- 1.6 - Other Types of Equations & Applications

Objectives: Learn how to solve

- Rational equations
- Equations with radicals
- " " rational exponents
- Quadratic in form Equations

Rational Equations:

Exp1: Solve a) $\frac{2x+5}{2} - \frac{3x}{x-2} = x$

b) $\frac{x}{x-3} = \frac{3}{x-3} + 3$

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a) Multiply by LCD $= 2(x-2)$ $\quad x \neq 2$

$$2(x-2)\left(\frac{2x+5}{2} - \frac{3x}{x-2}\right) = 2(x-2)x$$

$$(x-2)(2x+5) - 6x = 2x^2 - 4x$$

$$2x^2 + x - 10 - 6x = 2x^2 - 4x$$

$$-5x - 10 + 4x = 0$$

$$-x = 10 \Rightarrow \boxed{x = -10}$$

Check

$x = -10 \neq 2 \Rightarrow$ SS $= \{-10\}$

b) $\frac{x}{x-3} = \frac{3}{x-3} + 3$

Domain $= \{x / x \neq 3\}$.

$(x-3)\ \frac{x}{x-3} = (x-3)\left(\frac{3}{x-3} + 3\right)$

$x = 3 + 3(x-3) = 3 + 3x - 9 = 3x - 6$

$-2x = -6 \Rightarrow \underline{x = 3} \notin \mathcal{D}$

$\Rightarrow \boxed{SS = \phi}$

Exp 2. Solve

a) $\frac{2x+1}{x-2} + \frac{3}{x} = \frac{-6}{x^2-2x}$

b) $\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$

Equations with Radicals.

Power Property. $n \in \mathbb{N}$

Any solution of $P(x) = Q(x)$ is a sol^n of $P^n(x) = Q^n(x)$.

Caution

$P^n(x) = Q^n(x) \not\Rightarrow P(x) = Q(x)$.

Eg. $P(x) = 2$ $\quad Q(x) = x$

$P(x) = Q(x)$

$2 = x$

$P^2(x) = Q^2(x)$

$4 = x^2$

$x = \pm 2$

Exp 3: Solve

$\sqrt{4x+13} - 2x = -1$

$\sqrt{4x+13} = 2x - 1$

Square both sides $\quad 4x + 13 = (2x - 1)^2$

$4x + 13 = 4x^2 - 4x + 1$

$4x^2 - 8x - 12 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, \quad x = -1$

Check $\quad x = 3 \quad \sqrt{4(3)+13} - 2(3) = -1$ ✓

$x = -1 \quad \sqrt{4(-1)+13} - 2(-1) = -1$ ✗

$SS = \{3\}$

Exp 4 Solve $\sqrt{4x+1} - \sqrt{x-1} = 2$

Exp 5 $\sqrt[3]{4x+3} = \sqrt[3]{2x-1}$

Exp 6. $x^{3/5} = 27$, $(x^2+24x)^{1/4} = 3$

Quadratic In form Equations.

An eqn that is of the form

$$a(u(x))^2 + b(u(x)) + c = 0$$

Exp. Solve

a) $2x^4 - 7x^2 + 5 = 0$

b) $(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$

c) $7x^{-2} - 10x^{-1} - 8 = 0$

Exp. Solve for the specified

1) $x^{2/3} + y^{2/3} = a^{2/3}$ for y

2) $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$