– R7 – Radical Expressions –

Objectives.

- How to simplify radicals & radical expressions
- How to perform & simplify operations on radical expressions
- Rationalizing Denominators.

Definition of $\sqrt[n]{a}$:

If $a \in \mathbb{R}$, $n \in \mathbb{N}$ & $a^{1/n}$ is a defined real nbr,

then $\boxed{\sqrt[n]{a} = a^{1/n}}$

$\sqrt[n]{a}$: principal n^{th} root.

$\sqrt[n]{\ }$ radical   n : index

radicand

For n : odd, $\sqrt[n]{a}$ is defined for $a \in \mathbb{R}$

For n : even, $\sqrt[n]{a}$ is defined for $a \geq 0$.

Exp

i) $\sqrt[5]{-32} = -2$

ii) $\sqrt[4]{16} = 2$

iii) $\sqrt[6]{\frac{64}{729}} = \sqrt[6]{\frac{2^6}{3^6}} = \frac{2}{3}$

iv) $\sqrt[4]{-16}$ undefined as a real nbr.

R7 p2

Properties of nth Roots. If $\sqrt[n]{a}$, $\sqrt[n]{b}$ are defined,

1. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$ Eg. $\sqrt[3]{-8 \cdot 27} = \sqrt[3]{-8}\,\sqrt[3]{27} = (-2)\cdot 3$
$= -6$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ Eg. $\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ Eg. $\sqrt{\sqrt[3]{729}} = \sqrt[6]{729} = 3$

4. $\boxed{\sqrt[n]{a^n} = a \quad \text{if } n \text{ is } \underline{\text{odd}}}$ $\sqrt[3]{(-5)^3} = -5$

5. $\boxed{\sqrt[n]{a^n} = |a| \quad \text{if } n \text{ is } \underline{\text{even}}}$ $\sqrt[4]{(-3)^4} = |-3| = 3$

Exp. Simplify (the variables are real nbrs)

1) $\sqrt[4]{x^4}$ 2) $\sqrt[4]{x^8}$ 3) $\sqrt{16m^8r^6}$

4) $\sqrt{(3x+4)^2}$ 5) $\sqrt{x^2-2x+1}$ 6) $\sqrt[3]{(2-x)^3}$

1) $\sqrt[4]{x^4} = |x|$ 2) $\sqrt[4]{x^8} = \sqrt[4]{(x^2)^4} = |x^2| = x^2$

3) $\sqrt{16m^8r^6}$ 4) $\sqrt{(3x+4)^2} = |3x+4|$

5) $\sqrt{x^2-2x+1} = \sqrt{(x-1)^2} = |x-1|$

Caution.

$\sqrt{a^2+b^2} \neq a+b \quad \because (a+b)^2 \neq a^2+b^2$

$\sqrt[3]{a^3 \pm b^3} \neq a \pm b \quad \because (a \pm b)^n \neq a^n \pm b^n$

Exp. If the variables are positive, simplify

i) $\sqrt{24 m^6 n^5}$ ii) $\sqrt[6]{\frac{r}{s^6}}$ iii) $\sqrt[4]{x^4+y^4}$

A radical expression is an expression with radicals.

A radical expression is simplified if

1) The powers inside the radicand are less than the index
2) No fractions under the radical
3) No radical in denominator (Rationalizing the deno)
4) powers & index have no common factor.

5) Operations performed.

Exp. Simplify

$\sqrt[3]{250} = \sqrt[3]{2 . 5^3} = 5\sqrt[3]{2}$

$\sqrt[3]{81 x^5 y^7 z^6} = \sqrt[3]{3^4 x^3 x^2 y^6 y (z^2)^3} = \boxed{3 x y^2 z^2 \sqrt[3]{3x^2 y}}$

Exp Simplify $x, y > 0$

1) $\sqrt{98x^3y} + 3x\sqrt{32xy}$

2) $\sqrt[3]{64m^4n^5} - \sqrt[3]{-27m^{10}n^4}$

$$\begin{array}{r|l} 98 & 2 \\ 49 & 7 \\ 7 & 1 \\ 1 & \end{array}$$

$32 = 2^5$

1) $\sqrt{98x^3y} + 3x\sqrt{32xy} = \sqrt{2 \cdot 7^2 x^2 \cdot x \cdot y} + 3x\sqrt{2^5 xy}$

$= \cancel{7}x\sqrt{2xy} + 3x(2^2)\sqrt{2xy} = (7x + 12x)\sqrt{2xy}$

$= \boxed{19x\sqrt{2xy}}$

2) $\sqrt[3]{4^3 m^3 m\, n^3 n^2} - \sqrt[3]{(-3)^3 (m^3)^3 m \cdot n^3 n^2} = 4mn\sqrt[3]{mn^2}$

$-(-3)m^3 n\sqrt[3]{mn^2} = (4mn + 3m^3n^4)\sqrt[3]{mn^2}$

Exp.

1) $(\sqrt{7} - \sqrt{11})(\sqrt{7} + \sqrt{1\cancel{0}})$

2) $(\sqrt{2} + 3)(\sqrt{8} - 5)$

R7p5

Rationalizing The denominator.

1. $\dfrac{a}{\sqrt{b}} = \dfrac{a}{\sqrt{b}} \cdot \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{a\sqrt{b}}{b}$

2. $\dfrac{a}{\sqrt[3]{b}} = \dfrac{a}{\sqrt[3]{b}} \cdot \dfrac{\sqrt[3]{b^2}}{\sqrt[3]{b^2}} = \dfrac{a\sqrt[3]{b^2}}{b}$

3) $\dfrac{a}{b-\sqrt{c}} = \dfrac{a\cdot(b+\sqrt{c})}{(b-\sqrt{c})(b+\sqrt{c})} = \dfrac{a(b+\sqrt{c})}{b^2-c}$

Exp. Rationalize the denominators.

1) $\dfrac{4}{\sqrt{3}}$ 2) $\sqrt[4]{\dfrac{3}{5}}$, 3) $\dfrac{\sqrt[4]{xy^3}}{\sqrt[4]{x^3y^2}}$

4) $\sqrt[3]{\dfrac{5}{x^6}} - \sqrt[3]{\dfrac{4}{x^9}}$ 5) $\dfrac{\sqrt{7}-1}{2\sqrt{7}+4\sqrt{2}}$

6) $\dfrac{3m}{2-\sqrt{m+n}}$

1) $\dfrac{4\sqrt{3}}{3}$ 2) $\dfrac{\sqrt[4]{3}}{\sqrt[4]{5}} \cdot \dfrac{\sqrt[4]{5^3}}{\sqrt[4]{5^3}} = \dfrac{\sqrt[4]{3\times 5^3}}{5}$

3) $\dfrac{\sqrt[4]{xy^3}\cdot\sqrt[4]{xy^2}}{\sqrt[4]{x^3y^2}\cdot\sqrt[4]{xy^2}} = \dfrac{\sqrt[4]{x^2y^5}}{xy} = \dfrac{y\sqrt[4]{x^2y}}{xy} = \boxed{\dfrac{\sqrt[4]{x^2y}}{x}}$

5) $\dfrac{\sqrt{7}-1}{2\sqrt{7}+4\sqrt{2}} \cdot \dfrac{(2\sqrt{7}-4\sqrt{2})}{(2\sqrt{7}-4\sqrt{2})} = \dfrac{2\cdot 7 - 4\sqrt{14} - 2\sqrt{7} + 4\sqrt{2}}{(2\sqrt{7})^2 - (4\sqrt{2})^2}$

$= \dfrac{14 - 4\sqrt{14} - 2\sqrt{7} + 4\sqrt{2}}{28 - 32} = \dfrac{7 - 2\sqrt{14} - \sqrt{7} + 2\sqrt{2}}{-2}$