduced into a colony of bacteria. After *t* minutes, the number of bacteria is given approximately by
$N(t) = 1500 + 27t^2 - t^3,\ 0 \le t \le 50$
At which value of *t* is the rate of growth maximum?

(a) 36 min (b) 18 min (c) 27 min (d) 9 min

Real Zeros of a Polynomial Function

Find all real zeros of the function.

25. $f(x) = -7x^4 + 112x^2$

(a) $x = 0,\ x = \pm 16$ (b) $x = 0,\ x = 4$ (c) $x = 0,\ x = \pm 4$ (d) $x = 0,\ x = 16$ (e) None of these

Find all real zeros of the function.

26. $f(x) = x^3 - 9x^2 + 23x - 15$

(a) $x = 1,\ x = -3,\ x = -5$
(b) $x = 1,\ x = 3,\ x = 5$
(c) $x = -1,\ x = -3,\ x = -5$
(d) $x = -1,\ x = 3,\ x = -5$
(e) None of these

27. $f(x) = x^4 - 11x^2 + 10$

(a) $x = \pm 1,\ x = \pm 10$
(b) $x = \pm 1,\ x = \pm\sqrt{10}$
(c) $x = 1,\ x = 10$
(d) $x = 1,\ x = \sqrt{10}$
(e) None of these

28. Use the Zero Location Theorem to determine whether the given polynomial has a zero between 0 and 2.
$P(y) = y^2 - 6y + 3$

(a) $P(y)$ does not have a zero between 0 and 2.
(b) $P(y)$ has a zero at 0.
(c) $P(y)$ has a zero at 2.
(d) $P(y)$ has a zero between 0 and 2.

Even and Odd Powers of (x - c) Theorem

Use the Even and Odd Powers of $(x - c)$ Theorem to determine where the graph of the given polynomial will cross the *x*-axis and where the graph will intersect but not cross the *x*-axis.

29. $y = (x+7)(x+5)(x+1)^2(x-4)$

(a) The graph of *y* will cross the *x*-axis at the *x*-intercepts $(-1,\ 0)$ and $(-7,\ 0)$.
The graph of *y* will intersect but not cross the *x*-axis at $(-5,\ 0)$ and $(4,\ 0)$.

(b) The graph of *y* will cross the *x*-axis at the *x*-intercepts $(-7,\ 0)$, $(-5,\ 0)$, and $(4,\ 0)$.
The graph of *y* will intersect but not cross the *x*-axis at $(-1,\ 0)$.

(c) The graph of *y* will cross the *x*-axis at the *x*-intercept $(-1,\ 0)$.
The graph of *y* will intersect but not cross the *x*-axis at $(-7,\ 0)$, $(-5,\ 0)$, and $(4,\ 0)$.

(d) The graph of *y* will cross the *x*-axis at the *x*-intercepts $(-7,\ 0)$ and $(-5,\ 0)$.
The graph of *y* will intersect but not cross the *x*-axis at $(-1,\ 0)$ and $(4,\ 0)$.

Use the Even and Odd Powers of $(x-c)$ Theorem to determine where the graph of the given polynomial will cross the x-axis and where the graph will intersect but not cross the x-axis.

30. $y = x(x-2)^2$

(a) The graph of y will cross the x-axis at the x-intercept $(0,\ 0)$.
The graph of y will intersect but not cross the x-axis at $(2,\ 0)$.

(b) The graph of y will cross the x-axis at the x-intercept $(0,\ 0)$.
The graph of y will cross the x-axis at the x-intercept $(2,\ 0)$.

(c) The graph of y will intersect but not cross the x-axis at $(0,\ 0)$.
The graph of y will intersect but not cross the x-axis at $(2,\ 0)$.

(d) The graph of y will intersect but not cross the x-axis at $(0,\ 0)$.
The graph of y will cross the x-axis at the x-intercept $(2,\ 0)$.

31. $P(x) = x(x+2)^3$

(a) The graph of P will cross the x-axis at the x-intercept $(0,\ 0)$ and will intersect but not cross the x-axis at $(-2,\ 0)$.

(b) The graph of P will intersect but not cross the x-axis at $(0,\ 0)$ and $(-2,\ 0)$.

(c) The graph of P will intersect but not cross the x-axis at $(0,\ 0)$ and will cross the x-axis at the x-intercept $(-2,\ 0)$.

(d) The graph of P will cross the x-axis at the x-intercepts $(0,\ 0)$ and $(-2,\ 0)$.

32. $P(x) = (x+5)(x+2)^2(3x+1)$

(a) The graph of P will cross the x-axis at the x-intercepts $(-5,\ 0)$ and $(-2,\ 0)$.
The graph of P will intersect but not cross the x-axis at $\left(-\frac{1}{3},\ 0\right)$.

(b) The graph of P will cross the x-axis at the x-intercepts $(-5,\ 0)$ and $\left(-\frac{1}{3},\ 0\right)$.
The graph of P will intersect but not cross the x-axis at $(-2,\ 0)$.

(c) The graph of P will cross the x-axis at the x-intercept $(-2,\ 0)$.
The graph of P will intersect but not cross the x-axis at $(-5,\ 0)$ and $\left(-\frac{1}{3},\ 0\right)$.

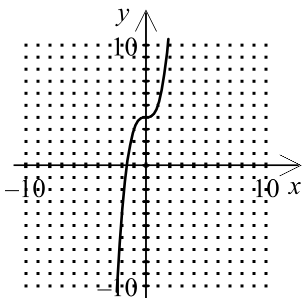
(d) The graph of P will cross the x-axis at the x-intercepts $(-2,\ 0)$ and $\left(-\frac{1}{3},\ 0\right)$.
The graph of P will intersect but not cross the x-axis at $(-5,\ 0)$.

A Procedure for Graphing Polynomial Functions

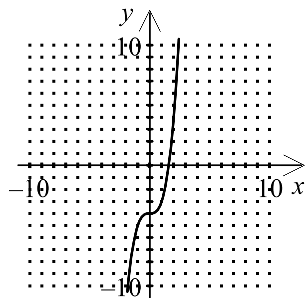
Identify the graph of the function.

33. $f(x) = -x^3 + 4$

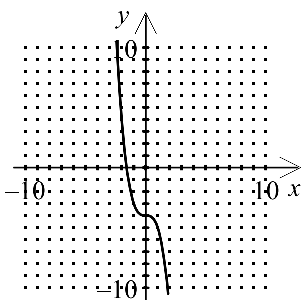
(a)



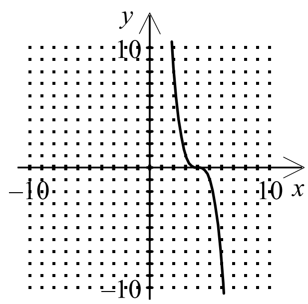
(b)



(c)



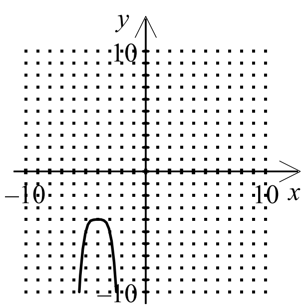
(d)



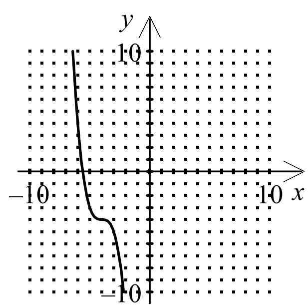
(e) None of these

34. $f(x) = (x+4)^4 + 4$

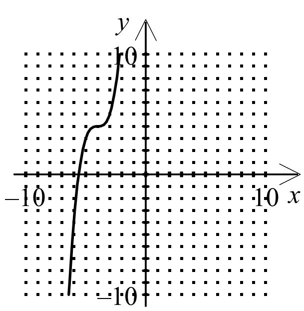
(a)



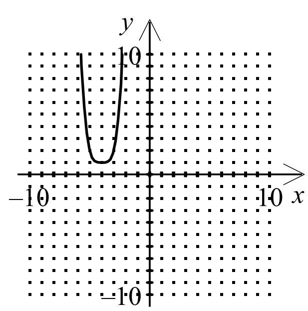
(b)



(c)



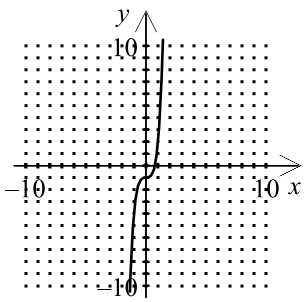
(d)

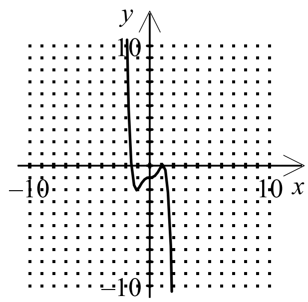


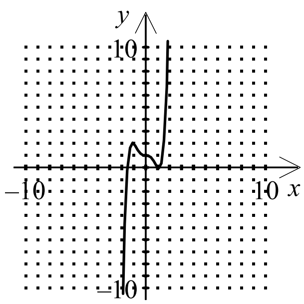
(e) None of these

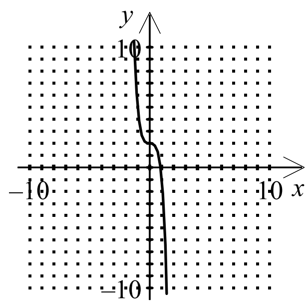
Identify the graph of the function.

35. $f(x) = -x^5 - 2x^3 + 1$

(a)


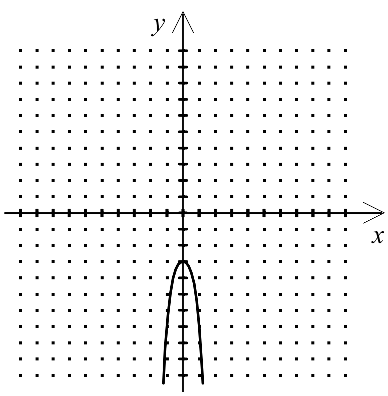
(b)


(c)


(d)


(e) None of these

36. Identify the equation that matches the graph.



(a) $f(x) = -2x^5 - 2x^3 + 3$ (b) $f(x) = -2x^4 - 2x^2 - 3$ (c) $f(x) = 2x^4 + 2x^2 - 3x$

(d) $f(x) = 2x^5 - 2x^3 - 3$ (e) None of these

Section 3.3 Zeros of Polynomial Functions

Multiple Zeros of a Polynomial Function

List the zeros of the cubic function and tell which, if any, are double or triple zeros.

37. $y = x(x+9)^2$ (a) 1, –9 (double) (b) 0, –9 (double) (c) 0, 9 (double) (d) 1, 9 (double)

38. $y = (x+2)^2(x-8)$

(a) –8, 2 (double) (b) –8 (double), 2 (c) –2 (double), 8 (d) –2, 8 (double)

List the zeros of the cubic function and tell which, if any, are double or triple zeros.

39. $y = (3x+1)^3$

(a) 3 (triple) (b) $\frac{1}{3}$ (triple) (c) -3 (triple) (d) $-\frac{1}{3}$ (triple)

40. Find the roots of the polynomial equation, and state the multiplicity of each root.

$P(x) = -\frac{1}{4}(x+4)(x-4)(4x+1)$

(a) $4,\ -4,$ and $\frac{1}{4}$ are roots each of multiplicity 1.

(b) $-4,\ 4,$ and $-\frac{1}{4}$ are roots each of multiplicity 1.

(c) –4 and 4 are roots each of multiplicity 1.
$-\frac{1}{4}$ is a root of multiplicity 2.

(d) none of these

The Rational Zero Theorem

Use the Rational Zero Theorem to find all possible rational zeros of the polynomial.

41. $f(x) = -6x^4 + 6x^3 - 2x^2 + 3x - 77$

(a) $\pm1,\ \pm7,\ \pm11,\ \pm77,\ \pm\frac{7}{3},\ \pm\frac{11}{3},\ \pm\frac{77}{3},\ \pm\frac{1}{6},\ \pm\frac{7}{6},\ \pm\frac{11}{6},\ \pm\frac{77}{6}$

(b) $\pm1,\ \pm7,\ \frac{7}{2},\ \pm\frac{11}{2},\ \pm\frac{7}{3},\ \pm\frac{11}{3}$

(c) $\pm1,\ \pm7,\ \pm11,\ \pm77,\ \pm\frac{1}{2},\ \pm\frac{7}{2},\ \pm\frac{11}{2},\ \pm\frac{77}{2},\ \pm\frac{1}{3},\ \pm\frac{7}{3},\ \pm\frac{11}{3},\ \pm\frac{77}{3},\ \pm\frac{1}{6},\ \pm\frac{7}{6},\ \pm\frac{11}{6},\ \pm\frac{77}{6}$

(d) $\pm\frac{7}{2},\ \pm\frac{11}{2},\ \pm\frac{77}{2},\ \pm\frac{1}{3},\ \pm\frac{7}{3},\ \pm\frac{11}{3},\ \pm\frac{77}{6}$

42. $g(x) = -15x^3 - 5x^2 - x - 1$

(a) $\pm1,\ \pm\frac{1}{4},\ \pm\frac{1}{6},\ \pm\frac{1}{15}$

(b) $\pm1,\ \pm\frac{1}{3},\ \pm\frac{1}{5},\ \pm\frac{1}{15}$

(c) $\pm1,\ -\frac{1}{3},\ -\frac{1}{5},\ \pm\frac{1}{17}$

(d) $-2,\ -\frac{1}{3},\ \pm\frac{1}{7},\ -\frac{1}{15}$

Use the Rational Zero Theorem to determine all possible rational zeros of f.

43. $f(x) = 2x^3 + 6x^2 + 5x + 8$

(a) $\pm\frac{1}{2},\ \pm2,\ \pm4,\ \pm8,\ \pm16$

(b) $\pm\frac{1}{2},\ \pm1,\ \pm2,\ \pm4,\ \pm8$

(c) $\pm\frac{1}{2},\ \pm2,\ \pm4,\ \pm8$

(d) $0,\ \pm\frac{1}{2},\ \pm1,\ \pm2,\ \pm4$

Use the Rational Zero Theorem to determine all possible rational zeros of *f*.

44. $f(x) = 2x^4 - 3x^2 - 8$

(a) $\pm 1,\ \pm 2,\ \pm 4,\ \pm 8,\ \pm\frac{1}{2}$

(b) $\pm 2,\ \pm 4,\ \pm 8,\ \pm\frac{1}{2},\ \pm\frac{7}{2}$

(c) $\pm 2,\ \pm 4,\ \pm 8,\ \pm 16,\ \pm\frac{1}{2}$

(d) $0,\ \pm 1,\ \pm 2,\ \pm 4,\ \pm\frac{1}{2}$

Upper and Lower Bounds for Real Zeros

45. Use synthetic division to determine which pair of integers provide both a lower and an upper bound for the zeros of $f(x) = 2x^4 - 8x^3 - 2x^2 + 32x - 24$.

(a) –2, 5 (b) –3, 4 (c) 0, 6 (d) none of these

Use synthetic division to find upper and lower bounds of the real zeros of *f*.

46. $f(x) = x^5 - 2x^4 - 10x^3 + 20x^2 + 9x - 20$

(a) Upper: $x = 4$ Lower: $x = 1$

(b) Upper: $x = 5$ Lower: $x = -4$

(c) Upper: $x = 4$ Lower: $x = -2$

(d) Upper: $x = 4$ Lower: $x = -3$

47. $f(x) = 6x^3 - 11x^2 - 51x + 56$

(a) Upper: $x = 4$ Lower: $x = -3$

(b) Upper: $x = 3$ Lower: $x = 1$

(c) Upper: $x = 4$ Lower: $x = -2$

(d) Upper: $x = 1$ Lower: $x = -2$

48. $f(x) = x^5 + x^4 - 5x^3 - 3x^2 + 3x - 3$

(a) Upper: $x = 3$ Lower: $x = -3$

(b) Upper: $x = 4$ Lower: $x = -4$

(c) Upper: $x = 2$ Lower: $x = -2$

(d) Upper: $x = 1$ Lower: $x = -1$

Descartes' Rule of Signs

Use Descartes' Rule of Signs to determine the possible number of positive real zeros of the function.

49. $f(x) = -5x^3 + 2x^2 - 5x - 5$ (a) 4, 2, or 0 (b) 3 or 1 (c) 1 (d) 2 or 0

50. $f(x) = -4x^4 - x^3 - 5x^2 + 4x + 1$ (a) 4, 2, or 0 (b) 3 or 1 (c) 5, 3, or 1 (d) 1

51. Use Descartes' Rule of Signs to determine the possible number of negative real zeros of the function.
$f(x) = 5x^5 + 2x^4 + x^3 + 5x^2 + 2x - 1$

(a) 5, 3, or 1 (b) 4, 2, or 0 (c) 1 (d) 2 or 0

52. Use Descartes's Rule of Signs to determine the possible number of positive and negative zeros of the function.
$f(x) = x^6 - 4x^5 - x^4 + 2x^3 + 5x^2 + x + 3$

(a) Four, two, or no positive zeros
Two or no negative zeros

(b) Two or no positive zeros
Five, three, or one negative zeros

(c) Two or no positive zeros
Four, two, or no negative zeros

(d) Three or one positive zeros
Four, two, or no negative zeros

Zeros of a Polynomial Function

Find all the zeros of the function.

53. $x^3 + 5x^2 + x - 10$

(a) $-2,\ \dfrac{-3+\sqrt{29}}{2},\ \dfrac{-3-\sqrt{29}}{2}$

(b) $-3+\sqrt{29},\ -3-\sqrt{29}$

(c) $-2,\ \dfrac{2+\sqrt{27}}{2},\ \dfrac{2-\sqrt{27}}{2}$

(d) none of these

54. $f(x) = 6x^4 - 13x^3 + 13x - 6$

(a) $-1,\ 1,\ -\dfrac{3}{2},\ \dfrac{3}{2}$ (b) $-1,\ 1,\ -\dfrac{3}{2},\ -\dfrac{2}{3}$ (c) $-1,\ 1,\ -3,\ -\dfrac{3}{2}$ (d) $-1,\ 1,\ \dfrac{2}{3},\ \dfrac{3}{2}$

55. $p(x) = x^3 - 4x^2 - 15x + 18$ (a) –3, 1, and 6 (b) 3, 1, and 6 (c) –3, 1, and –6 (d) 3, 1, and –6

56. $y = 4x^4 - 4x^3 - 224x^2$ (a) 0, 4, 8 (b) –7, 8 (c) 0, 8 (d) –7, 0, 8

Applications of Polynomial Functions

57. A cubic model for the yearly worldwide carbon emissions is
$W = -0.051x^3 + 2.93x^2 + 74.6x + 1589$
where W is the weight of the emissions in millions of tons and x is the number of years since 1950. Use synthetic division to evaluate the model for the year 1960.

(a) 2029 million tons (b) 2835 million tons (c) 2679 million tons (d) 2577 million tons

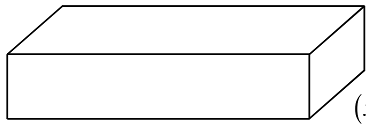
58. A cubic model for the weight of an alligator is
$W = 0.001l^3 - 0.22l^2 + 17.4l - 426$
where W is the weight of the alligator in pounds and l is the length in inches. Use synthetic division to evaluate the model for an alligator 72 inches long.

(a) 46 lb (b) 66 lb (c) 60 lb (d) 54 lb

59. A cubic model for the increase in population of a certain town is
$P = -0.003x^3 + 6.8x^2 - 79x + 4972$
where x is the number of years since 1950. Use synthetic division to evaluate the population in 2050.

(a) 74,569 (b) 50,755 (c) 62,072 (d) 71,976

60. The volume of the small box below is 60 cubic inches. Find the dimensions of the box.



(a) 10 in., by 3 in., by 2 in.
(b) 8 in., by 4 in., by 2 in.
(c) 5 in., by 3 in., by 4 in.
(d) 9 in., by 3 in., by 2 in.

Section 3.4 The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra

Find all the zeros of the function.

61. $r(x) = x^3 - 4x^2 + 9x - 10$

(a) $1+2i,\ 1-2i,\ 2$
(b) $1+2i,\ -1-2i,\ 2$
(c) $1+2i,\ 1-2i,\ -2$
(d) $1+2i,\ -1-2i,\ -2$

62. $f(x) = x^4 - 8x^3 + 18x^2 + 8x - 19$

(a) $1,\ -1,\ 4+\sqrt{3}i,\ 4-\sqrt{3}i$
(b) $1,\ -1,\ -4+\sqrt{3}i,\ -4-\sqrt{3}i$
(c) $4,\ -4,\ -4+\sqrt{3}i,\ -4-\sqrt{3}i$
(d) $3,\ -3,\ 4+\sqrt{3}i,\ 4-\sqrt{3}i$

63. $f(x) = 5x^2 + 3x + 4$

(a) $\dfrac{3 \pm \sqrt{89}i}{10}$
(b) $\dfrac{3 \pm \sqrt{71}i}{10}$
(c) $\dfrac{-3 \pm \sqrt{71}i}{10}$
(d) $\dfrac{-3 \pm \sqrt{89}i}{10}$

64. Find all the real zeros of the function.
$y = -5x^4 + 40x^3 - 60x^2$

(a) $0,\ 2,\ 6$
(b) $0,\ 6$
(c) $-5,\ 0,\ 6$
(d) $2,\ 6$

The Number of Zeros of a Polynomial Function

Identify the polynomial written as a product of linear factors.

65. $f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$

(a) $f(x) = (x+2)(x-3)(x+4)(x-1)$
(b) $f(x) = (x+2)(x-3)(x+4)(x+1)$
(c) $f(x) = (x-2)(x-3)(x-4)(x-1)$
(d) $f(x) = (x-2)(x+3)(x-4)(x+1)$
(e) None of these

66. $f(x) = 2x^3 + 21x^2 + 72x + 80$

(a) $f(x) = (2x-5)(x-4)(x-4)$
(b) $f(x) = (2x-5)(x+4)(x+4)$
(c) $f(x) = (2x+5)(x+4)(x+4)$
(d) $f(x) = (2x+5)(x-4)(x+4)$
(e) None of these

67. $f(x) = 2x^4 - 3x^3 - 20x^2 + 27x + 18$

(a) $f(x) = (2x+1)(x+3)(x-2)(x-3)$
(b) $f(x) = (2x-1)(x+3)(x+2)(x-3)$
(c) $f(x) = (2x+1)(x-3)(x-2)(x+3)$
(d) $f(x) = (2x-1)(x-3)(x+2)(x-3)$
(e) None of these

Identify the polynomial written as a product of linear factors.

68. $f(x)=x^2+36$

(a) $f(x)=(-x+6i)(x-6i)$ (b) $f(x)=(x+6i)(x-6i)$ (c) $f(x)=(x+6i)^2$

(d) $f(x)=(x-i)(x+36i)$ (e) None of these

The Conjugate Pair Theorem

69. Solve $z^3-5z^2+4z+10$ given that $3+i$ is a root.

(a) $-1, 3+i, 3-i$ (b) $7, 2+i, 2-i$ (c) $-7, 2+i, 2-i$ (d) $1, 3+i, -3-i$

Use the given zero of *f* to find all the zeros of *f*.

70. $f(x)=x^3+5x^2+11x+15,\ -1+2i$ (a) $-1\pm 2i,\ -3$ (b) $-1-2i,\ 4$ (c) $1\pm 2i,\ -3$ (d) $1+2i,\ 4$

71. $f(x)=x^4-4x^3+6x^2+4x-7,\ 2+\sqrt{3}i$

(a) $2,\ -2,\ -2+\sqrt{3}i,\ -2-\sqrt{3}i$ (b) $-3,\ 3,\ 2+\sqrt{3}i,\ 2-\sqrt{3}i$

(c) $1,\ -1,\ 2+\sqrt{3}i,\ 2-\sqrt{3}i$ (d) $1,\ -1,\ -2+\sqrt{3}i,\ -2-\sqrt{3}i$

72. $f(x)=x^4+2x^3-2x^2-8x-8,\ -1+i$

(a) $-3,\ 3,\ -1+i,\ -1-i$ (b) $2,\ -2,\ -1+i,\ -1-i$ (c) $-1,\ 1,\ 1+i,\ 1-i$ (d) $2,\ -2,\ 1+i,\ 1-i$

Find a Polynomial Function with Given Zeros

Find a polynomial with integer coefficients that has the given zeros.

73. $2,\ 3+i$

(a) $P(x)=x^3+8x^2+22x+20$ (b) $P(x)=x^3-8x^2+22x-20$

(c) $P(x)=x^3-8x^2-2x-20$ (d) $P(x)=x^3-4x^2-2x+20$

74. $1,\ -4+i,\ -4-i$

(a) $f(x)=x^3+7x^2+9x-17$ (b) $f(x)=x^3-7x^2+9x+17$

(c) $f(x)=x^3+7x^2+25x-17$ (d) $f(x)=x^3+9x^2+25x+17$

75. $2,\ 3i,\ -3i,\ 4i,\ -4i$

(a) $f(x)=x^5+2x^4+50x^2-144x+288$ (b) $f(x)=x^5-2x^4+25x^3-50x^2+144x-288$

(c) $f(x)=x^5-2x^4-7x^3-12x+288$ (d) $f(x)=x^5-7x^4-12x^3-50x^2-144x-288$

Find a polynomial with integer coefficients that has the given zeros.

76. $2,\ 3,\ 1-\sqrt{3}i$

(a) $f(x)=x^4+7x^2-24x+36$

(b) $f(x)=x^4-5x^3+6x+36$

(c) $f(x)=x^4-7x^3+20x^2-32x+24$

(d) $f(x)=x^4+6x^3-32x^2-24x-36$

Section 3.5 Graphs of Rational Functions and Their Applications

Vertical and Horizontal Asymptotes

77. Find the horizontal asymptote of the graph of $f(x)=\dfrac{3}{x-6}$.

(a) $y=3$ (b) $x=6$ (c) $x=0$ (d) $y=0$

78. Find the vertical asymptote(s), if any, for $f(x)=\dfrac{3x+4}{x^2-9x+18}$.

(a) $x=-4,\ x=6$ (b) $x=6,\ x=3,\ x=-4$ (c) $x=6,\ x=3$ (d) No vertical asymptotes

79. Find all vertical asymptotes of the function.

$f(x)=\dfrac{x+2}{3x^2+7x+2}$

(a) $x=-\dfrac{1}{3}$ (b) $x=\dfrac{3}{2},\ x=-\dfrac{1}{3}$ (c) $x=-2$ (d) The function has no vertical asymptotes.

80. Find the vertical and horizontal asymptotes for the rational function.

$f(x)=\dfrac{2x^2-5x-3}{x^2-4}$

(a) $x=-1,\ x=3,\ y=3$ (b) $x=-2,\ x=2,\ y=2$ (c) $x=-3,\ x=2,\ y=3$ (d) $x=-3,\ x=3,\ y=2$

A Sign Property of Rational Functions

Which shows the true statement for the graph of the rational function g?

81. $g(x)=\dfrac{x+2}{x^2+2x-3}$

(a) The graph of g is negative for all x such that $-3<x<-2$.

(b) The graph of g is negative for all x such that $x<-3$.

(c) The graph of g is positive for all x such that $x<1$.

(d) The graph of g is positive for all x such that $-2<x<1$.

Which shows the true statement for the graph of the rational function *g*?

82. $g(x) = \dfrac{x}{x^2 + 3x - 4}$

(a) The graph of *g* is positive for all *x* such that $x < -4$.

(b) The graph of *g* is negative for all *x* such that $-4 < x < 0$.

(c) The graph of *g* is positive for all *x* such that $0 < x < 1$.

(d) The graph of *g* is negative for all *x* such that $x < -4$.

83. $g(x) = \dfrac{x - 1}{x^2 - 2x - 8}$

(a) The graph of *g* is positive for all *x* such that $x < -2$.

(b) The graph of *g* is negative for all *x* such that $x > 4$.

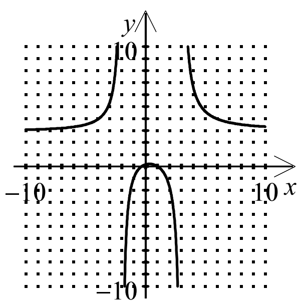
(c) The graph of *g* is positive for all *x* such that $-2 < x < 1$.

(d) The graph of *g* is positive for all *x* such that $1 < x < 4$.

84. Identify the graph of the rational function. Find any vertical and horizontal asymptotes.

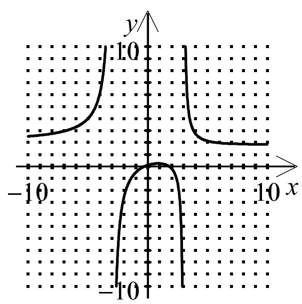
$f(x) = \dfrac{3x^2 - 4x + 1}{x^2 + x - 6}$

(a)



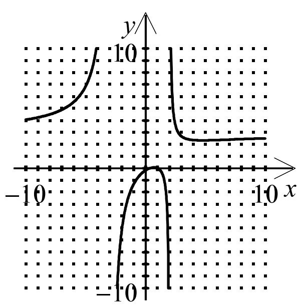
Asymptotes: $x = -2$, $x = 3$, $y = 3$

(b)



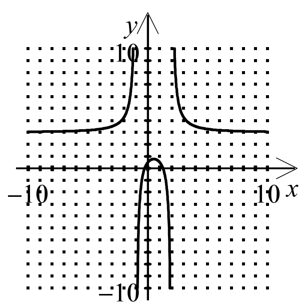
Asymptotes: $x = -3$, $x = -3$, $y = 2$

(c)



Asymptotes: $x = -3$, $x = 2$, $y = 3$

(d)

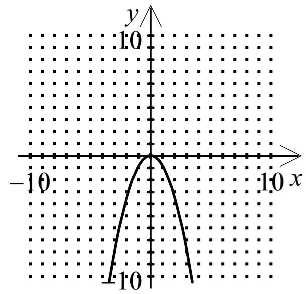


Asymptotes: $x = -1$, $x = 2$, $y = 3$

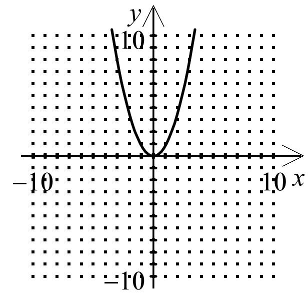
A General Graphing Procedure

85. Graph: $f(x) = -\dfrac{41}{x^2 - 36}$

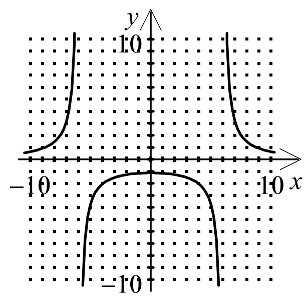
(a)



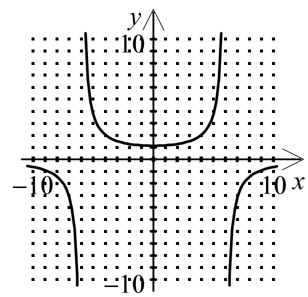
(b)



(c)



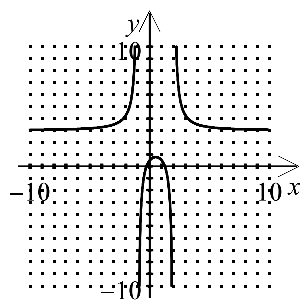
(d)



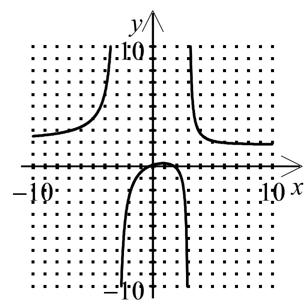
86. Graph:

$f(x) = \dfrac{3x^2 - 3x - 1}{x^2 - x - 2}$

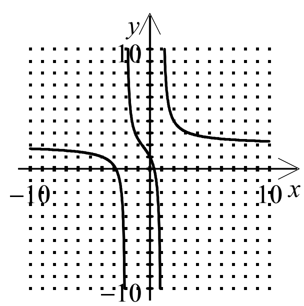
(a)



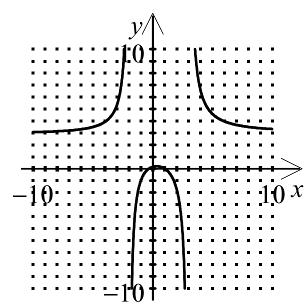
(b)



(c)

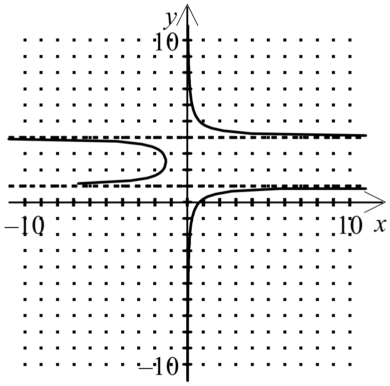


(d)

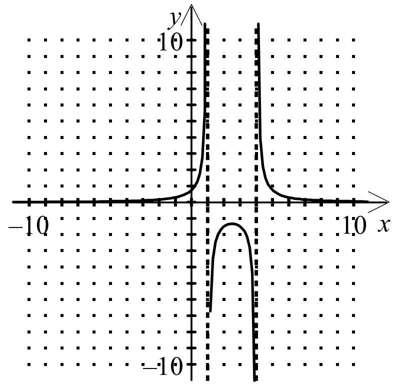


87. Graph: $y = \dfrac{3}{(x-4)(x-1)}$

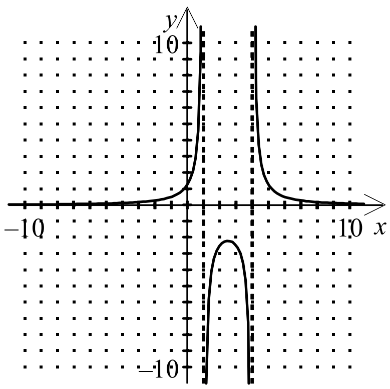
(a)



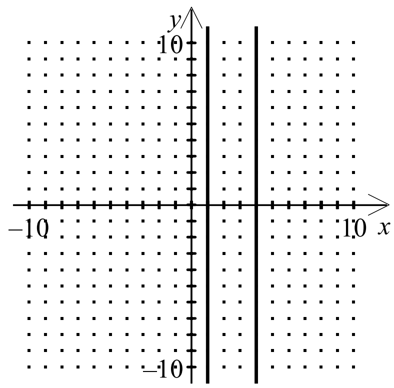
(b)



(c)



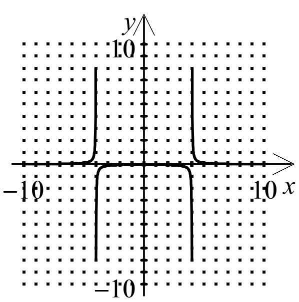
(d)



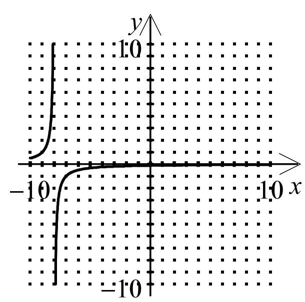
88. Graph:

$f(x) = \dfrac{x^2}{x^2 - 16}$

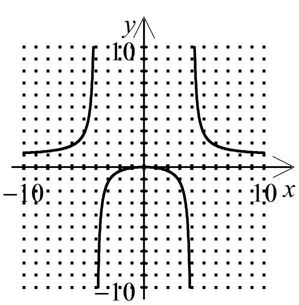
(a)



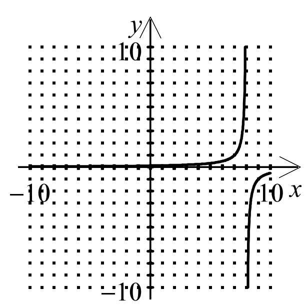
(b)



(c)



(d)



Slant Asymptotes

89. Find the slant asymptote(s), if any, of $f(x) = x + 6 + \dfrac{8}{7x+1}$.

(a) $y = x + 6$ (b) $y = 7x + 1$ (c) $y = -x + 6$ (d) $y = -7x + 1$

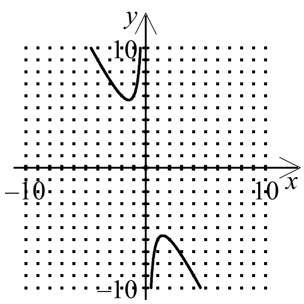
90. Which of the following rational functions has a graph with a slant asymptote?

(a) $f(x) = \dfrac{2x^5 - 9x^2 - 9}{8x^4 + 8x^2 + 3}$ (b) $f(x) = \dfrac{8x^4 + 8x^2 + 3}{\left(-2x^2 - 3\right)^2}$ (c) $f(x) = \dfrac{3 - x}{x + 3}$ (d) $f(x) = \dfrac{8x^4 + 8x^2 + 3}{2x^5 - 9x^2 - 9}$

Identify the graph of the rational function and find the equation of the slant asymptote.

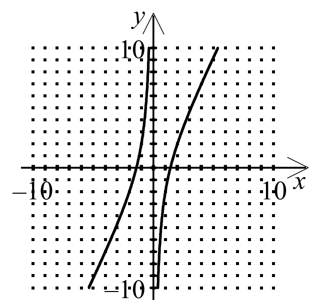
91. $f(x) = \dfrac{-2x^2 + 4}{x}$

(a)



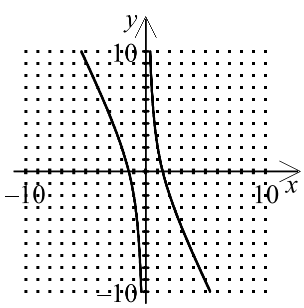
Slant asymptote: $y = -2x$

(b)



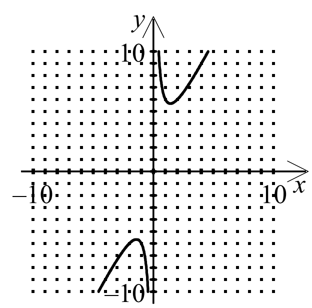
Slant asymptote: $y = 2x$

(c)



Slant asymptote: $y = -2x$

(d)

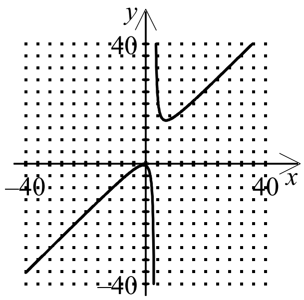


Slant asymptote: $y = 2x$

Identify the graph of the rational function and find the equation of the slant asymptote.

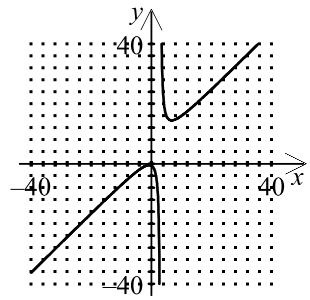
92. $f(x) = \dfrac{-x^2 - x - 2}{x - 3}$

(a)



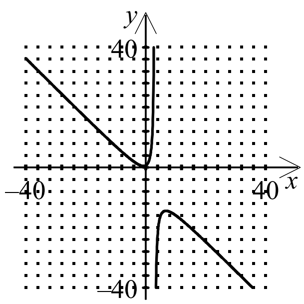
Slant asymptote: $y = x + 4$

(b)



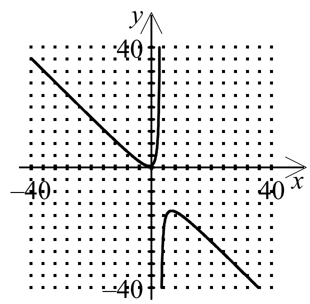
Slant asymptote: $y = -x - 4$

(c)



Slant asymptote: $y = x + 4$

(d)



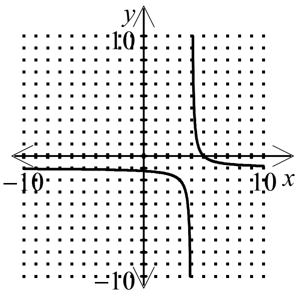
Slant asymptote: $y = -x - 4$

Graph Rational Functions That Have a Common Factor

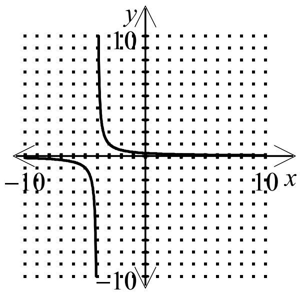
Graph:

93. $f(x)=\dfrac{x-3}{x^2+x-12}$

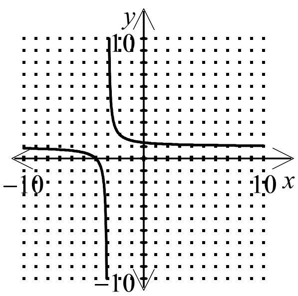
(a)



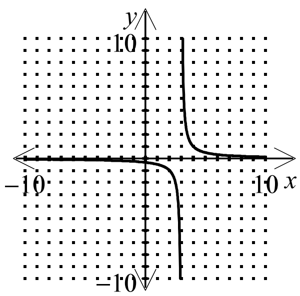
(b)



(c)

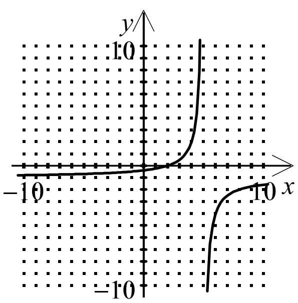


(d)

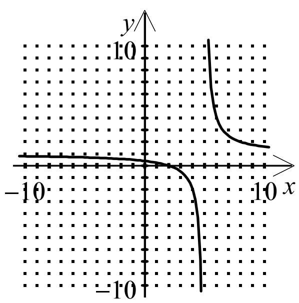


94. $f(x)=\dfrac{x^2+2x-8}{x^2-x-20}$

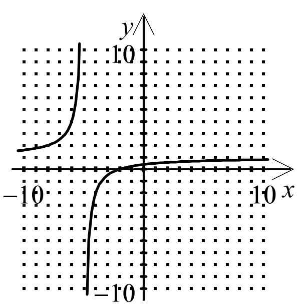
(a)



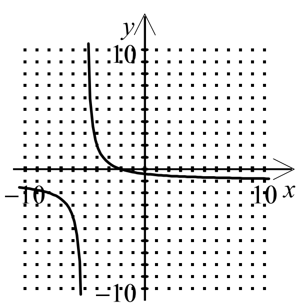
(b)



(c)



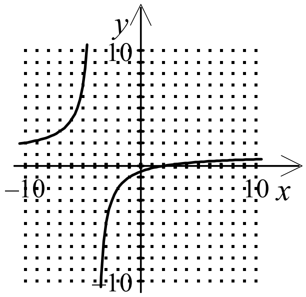
(d)



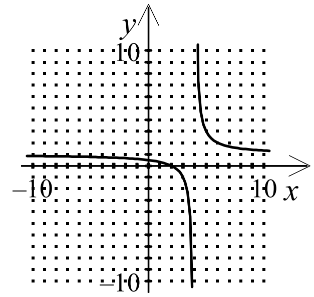
Graph:

95. $f(x)=\dfrac{x^2+4x-12}{x^2+2x-24}$

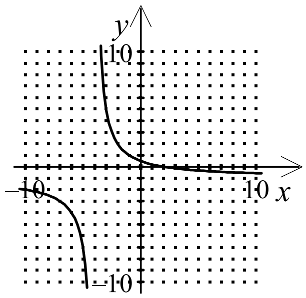
(a)



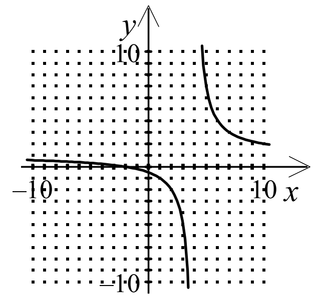
(b)



(c)

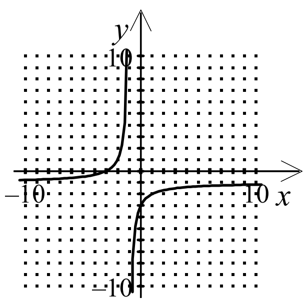


(d)

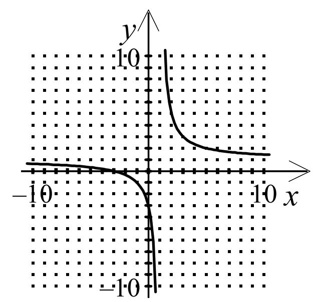


96. $f(x)=\dfrac{x^2+8x+15}{x^2+4x-5}$

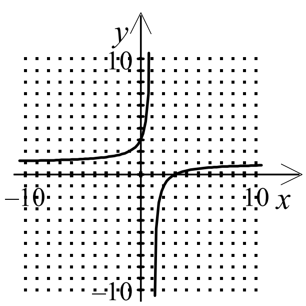
(a)



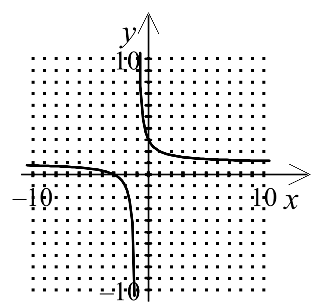
(b)



(c)



(d)



Applications of Rational Functions

97. The total revenue R from the sale of a popular CD is approximately given by the function

$$R(x)=\frac{120x^2}{x^2+6}$$

where x is the number of years since the CD has been released and revenue R is in millions of dollars.
a. Find the total revenue generated by the end of the first year.
b. Find the total revenue generated by the end of the second year.
c. Find the total revenue generated in the second year only.
Round answers to the nearest tenth.

(a) a. \$17.1 million
b. \$48.0 million
c. \$30.9 million

(b) a. \$33.1 million
b. \$64.0 million
c. \$30.9 million

(c) a. \$33.1 million
b. \$48.0 million
c. \$36.9 million

(d) a. \$17.1 million
b. \$64.0 million
c. \$30.9 million

98. A sweatshirt printing business spends \$350 on equipment and supplies for each new batch of sweatshirts. In addition to these one time charges, the cost of printing each sweatshirt is \$1.25. The average cost per sweatshirt when x sweatshirts are printed is modeled by the formula $A=\frac{1.25x+350}{x}$. Determine how many sweatshirts the company must print to have an average cost per sweatshirt of \$4.75.

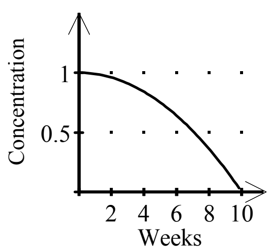
(a) 250 (b) 200 (c) 100 (d) 150

99. After an accident in which organic waste fell into a pond, the decomposition process included oxidation whereby oxygen that was dissolved in the pond water combined with the decomposing waste. The oxygen level of the pond is

$$O_2=\frac{t^2-t+1}{t^2+1}$$

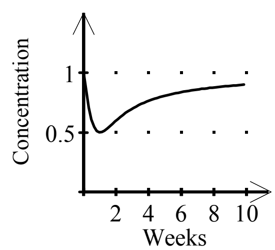
where $O_2 = 1$ represents the normal oxygen level of the pond and t represents the number of weeks after the accident. Identify the graph of this model and find the concentration of oxygen after 9 weeks.

(a)



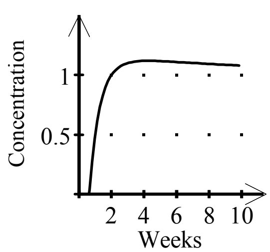
The concentration of O_2 is 0.45.

(b)



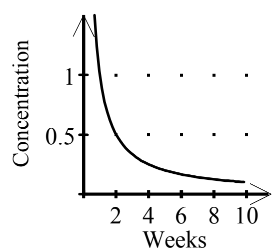
The concentration of O_2 is 0.89.

(c)



The concentration of O_2 is 1.12.

(d)



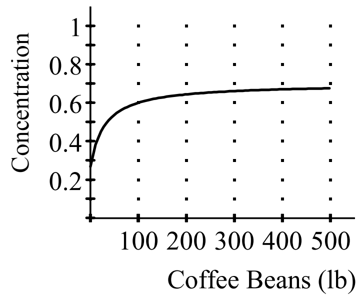
The concentration of O_2 is 0.11.

100. Calypso Coffee mixes 50 pounds of a standard coffee containing 10% Columbian coffee beans with x pounds of a premium coffee containing 60% Columbian coffee beans. The concentration of Columbian coffee beans in the final mix is given by

$$C = \frac{6x + 50}{10(x + 50)}.$$

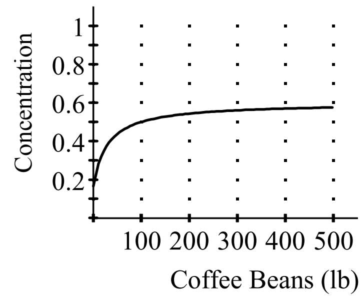
Identify the graph of the concentration function and find the concentration of Columbian coffee beans the graph approaches as x increases.

(a)



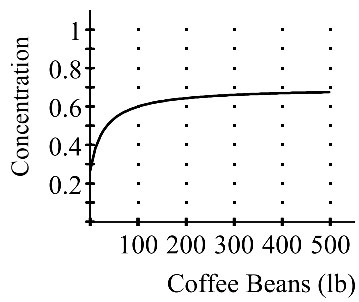
The concentration approaches 70%.

(b)



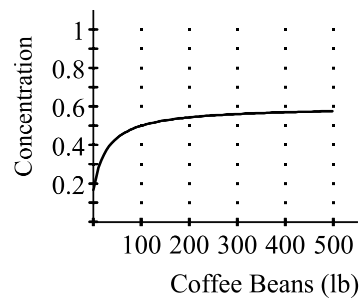
The concentration approaches 35%.

(c)



The concentration approaches 60%.

(d)



The concentration approaches 60%.

Chapter 3: Polynomial and Rational Functions (Answer Key)

Section 3.1 The Remainder Theorem and the Factor Theorem

Division of Polynomials

[1] (b)

[2] (c)

[3] (d)

[4] (c)

The Remainder Theorem

[5] (d)

[6] (d)

[7] (d)

[8] (a)

The Factor Theorem

[9] (a)

[10] (c)

[11] (c)

[12] (d)

Reduced Polynomials

[13] (b)

[14] (a)

[15] (d)

[16] (a)

Section 3.2 Polynomial Functions of Higher Degree

Far-Left and Far-Right Behavior

[17] (a)

[18] (c)

[19] (b)

[20] (b)

Maximum and Minimum Values

[21] (c)

[22] (b)

[23] (d)

[24] (b)

Real Zeros of a Polynomial Function

[25] (c)

[26] (b)

[27] (b)

[28] (d)

Even and Odd Powers of (x - c) Theorem

[29] (b)

[30] (a)

[31] (d)

[32] (b)

A Procedure for Graphing Polynomial Functions

[33] (e)

[34] (e)

[35] (e)

[36] (b)

Section 3.3 Zeros of Polynomial Functions

Multiple Zeros of a Polynomial Function

[37] (b)

[38] (c)

[39] (d)

[40] (b)

The Rational Zero Theorem

[41] (c)

[42] (b)

[43] (b)

[44] (a)

Upper and Lower Bounds for Real Zeros

[45] (a)

[46] (b)

[47] (a)

[48] (a)

Descartes' Rule of Signs

[49] (d)

[50] (d)

[51] (b)

[52] (c)

Zeros of a Polynomial Function

[53] (a)

[54] (d)

[55] (a)

[56] (d)

Applications of Polynomial Functions

[57] (d)

[58] (c)

[59] (c)

[60] (a)

Section 3.4 The Fundamental Theorem of Algebra

The Fundamental Theorem of Algebra

[61] (a)

[62] (a)

[63] (c)

[64] (a)

The Number of Zeros of a Polynomial Function

[65] (e)

[66] (c)

[67] (c)

[68] (b)

The Conjugate Pair Theorem

[69] (a)

[70] (a)

[71] (c)

[72] (b)

Find a Polynomial Function with Given Zeros

[73] (b)

[74] (a)

[75] (b)

[76] (c)

Section 3.5 Graphs of Rational Functions and Their Applications

Vertical and Horizontal Asymptotes

[77] (d)

[78] (c)

[79] (a)

[80] (b)

A Sign Property of Rational Functions

[81] (b)

[82] (d)

[83] (c)

[84] (c)

A General Graphing Procedure

[85] (d)

[86] (a)

[87] (b)

[88] (c)

Slant Asymptotes

[89] (a)

[90] (a)

[91] (c)

[92] (d)

Graph Rational Functions That Have a Common Factor

[93] (b)

[94] (b)

[95] (b)

[96] (b)

Applications of Rational Functions

[97] (a)

[98] (c)

[99] (b)

[100] (d)