KING FAHD UNIVERSITY OF PETROLUEM AND MINERALS
Faculty of Science, Prep-Year Math Program
Math 001 - Term 041

CLASS TEST 2

Code 1

Name: **ID#:** **Sr. #:** **Section:**

[Provide neat and complete solution. Show all necessary steps for full credit.]

Question1

Given the function $f(x) = 4x - x^2,\ x \le 2.$.

a) Write $f^{-1}(x)$.

4 points

let $f(y) = x \Rightarrow 4y - y^2 = x$, complete the square for y

$\Rightarrow y^2 - 4y = -x$, add 4 to both sides

$\Rightarrow (y-2)^2 = -x + 4$

$\Rightarrow y - 2 = \pm\sqrt{-x+4}$

$\Rightarrow y = 2 \pm \sqrt{-x+4}, y \le 2$

$\Rightarrow y = 2 - \sqrt{-x+4}$

$\Rightarrow f^{-1}(x) = 2 - \sqrt{-x+4}$

b) State the **domain** and **range** of f^{-1}

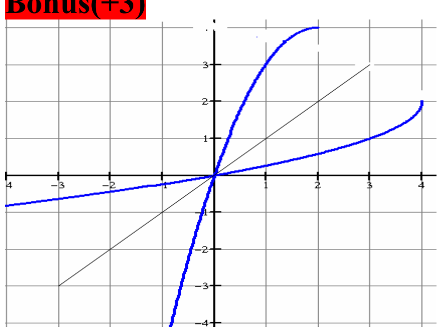
2 points

$D_{f^{-1}} : -x + 4 \ge 0 \Rightarrow x \le 4 \Rightarrow D_{f^{-1}} = (-\infty.4]$

$R_{f^{-1}} = D_f = (-\infty,2]$

c) Sketch the graphs of f and f^{-1} in the same coordinate system.

Bonus(+3)



Question2

If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the graph of fog crosses the $y - axis$ at 23.

4 points

$(f \circ g)(x) = f(g(x)) = f(3x + a) = 2(3x + a)^2 + 5$

$f \circ g$ crosses the $y - axis$ when $x = 0 \Rightarrow 2a^2 + 5 = 23 \Rightarrow 2a^2 = 18 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$

Question3 If $f(x) = x^2 + 1$ and $g(x) = |x| - 4$, find:

$a)$ $\dfrac{f(1+h) - f(1)}{h}$

3 points

$$\frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 + 1 - 2}{h} = \frac{1 + 2h + h^2 - 1}{h} \equiv \frac{2h + h^2}{h} = \frac{h(2+h)}{h} = 2 + h$$

$b)$ $(f\,o\,g)(2) + (g\,o\,g)(-1)$

3 points

$f(g(2)) + g(g(-1)) = f(-2) + g(-3) = 5 - 1 = 4$

$c)$ The Domain of $\left(\dfrac{f}{\sqrt{g}}\right)(x)$

3 points

$$\frac{f}{\sqrt{g}} = \frac{x^2 + 1}{\sqrt{|x| - 4}}$$

$|x| - 4 > 0 \Rightarrow |x| > 0 \Rightarrow x < -4 \;\; or \;\; x > 4$

$\therefore D = (-\infty, 4) \cup (4, \infty)$

Question 4 If the line L passes through the vertex of the quadratic function $f(x) = -2x^2 + 4x + 1$ and is perpendicular to the line $3y - 2x + 1 = 0$. Write the equation of the line L.

5 points

$* h = \frac{-b}{2a} = 1, k = f(h) = f(1) = 3 \Rightarrow \text{The vertex is (h, k)} = (1,3)$

$* 3y - 2x + 1 = 0 \Rightarrow 3y = 2x - 1 \Rightarrow y = \frac{2}{3}x - \frac{1}{3} \Rightarrow m = \frac{2}{3}$

$* \text{The slope of the required line is } -\frac{3}{2}$

$* \text{ Eq.: } y - 3 = -\frac{3}{2}(x-1)$

Question 5 If the quadratic function $f(x) = 2x^2 + bx + c$ decreases on $(-\infty, 2]$ and has x-intercept $(3,0)$ find the value of $b + c$

5 points

$* h = 2 = \frac{-b}{2a} = \frac{-b}{2(2)} = \frac{-b}{4} \Rightarrow b = -8$

$* f(3) = 0 \Rightarrow 2(9) - 24 + c \Rightarrow c = 6$

$* b + c = -8 + 6 = -2$

Question 6 The graph of the equation $y = |x-2| + 1$ is reflected across the $y-axis$, then shifted 2 units left, then shifted 1 unit down. The equation of the new graph is $y = |ax + b| + c$. Find the value of $a + b + c$.

5 points

$y_1 = |-x-2| + 1$

$y_2 = |-(x+2) - 2| + 1 = |-x-4| + 1 =$

$y_{new} = |-x-4| + 1 - 1 = |-x-4|$

Hence,

$a = -1, b = -3, c = 0 \Rightarrow a + b + c = -5$

Note:

$y_{new} = |-x-4| = |-(x+4)| = |x+4|$

Hence,

$a = 1, b = 3, c = 0 \Rightarrow a + b + c = 5$

One solution is enough.

Question7 Given the equation of a circle $2x^2 + 2y^2 + 8x - 4y + 2 = 0$.

d) Write this equation in the standard form.

3 points

$$x^2 + y^2 + 4x - 2y = -1$$
$$(x^2 + 4x + 4) + (y^2 - 2y + 1) = -1 + 4 + 1$$
$$(x + 2)^2 + (y - 1)^2 = 4$$

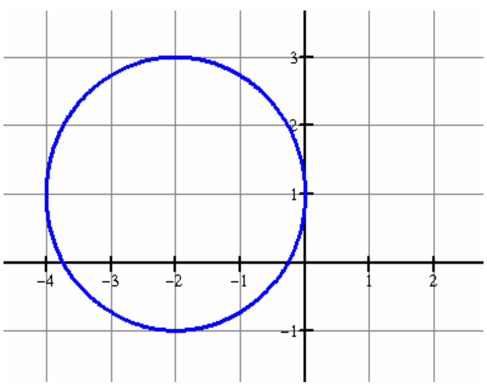
e) Find the center and the radius of this circle.

1 point

$C : (-2,1)$
$r = 2$

f) Determine whether the circle is tangent to the $x-$axis, $y-$axis or both.

2 points



so, $y-$axis only

Question 8

Identify the equation or the set of ordered pair that define y as a function of x.
3 points

a) $\{(2,5),,(-4,3),(-2,5)\}$

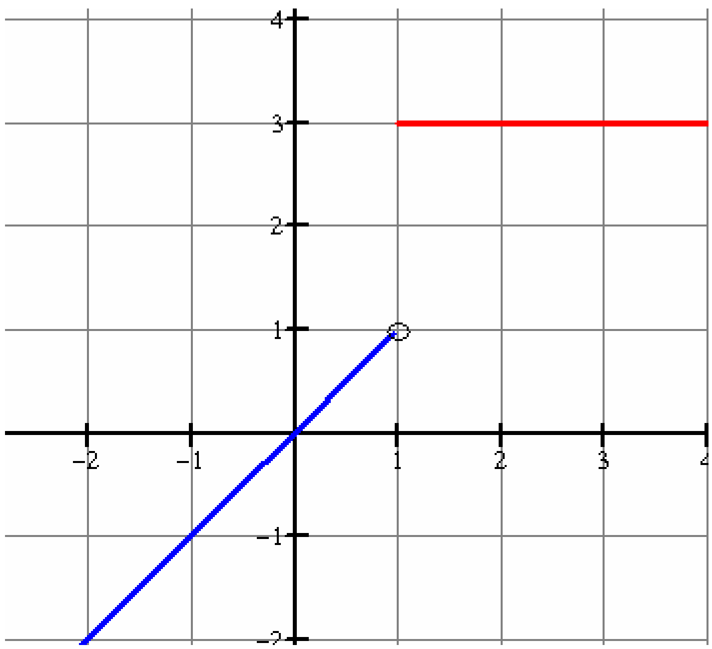
function

b) $|x| + y^3 = 1$
function

<u>Question 9</u>

a) If $g(x)=\begin{cases} x & if \quad x<1 \\ 3 & if \quad x\geq 1\end{cases}$, then find the range of $g(x)$.

3 points



$R=(-\infty,1)\cup\{3\}$

b) If $h(x)=[x]$, where $[x]$ is the greatest integer of less than or equal to x, then find the value of $h(-0.5)+h^2(\sqrt{8})$.

4 points

$[-0.5]+([2.?])^2=-1+(2)^2=-1+4=3$