

## 1.7 - Inequalities -

Let  $E(x)$  &  $G(x)$  be 2 expressions,

the  $E(x) < G(x)$  or  $E(x) \leq G(x)$  are inequalities

Objectives: To learn how to solve effectively:

- 1) linear inequalities.
- 2) Three part inequalities.
- 3) Quadratic Inequalities, polynomial inequalities
- 4) Rational inequalities.

Order Properties  $a, b, c \in \mathbb{R}$

$$1) a < b \Rightarrow a + c < b + c \\ c \in \mathbb{R}$$

$$2) a < b \Rightarrow ac < bc \\ c > 0$$

$$3) a < b \Rightarrow ac > bc \\ c < 0 \quad \hookrightarrow \text{change of sign.}$$

Linear Inequalities.

A linear inequality is an inequality

that can be written as

$$ax + b > 0$$

( $a \neq 0$ )

Exp 1. Solve & write the solution in interval notation.

a)  $-3x - 8 \leq 7$

b)  $+\frac{1}{3}x + \frac{2}{5}x - \frac{1}{2}(x+3) \leq \frac{1}{10}$

c)  $2 - 4x + 5(x-1) > -6(x-2)$

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b)

Three part Inequalities.

This is an inequality of the form  $E(x) \leq G(x) \leq F(x)$

Exp 2. Solve

a)  $1 \leq \frac{4x-5}{-2} < 9$

b)  $\frac{1}{3} < 2x-1 \leq \frac{5}{2}$

$x-2$        $-2 \geq 4x-5 > -9$

add 5       $-2+5 \geq 4x > -9+5$

$3 \geq 4x > -4$

$\frac{3}{4} \geq x > -1$

SS =  $(-1, \frac{3}{4}]$

## Quadratic & Polynomial Inequalities.

1) Let  $ax + b$  be a polynomial of degree 1 and let  $r$  be its zero.

$x$	$-\infty$	$r$	$+\infty$	
Sign of $ax + b$ if $a > 0$		-	0	+
Sign of $ax + b$ if $a < 0$		+	0	-

2) Let  $p(x) = ax^2 + bx + c$  have 2 real zeros  $r_1, r_2$  ( $r_1 < r_2$ )

$x$	$-\infty$	$r_1$	$r_2$	$+\infty$		
$ax^2 + bx + c$ if $a > 0$		+	0	-	0	+
$ax^2 + bx + c$ if $a < 0$		-	0	+	0	-

Exp 3. Solve

a)  $3x^2 + x \leq 4$

c)  $x^2 < 9$

e)  $(x+2)^2 > 0$

b)  $x(x+1) \geq 12$

d)  $(x+1)^2 \geq 16$

f)  $(x+2)^2 \leq 0$

Exp 4. Solve

a)  $(3x-4)(x+2)(x+6) \leq 0$

b)  $-(x+1)^2(x+2) < 0$

### Rational Inequalities

If  $R(x)$  &  $S(x)$  are 2 rational Expressions  
 $R(x) \leq S(x)$  or  $R(x) < S(x)$  are  
rational inequalities.

Exp 5. Solve:

a)  $\frac{1-x}{x+2} < -1$

b)  $\frac{7}{x+2} \geq 3$

c)  $\frac{5}{x+1} > \frac{12}{x+1}$

Exp 6. Solve

$$a) \frac{(5-3x)^2}{(2x+5)^3} > 0$$

$$b) \frac{(9x-11)(2x+7)}{(3x-8)^3} < 0$$