

King Fahd University of Petroleum and Minerals  
Prep Year Math Program  
Math002, Quiz # 2

Name:.....ID:..... Sec:.....

(1) Given  $\csc \alpha = -\frac{5}{4}$ ,  $\alpha$  in quadrant III, and  $\cos\left(\frac{\pi}{2} - \beta\right) = \frac{5}{13}$ ,  $\beta$  in quadrant II, find  $\tan(\alpha + \beta)$

( see the recitation problems of sections 6.16-6.2 for more details)

$$\sin \alpha = -\frac{4}{5} \text{ and } \sin \beta = \frac{5}{13}$$

$$\begin{aligned} \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ &= \frac{-4}{5} \frac{12}{13} + \left(\frac{-3}{5}\right) \frac{5}{13} = \frac{33}{65} \end{aligned}$$

$$\Rightarrow \csc(\alpha + \beta) = \frac{65}{33}$$

(2) Find the exact value of  $\sin 195^\circ + [\sin 13^\circ \sin 73^\circ + \sin 77^\circ \sin 17^\circ]$

$$\begin{aligned} &= -\sin 15 + \cos(90^\circ - 13^\circ) \cos(90^\circ - 73^\circ) + \sin 77^\circ \sin 17^\circ \\ &= -\sin(45^\circ - 30^\circ) + \cos 77^\circ \cos 17^\circ + \sin 77^\circ \sin 17^\circ \\ &= -[\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ] + \cos(77^\circ - 17^\circ) \\ &= -\left[\frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \frac{1}{2}\right] + \cos 60^\circ \\ &= \frac{-\sqrt{6} - \sqrt{2} + 1}{2} \end{aligned}$$

3) Verify the following identity  $\sqrt{\frac{1 + \sin x}{1 - \sin x}} = -\frac{1 + \sin x}{\cos x}$ ,  $\cos x < 0$

$$\sqrt{\frac{1 + \sin x}{1 - \sin x} \frac{1 + \sin x}{1 + \sin x}} = \sqrt{\frac{(1 + \sin x)^2}{\cos^2 x}} = \frac{|1 + \sin x|}{|\cos x|} = \frac{1 + \sin x}{-\cos x}$$