King Fahd University of Petroleum and Minerals
Faculty of Science – Math Prep Year program
Math 002 -042
Quiz #3 (5.4-5.6)

Name: ID: **Sr#:**   Sec.:

Question1   (4 points)

If W is the wrapping function, then find $W(-\frac{5\pi}{3})$.

Solution

$$W(-\frac{5\pi}{3}) = \left(\cos(-\frac{5\pi}{3}), \sin(-\frac{5\pi}{3})\right) = \left(\cos(-300°), \sin(-300°)\right) = \left(\cos 60°, \sin 60°\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

Question2   (3 points)

Write $\tan t$ in terms of $\cos t$, $\pi < t < \frac{3\pi}{2}$.

Solution

$$\tan t = \frac{\sin t}{\cos t}, \text{ but } \sin^2 t + \cos^2 t = 1 \Rightarrow \sin t = \pm\sqrt{1-\cos^2 t}, \text{ but } \pi < t < \frac{3\pi}{2} \Rightarrow \sin t = -\sqrt{1-\cos^2 t}$$

$$\therefore \tan t = -\frac{-\sqrt{1-\cos^2 t}}{\cos t}$$

Question3

Let $f(x) = a\cos bx$ with period $= 8$ and $f(4) = 3$.
    a) Find the values of a and b.   **(2 points)**
    b) Find the range of $f(x)$.   **(2 points)**

Solution

$$a)\quad P = \frac{2\pi}{|b|} = 8 \Rightarrow |b| = \frac{\pi}{4} \Rightarrow b = \pm\frac{\pi}{4}$$

$$\therefore f(x) = a\cos(\pm\frac{\pi}{4}x) = a\cos(\frac{\pi}{4}x)$$

$$f(4) = 3 \Rightarrow a\cos\pi = 3 \Rightarrow -a = 3 \Rightarrow a = -3$$

$$b)\quad R = \left[-|a|, |a|\right] = \left[-3, 3\right]$$

Question4

For the function $f(x) = -2\tan\dfrac{2x}{3}$,

    a) Find the amplitude.   (1 point)

    b) Find $f(-\pi)$.   (2 points)

    c) Find the equation of all vertical asymptotes over the interval $[-3\pi, 3\pi]$.   (3 points)

    d) Find the x – intercepts (the zeros) over the interval $[-3\pi, 3\pi]$.   (3 points)

Solution

$a)$ No amplitude

$b)$ $f(-\pi) = -2\tan(\dfrac{-2\pi}{3}) = 2\tan(\dfrac{2\pi}{3}) = 2\tan 120° = -2\tan 60° = -2\sqrt{3}$

$c)$ $\cos\dfrac{2x}{3} = 0 \Rightarrow \dfrac{2x}{3} = \dfrac{2n+1}{2}\pi, \text{n is integer} \Rightarrow x = \dfrac{6n+3}{4}\pi$

$n = 0 \Rightarrow x = \dfrac{3\pi}{4}$

$n = 1 \Rightarrow x = \dfrac{9\pi}{4}$

$n = -1 \Rightarrow x = -\dfrac{3\pi}{4}$

$n = -2 \Rightarrow x = -\dfrac{9\pi}{4}$

$\therefore$ V.A. are: $x = \pm\dfrac{3\pi}{4}$ and $x = \pm\dfrac{9\pi}{4}$

$d)$ $\sin\dfrac{2x}{3} = 0 \Rightarrow \dfrac{2x}{3} = n\pi, \text{n is integer} \Rightarrow x = \dfrac{3n\pi}{2}, \text{n is integer}$

$n = 0 \Rightarrow x = 0$

$n = 1 \Rightarrow x = \dfrac{3\pi}{2}$

$n = 2 \Rightarrow x = 3\pi$

$n = -1 \Rightarrow x = -\dfrac{3\pi}{2}$

$n = -2 \Rightarrow x = -3\pi$

$\therefore x$ – intercepts are : $(0,0), (\pm\dfrac{3\pi}{2}, 0)$ and $(\pm 3\pi, 0)$