1.4. Quadratic Equations

Objectives:

1. Quadratic Equations, Zero-Product Property, Square root Property.
2. Completing the square.
3. Quadratic Formula.
4. Discriminant.

A quadratic equation is an eq^n of the form

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

Zero-Product Property.

If A & B are 2 numbers (real or complex)

$$A \cdot B \Rightarrow A = 0 \quad \text{or} \quad B = 0$$

Application: (Solving by factoring)

Exp 1. Solve $6x^2 + 7x = 3$

$$6x^2 + 7x - 3 = 0$$

$$(3x - 1)(2x + 3) = 0$$

$$\Rightarrow 3x - 1 = 0 \quad \text{or} \quad 2x + 3 = 0$$

$$\boxed{x = 1/3} \qquad \boxed{x = -2/3}$$

Square Root Property.

If X & k are 2 numbers, then

$$X^2 = k \implies \underline{x = \sqrt{k}} \quad \text{or} \quad x = -\sqrt{k}$$

Exp2 Solve

i) $x^2 = 32$ ii) $x^2 = \underline{-8}$ iii) $(x+3)^2 = 12$

Completing the square method.

We use the SQP to solve ~~a~~ quadratic eq^n

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = k$$

$$\left(x + \frac{b}{2a}\right)^2 = k$$

Apply the square root Property.

Exp 3. Solve by completing the square.

a) $x^2 - 7x + 12 = 0$

b) $9x^2 - 12x + 9 = 0$

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a) $x^2 - 7x = -12$

$$x^2 - 7x + \left(\frac{7}{2}\right)^2 = -12 + \left(\frac{7}{2}\right)^2$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{-48 + 49}{4} = \frac{1}{4}$$

$$x - \frac{7}{2} = \pm \frac{1}{2}$$

$$x = \frac{7}{2} \pm \frac{1}{2} \begin{cases} 4 \\ 3 \end{cases}$$

b) $9x^2 - 12x + 9 = 0$

$$9x^2 - 12x = -9$$

$$x^2 - \frac{4}{3}x = -1$$

$$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = -1 + \left(\frac{2}{3}\right)^2$$

$$\left(x - 2/3\right)^2 = \frac{-9 + 4}{9} = -\frac{5}{9}$$

$$x - 2/3 = \pm \frac{\sqrt{-5}}{3} = \pm i \frac{\sqrt{5}}{3}$$

$$x = \frac{2}{3} \pm i \frac{\sqrt{5}}{3} = \frac{2 \pm i\sqrt{5}}{3}$$

Quadratic Formula & Discriminant

$$ax^2 + bx + c = 0 \qquad (a \neq 0)$$

Let's solve it

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left[\frac{\left(\frac{b}{a}\right)}{2} = \frac{b}{2a}\right]$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Let $\Delta = b^2 - 4ac$ called the discriminant of the equation $ax^2+bx+c=0$

1) If $\Delta = 0$ we have one real solⁿ $\boxed{x = -\frac{b}{2a}}$

2) If $\Delta > 0$ " " 2 real solⁿ $\boxed{x = \frac{-b \pm \sqrt{\Delta}}{2a}}$

3) if $\Delta < 0$ " " 2 nonreal complex solⁿˢ

$$\boxed{x = \frac{-b \pm i\sqrt{|\Delta|}}{2a}}$$

Exp. Solve using the quadratic Formula:

a) $-4x^2 = -12x + 11$

b) $x^3 - 8 = 0$

b) $x^3 - 8 = (x-2)(x^2 + 2x + 4) = 0 \Rightarrow \boxed{x = 2}$ or $x^2 + 2x + 4 = 0$

$a = 1,\ b = 2,\ c = 4 \Rightarrow \Delta = 4 - 4(4) = -12 = (i\sqrt{12})^2 = (2i\sqrt{3})^2$

$$x = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 + i\sqrt{3}}$$

Exp. Solve for the specified variable

a) $E = \dfrac{e^2 k}{2r}$ for e

b) $S = 2\pi r h + 2\pi r^2$ for r.

c) $h = -16t^2 + v_0 t + s_0$ for t

Sol^n

c) $16t^2 - v_0 t + h - s_0 = 0$

$a = 16,\quad b = -v_0,\quad c = h - s_0$

$\Delta = b^2 - 4ac = v_0^2 - 4(16)(h - s_0) = v_0^2 - 64(h - s_0)$

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 64(h - s_0)}}{32}$$

Rational & Irrational Roots

If $a, b, c \in \mathbb{Z}$ & $\Delta = b^2 - 4ac > 0$

the equation $ax^2 + bx + c = 0$ has

1) 2 rational sol^{ns} if Δ is a perfect square

2) 2 irrational sol^{n} if $\Delta > 0$ but not perfect square.

Exp. Determine the nbr of sol$^{\underline{ns}}$ & whether they are rational, irrational or nonreal complex.

a) $9x^2 + 11x + 4 = 0$

b) $3x^2 + 5x + 2 = 0$

c) $x^2 - 8x + 16 = 0$