Practice Exam I Math002

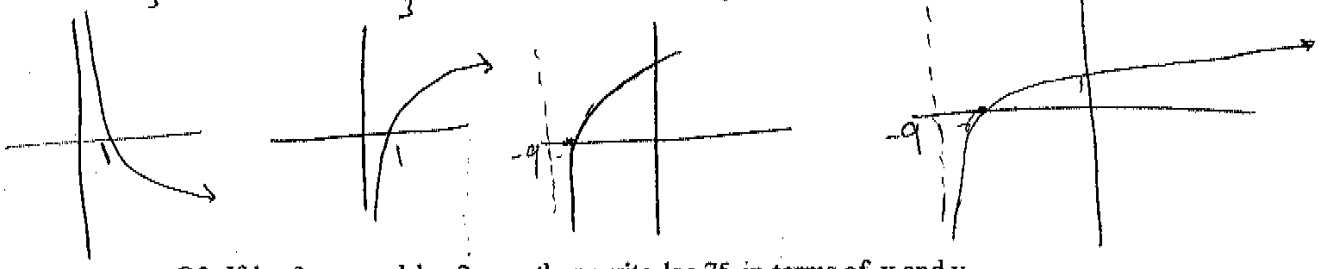
Q1. Given $f(x) = -\log_{\frac{1}{3}}(x+9) - 1$.

a) Find x-intercept and y-intercept.
b) Graph f(x).
c) Find the **domain** and the **range** of f. $D = (-9, \infty)$, $R = (-\infty, \infty)$
d) Find an equation of the asymptote of the graph of f. → V.A. $x = -9$

x-int $0 = -\log_{\frac{1}{3}}(x+9) - 1 \rightarrow \log_{\frac{1}{3}}(x+9) = -1 \rightarrow 3 = x+9 \rightarrow x = -6$

y-int $y = -\log_{\frac{1}{3}}(0+9) - 1 = -\log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{-2} - 1 = -(-2) - 1 = 1$

$y_1 = \log_{\frac{1}{3}} x$ $y_2 = -\log_{\frac{1}{3}} x$ $y_3 = -\log_{\frac{1}{3}}(x+9)$ $f(x) = -\log_{\frac{1}{3}}(x+9) - 1$



Q2. If $\log 3 = x$ and $\log 2 = y$, then write $\log 75$ in terms of x and y.

$\log 75 = \log 3 \cdot 25 = \log 3 + \log 5^2 = x + 2\log 5 =$
$= x + 2\log\frac{10}{2} = x + 2[\log 10 - \log 2]$
$= x + 2[1 - y] = x + 2 - 2y$.

Q3. Find the solution set of $\log(x+4) < 0$.

take antilog for base 10. $\Rightarrow 10^{\log x+4} < 10^0 \Rightarrow x + 4 < 1$

$x < -3$, but domain $\log(x+4)$ is $(-4, \infty)$

∴ Solution Set is $(-4, -3)$.

Q4. Solve $2\log_3(1-x)+\log_1(x-2)=2\log_3 2$

$2\log_3(1-x)+\frac{\log_3(x-2)}{\log_3 3}=\log_3 2^2$

$\Rightarrow \log_3(1-x)^2+\frac{\log_3(x-2)}{-1}=\log_3 4$

$\rightarrow \log_3(1-x)^2-\log_3(x-2)=\log_3 4$

$\rightarrow \log_3\frac{(1-x)^2}{x-2}=\log_3 4$

$\rightarrow \frac{(1-x)^2}{x-2}=4 \rightarrow 1-2x+x^2=4x-8$

$\rightarrow x^2-6x+9=0 \rightarrow (x-3)^2=0$

$x=3$, but 3 is not in domain $\log_3(1-x)$

$\rightarrow$ S.S. ϕ

or No solution.

Q5. Solve $\frac{5^x+5^{-x}}{5^x-5^{-x}}=3$

$\rightarrow 5^x+5^{-x}=3(5^x)-3(5^{-x}) \rightarrow -2\,5^x+4\,5^{-x}=0$ multiply by 5^x both sides

$\rightarrow -2\,5^{2x}+4\,5^0=0 \rightarrow -2\,5^{2x}=-4 \rightarrow 5^{2x}=2$, take ln both sides

$\ln 5^{2x}=\ln 2 \rightarrow 2x\ln 5=\ln 2 \rightarrow 2x=\frac{\ln 2}{\ln 5} \rightarrow x=\frac{\ln 2}{2\ln 5}$

Q6. a) Find the measure of the complement of the angle 54°23'37".

$90^0-54^0\,23'\,37'' \rightarrow$

$89^0\,59'\,60''$

$-\;54^0\,23'\,37''$

$35^0\,36'\,23''$

b) Find the length of an arc that subtends a central angle of 80° in a circle of radius 10 cm.

$S=r\theta$ θ must be in radian $\rightarrow \theta=80^0\cdot\frac{\pi}{180^0}=\frac{4\pi}{9}$ rad

$S=10\cdot\frac{4\pi}{9}=\frac{40\pi}{9}$ cm

Q7. A car with a wheel of radius 13 inches is moving with a speed of 50 mph. Find the angular speed of the wheel in radian per minute.

$V = r\omega$, $V = \frac{50\ mi}{h} = \frac{50 \cdot 5280 \cdot 12}{60}$ in/min

$\omega = \frac{V}{r} = \frac{50 \cdot 5280 \cdot 12}{60 \cdot 13}$ rad/min.

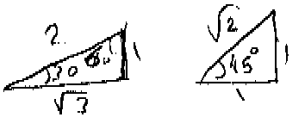
Q8. Find the exact value of each expression

a) $3\cot\frac{\pi}{6} + \sec\frac{\pi}{4}\sin 60°$.

$= 3 \cdot \sqrt{3} + \sqrt{2} \cdot \frac{\sqrt{3}}{2}$

$= 3\sqrt{3} + \frac{\sqrt{6}}{2}$

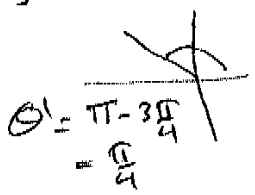
$= \frac{6\sqrt{3} + \sqrt{6}}{2}$



b) $\cos\left(-\frac{59\pi}{4}\right)$

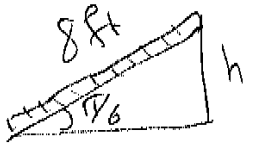
$= \cos\left(\frac{59\pi}{4}\right) = \cos\left[\frac{56\pi + 3\pi}{4}\right] = \cos\left[14\pi + \frac{3\pi}{4}\right] = \cos\frac{3\pi}{4}$

$= -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$



Q9. A 8 – foot ladder is resting against a wall and makes an angle of $\frac{\pi}{6}$ with the ground. Find the height to which the ladder will reach on the wall.

$\sin\frac{\pi}{6} = \frac{h}{8} \rightarrow h = 8\sin\frac{\pi}{6} = 8 \cdot \frac{1}{2} = 4$ ft



Q10 a) Is the function $f(x)=\dfrac{\cot x}{x}$ even, odd, or neither?

$f(-x)=\dfrac{\cot -x}{-x}=\dfrac{-\cot x}{-x}=\dfrac{\cot x}{x}=f(x)$

$\therefore f$ is even function

b) Write $\dfrac{\tan t-\cot t}{\tan t}$ in terms of a single trigonometric function.

$1-\dfrac{\cot t}{\tan t}=1-\dfrac{\cos t}{\sin t}\div\dfrac{\sin t}{\cos t}=1-\dfrac{\cos^2 t}{\sin^2 t}=1-\cot^2 t$

c) If $\cos\theta=-\dfrac{1}{\sqrt{6}}$ and $\tan\theta=\sqrt{5}$; find $\csc\theta$.

$\cos\theta=-\frac{1}{\sqrt{6}}\rightarrow\theta$ in QII, III, $\tan\theta=\sqrt{5}\rightarrow\theta$ in QI, III

$\rightarrow\theta$ in QIII

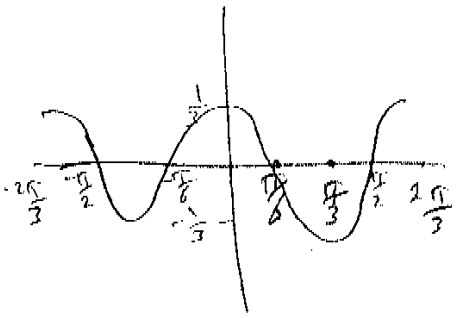
$\sin^2\theta+\cos^2\theta=1\rightarrow\sin^2\theta+\frac{1}{6}=1\rightarrow\sin^2\theta=1-\frac{1}{6}=\frac{5}{6}$

$\sin\theta=-\sqrt{5/6}\rightarrow\csc\theta=-\frac{\sqrt{6}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{-\sqrt{30}}{5}$

Q11. Given $f(x)=\left|\dfrac{1}{3}\cos 3x\right|$.

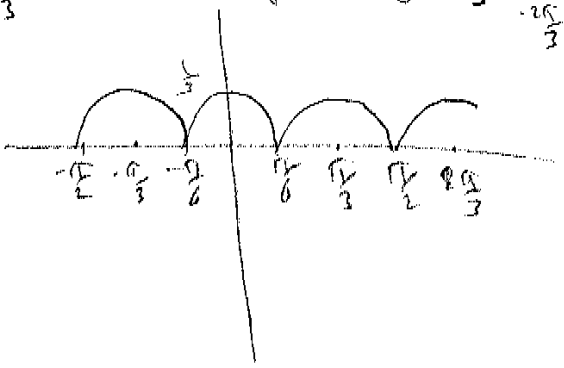
a) Graph f.
b) Find **period** and **amplitude** of the graph of f.
c) Find the range of f.

First graph $y=\frac{1}{3}\cos 3x$ period y $\frac{2\pi}{3}$



Now

(a) $f(x)=\left|\frac{1}{3}\cos 3x\right|$



(b) Period $\frac{\pi}{3}$

Amp $\frac{\frac{1}{3}-0}{2}=\frac{1}{6}$

(c) range $[0,\frac{1}{3}]$