3.5. Rational Functions.

Objectives.
To study the rational functions of the form $f(x) = \frac{ax+b}{cx+d}$, find its main features & graph them.

Def^n. A rational f^n is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials.

Eg. $f(x) = \frac{1}{x}$, $\frac{2x+3}{x^2+1}$, $\frac{7x+2}{1-x}$

The domain of $f(x) = \{x / q(x) \neq 0\}$

How does the graph look like near the point where f is not defined?

Another question, how does the graph look like at the far left & far right?

Exp $f(x) = \frac{1}{x}$

x	$f(x)$
-2	$-1/2$
0	U
2	$1/2$

$x \to 0^+$ $\quad f(x) \to +\infty$

$x \to 0^-$ $\quad f(x) \to -\infty$

~~little~~ $x=0$ is called A vertical Asymptote

$x \to +\infty \Rightarrow f(x) \to 0$

$x \to 0 \Rightarrow f(x) \to 0$

So line $y=0$ is called a horizontal asymptote.

Asymptotes are an important feature of all rational functions.

How to find the Vertical Asymptotes.

If $f(x) = \frac{p(x)}{q(x)}$ in lowest terms, Then the vertical asymptotes are at the zeros of $q(x)$.

How to find the Horizontal asymptotes.

If $\frac{p(x)}{q(x)} = f(x)$ in lowest terms, then

1. If $\deg p < \deg q(x) \Rightarrow y=0$ is H.A
2. If $\deg p(x) = \deg q(x) \Rightarrow y = \frac{a_n}{b_m}$ is HA where a_n, b_m leading coefficients of $p(x)$ & $q(x)$ respectively.
3. If $\deg p(x) > \deg q(x) \Rightarrow$ No H.A.

Exp. Find the V.A & H.A of the following functions

a) $f(x) = \frac{-2}{x+1}$

b) $f(x) = \frac{3x-1}{5-2x}$

c) $f(x) = \frac{x+1}{x^2+x-2}$

d) $g(x) = \frac{3x^2-1}{x+2}$

a) V.A $\quad x+1=0$

$x=-1 \rightarrow$ V.A

H.A. $\quad n=0 < m=1$

$y=0$ H.A

b) V.A : $x = \frac{5}{2}$

H.A : $y = -\frac{3}{2}$

c) $x^2+x-2 = (x+2)(x-1)$

$\Rightarrow x=-2, x=1$ V.A.

$n=1, < m=2 \Rightarrow y=0$

d) $x=-2$ V.A

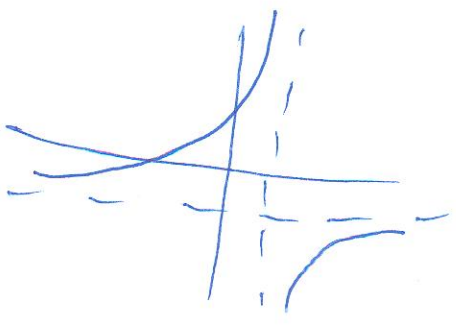
H.A : None.

Remark . One H.A only for any rational function.

. V.A can be many.

. Graph can cross H.A but never crosses a V.A.

. graph of $f(x) = \frac{ax+b}{cx+d}$ is



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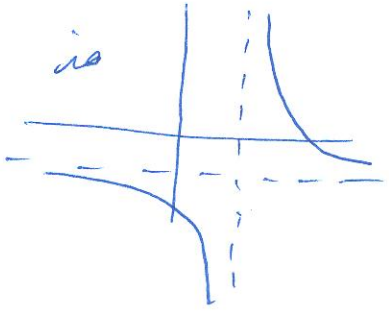
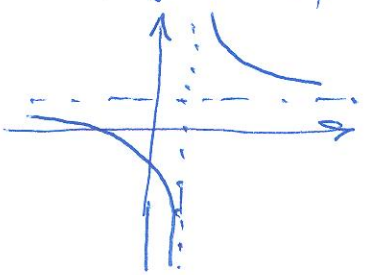
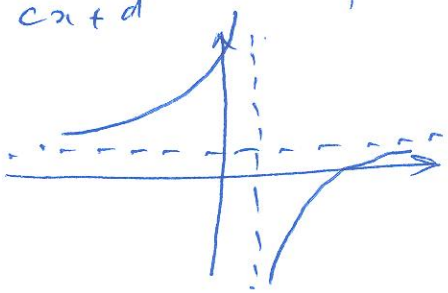


Steps for graphing a rational function $f(x)=\frac{ax+b}{cx+d}$ $(c\neq 0)$

1. Finding the V. A
2. Finding the H.A.
3. Finding the x-int & the y-int.
4. Finding some extra points * (for accuracy)
5. Putting the V.A, H A, x int, y-int, extra points in the coordinate system.
6. Because the graph of $f(x)=\frac{ax+b}{cx+d}$ is of the form  or 

the x-int, & other pts will show which form it is.

Exp. Graph $f(x)=\frac{2x+3}{x-3}$

1. V.A? $x-3=0$

$x=3$ V.A

2. H.A. $n=m=1$

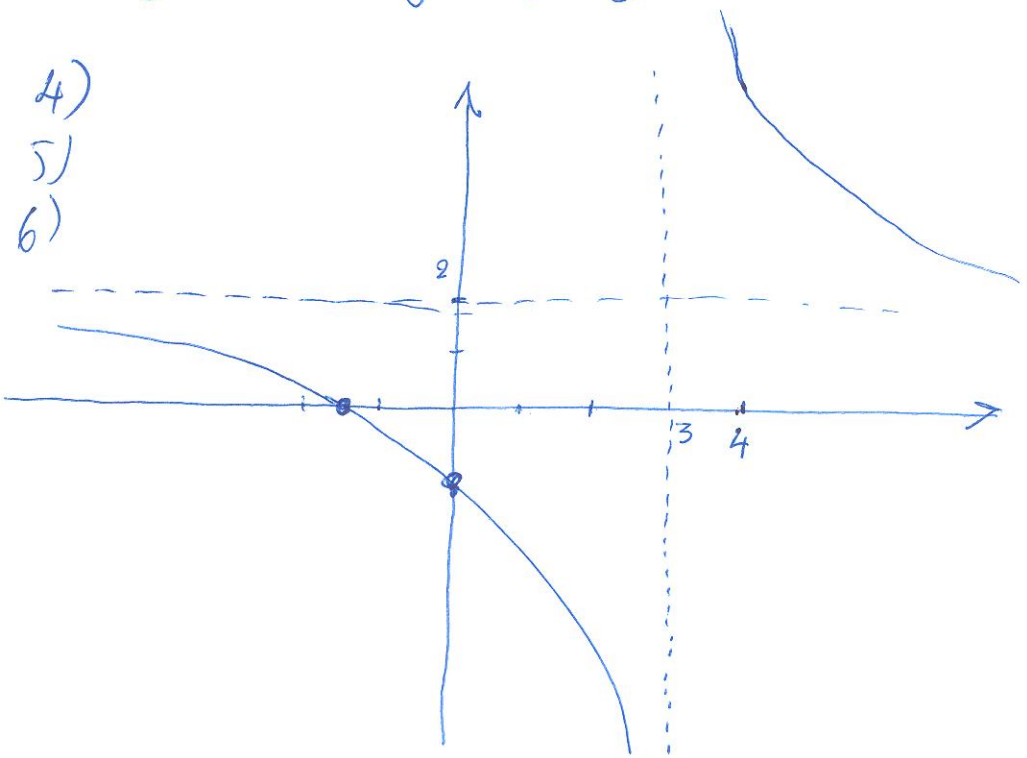
$\Rightarrow y=\frac{2}{1}=2$ H.A

3) a) x-int $\quad \frac{2x+3}{x-3} = 0 \iff 2x+3=0$

$\iff \boxed{x = -3/2} \longrightarrow$ x-int

b) y-int $\quad y = \frac{2(0)+3}{0-3} = \frac{3}{-3} = \boxed{-1}$ y-int

4)
5)
6)



Exp Graph $f(x) = \frac{3x+2}{1-x}$

V. A $\quad x = 1$

H. A $\quad y = -3$

x-int $\quad x = -2/3$

y-int $\quad y = 2$