

- R6. Rational Exponents -

R6/P1

Objectives.

- Learn to
- 1) Simplify multiplicative Exponential Expressions
 - 2) Simplify complex fractions with negative exponents.
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Integer Exponents.

We already defined a^n for n : natural nbr.

$a \neq 0$, $n \in \mathbb{N}$, $(-n)$ negative integer

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

Exponent Rules for integers

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

An exponential Expression is an expression containing exponents.

To simplify an "exponential expression" is to write it

- 1) Without negative powers.
- 2) Every base appears once.
- 3) No powers of zero.

Exercise. Simplify

$$\frac{(3x^2)^{-1} (3x^5)^{-2}}{(3^{-1} x^{-2})^2}$$

$$\frac{8x^3 y^{-5}}{4x^{-1} y^2}$$

$$\left(\frac{-30 a^{14} b^8}{10 a^{17} b^{-2}} \right)^3$$

Rational Exponents.

We start by defining $a^{1/n}$ for $n \in \mathbb{N}$.

1) $a \in \mathbb{R}$, n : odd

$a^{1/n}$ is the nbr x such that $x^n = a$.

2) $a \geq 0$, n : even

$a^{1/n}$ is the positive nbr x such that $x^n = a$.

If $a < 0$ & n : even, $a^{1/n}$ is not defined as a real nbr.

Exp.

$$(27)^{1/3} = 3$$

$$\text{as } 3^3 = 27$$

$$(-8)^{1/3} = -2$$

$$\text{as } (-2)^3 = -8$$

$$(16)^{1/2} = 4$$

$$4^2 = 16$$

$(-25)^{1/2}$: undefined as real nbr.

$$a^p : p \in \mathbb{Q}$$

Let $a \in \mathbb{R}$ & $p = \frac{m}{n}$ a rational nbr in lowest terms
 & $n \in \mathbb{N}$, $m \in \mathbb{Z}$ & given that $a^{1/n}$ is defined

$$a^{\frac{m}{n}} = (a^{1/n})^m$$

Rules of Exponents for Rational Numbers.

$$r, s \in \mathbb{Q}, \quad a, b > 0$$

$$a^r a^s = a^{r+s}$$

$$(a^r)^s = a^{rs}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$(ab)^r = a^r b^r$$

$$a^{-r} = \frac{1}{a^r}$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

Exp. Evaluate

$$(125)^{2/3}, \quad (32)^{7/5}, \quad -(81)^{3/2}, \quad (-27)^{2/3}, \quad (-4)^{5/2}$$

Exp. Simplify (all variables are positive)

$$\left(\frac{16m^3}{n}\right)^{1/4} \cdot \left(\frac{9n^{-1}}{m^2}\right)^{1/2}, \quad \frac{z^{1/3} z^{-2/3} z^{1/6}}{(z^{-1/6})^3}$$

Exp. Find each product

$$(2z^{1/2} + z)(z^{1/2} - z)$$

$$(r^{1/2} - r^{-1/2})^2$$

$$p^{11/5}(3p^{4/5} + 9p^{19/5})$$

Exp. Factor the given common factor. (all variables are positive)

$$y(p+4)^{-3/2} + (p+4)^{-1/2} + (p+4)^{1/2}$$

$$\text{Factor } (p+4)^{-3/2}$$

$$4x(2x+3)^{-5/9} + 6x^2(2x+3)^{4/9} - 8x^3(2x+3)^{13/9}$$

$$\text{Factor } 2x(2x+3)^{-5/9}$$

Complex Fraction with negative powers.

Exp. Simplify $\frac{x^{-2} + y^{-2}}{x^{-2} - y^{-2}} \cdot \frac{x+y}{x-y}$

$$\frac{a - 16b^{-1}}{(a + 4b^{-1})(a - 4b^{-1})} =$$

Exp Simplify each rational expression.

$$1) \frac{(y^2+2)^5 3y - y^3(6)(y^2+2)^4(3y)}{(y^2+2)^7}$$

$$2) \frac{7(3t+1)^{3/4} - (t-1)(3t+1)^{-3/4}}{(3t+1)^{3/4}}$$