R5. Rational Expressions

Objectives.

1. Define a rational expression and its domain
2. R.E in lowest term
3. Multiplication & division
4) Addition & subtraction & LCD
5. Complex Fractions

A Rational Expression (R.E.) is a quotient of 2 polynomials $P(x)$ & $Q(x)$

$$R(x) = \frac{P(x)}{Q(x)}$$

The domain of $R(x)$ is $\{x \,/\, Q(x) \neq 0\}$

Exp1. a) $\dfrac{x+6}{x-2}$ $\qquad D = \mathbb{R} - \{2\} = (-\infty, 2) \cup (2, \infty)$

b) $\dfrac{x^2+2}{5x(x+3)}$ $\qquad D = \mathbb{R} - \{0, -3\} = (-\infty, -3) \cup (-3, 0) \cup (0, \infty)$

Fundamental Principle of Fractions

$$\frac{ac}{bc} = \frac{a}{b} \qquad (b \neq 0,\ c \neq 0)$$

A fraction or a rational expression is in lowest term if it cannot be simplified anymore.

$$\frac{12}{21} = \frac{2^2 \cdot \cancel{3}}{\cancel{3} \cdot 5} = \frac{4}{5} \rightarrow \text{fraction in lowest terms}$$

$$R(x) = \frac{x^2 - 2x - 3}{x^2 + x} = \frac{(x-3)\cancel{(x+1)}}{x\cancel{(x+1)}} = \frac{x-3}{x} \qquad x \neq -1,\ x \neq 0$$

R(x) in lowest terms.

Exp 1. Write in lowest terms

a) $$\frac{x^3 + 64}{x^2 + 3x - 4} = \frac{(x+4)(x^2 - 4x + 16)}{(x+4)(x-1)} = \boxed{\frac{x^2 - 4x + 16}{x - 1}}$$

$x \neq -4$, $x \neq 1$

b) $$\frac{9 - 3t}{t^2 + 2t - 15} = \frac{3(3-t)}{(t+5)(t-3)} = \frac{(-3)(t-3)}{(t+5)(t-3)} = \boxed{-\frac{3}{(t+5)}}$$

Multiplication & Division.

We perform the operation and simplify.

Exp. Multiply or divide

a) $$\frac{6r-18}{9r^2+6r-24} \div \frac{4r-12}{12r-16}$$

$$\frac{(6r-18)}{(9r^2+6r-24)} \cdot \frac{(12r-16)}{(4r-12)} = \frac{\cancel{6}^{2}\,\cancel{(r-3)}}{\cancel{3}(3r^2+2r-8)} \cdot \frac{\cancel{4}(3r-4)}{\cancel{4}\,\cancel{(r-3)}}$$

$$= \frac{2\,\cancel{(3r-4)}}{\cancel{(3r-4)}(r+2)} = \boxed{\frac{2}{r+2}}$$

b) $$\frac{x^2-y^2}{(x-y)^2} \cdot \frac{x^2-xy+y^2}{x^2-2xy+y^2} \div \frac{x^3+y^3}{(x-y)^4}$$

$$\frac{(x-y)\cancel{(x+y)}}{\cancel{(x-y)^2}} \cdot \frac{\cancel{(x^2-xy+y^2)}}{\cancel{(x-y)^2}} \cdot \frac{\cancel{(x-y)^4}}{\cancel{(x+y)}\,\cancel{(x^2-xy+y^2)}} = \boxed{x-y}$$

Exp Which is -2

$$\frac{2x-2}{2+2x}, \quad \frac{4x-2}{-2+4x}, \quad \frac{2x+1}{2x-1}, \quad \frac{2x-2}{2-2x}$$

Addition & Subtraction

We know that

1) $\frac{P}{Q} + \frac{R}{Q} = \frac{P+R}{Q}$ (same denominator)

2) $\frac{P}{Q} + \frac{R}{S} = \frac{PS}{QS} + \frac{RQ}{QS} = \frac{PS+RQ}{QS}$

In (2) we put the 2 fractions under the same denominator QS, But sometimes there is a smaller or lower degree common denominator.

Least Common Denominator (LCD) of 2 or more fractions.

To find the LCD

1) Write each denominator in prime factors (ie. factor completely)

2) Form a product containing every factor with the greatest exponent.

Exp $\frac{7}{6} + \frac{4}{9}$

a) without use of LCD. $\frac{7\times 9}{6\times 9} + \frac{4\times 6}{6\times 9} = \frac{63+24}{54} = \frac{87}{54} = \frac{29}{18}$

b) with LCD. $6 = 2\times 3$ $\quad 9 = 3^2$ $\Rightarrow LCD = 2\times 3^2 = 18$

$$\frac{7}{6} + \frac{4}{9} = \frac{7\times 3}{18} + \frac{4\times 2}{18} = \frac{21+8}{18} = \boxed{\frac{29}{18}}$$

R5 p5

<u>Exp</u> Add or Subtract

a) $\frac{8}{3p} + \frac{5}{4p} + \frac{9}{2p}$

LCD $= 3 \cdot 2^2 p$
$= 12p$

so $= \frac{8 \times 4}{12p} + \frac{5 \times 3}{12p} + \frac{9 \times 6}{12p} = \frac{32 + 15 + 54}{12p}$

$= \frac{101}{12p}$

b) $\frac{4}{p-q} - \frac{2}{q-p} = \frac{4}{p-q} - \frac{2}{-(p-q)} = \frac{4+2}{p-q} = \boxed{\frac{6}{p-q}}$

c) $\frac{4}{x+1} + \frac{1}{x^2-x+1} - \frac{12}{x^3+1}$

$= \frac{4}{x+1} + \frac{1}{x^2-x+1} - \frac{12}{(x+1)(x^2-x+1)}$

LCD $= (x+1)(x^2-x+1)$

$= \frac{4(x^2-x+1)}{(x+1)(x^2-x+1)} + \frac{1x(x+1)}{(x+1)(x^2-x+1)} - \frac{12}{(x+1)(x^2-x+1)}$

$= \frac{4x^2-4x+4 \quad +x+1 \quad -12}{(x+1)(x^2-x+1)} = \frac{4x^2-3x-7}{(x+1)(x^2-x+1)}$

$= \frac{(4x-7)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)} = \boxed{\frac{4x-7}{x^2-x+9}}$

Complex Fractions

A complex fraction is a fraction containing fraction(s) in the numerator, denominator or both.

Simplifying a complex fraction means writing as a simplex fraction $\frac{P}{Q}$, P, Q polynomials.

To do that, we have 2 methods

1) Simplify the num & the deno and divide.
2) Multiply the num & deno by the LCD of all fractions.

Exp. Simplify

a) $$\frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x}} = \frac{\frac{x-(x+1)}{x(x+1)}}{1/x} = \frac{\frac{-1}{x(x+1)}}{1/x} = \frac{-1}{x(x+1)} \div \frac{1}{x}$$

$$= \frac{-1}{\cancel{x}(x+1)} \cdot \cancel{x} = \boxed{-\frac{1}{x+1}}$$ (1st Method)

b) $$\frac{m - \frac{1}{m^2-4}}{\frac{1}{m+2}}$$ LCD $=(m-2)(m+2)$

R5 p7

$$\frac{m - \frac{1}{m^2-4}}{\frac{1}{m+2}} = \frac{(m-2)(m+2)\left(m - \frac{1}{(m-2)(m-2)}\right)}{(m-2)\cancel{(m+2)}\left(\frac{1}{\cancel{m+2}}\right)}$$

$$= \frac{m(m-2)(m+2) - 1}{m-2} = \boxed{\frac{m^3 - 4m - 1}{m-2}}$$

c) $$\frac{\frac{1}{x^3-y^3}}{\frac{1}{x^2-y^2}} = \frac{1}{x^3-y^3}\cdot\left(x^2-y^2\right) = \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}(x^2+xy+y^2)}$$

$$= \boxed{\frac{x+y}{x^2+xy+y^2}}$$

LCD

d) $$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{(x+h)x\left(\frac{1}{x+h} - \frac{1}{x}\right)}{(x+h)(x)\ h}$$

$$= \frac{x - (x+h)}{x(x+h)h} = \frac{-h}{x(x+h)h} = \boxed{\frac{-1}{x(x+h)}}$$