3.1 Quadratic Functions & Models

Objectives

Learn how to graph a quadratic fn, find its vertex, its maximum, minimum, range.

A quadratic function is a function of the form

$$f(x) = ax^2 + bx + c \qquad a \neq 0$$

Exp $f(x) = x^2$ $\qquad a=1,\ b=0,\ c=0$

$g(x) = -2x^2 + 3x - 1$ $\qquad a=-2,\ b=3,\ c=-1$

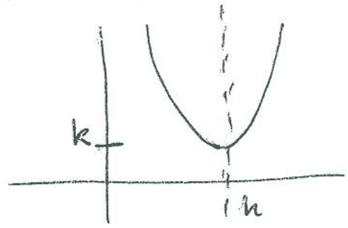
Standard form

Any quadratic function $f(x) = ax^2 + bx + c$ can be written, by completing the square, as

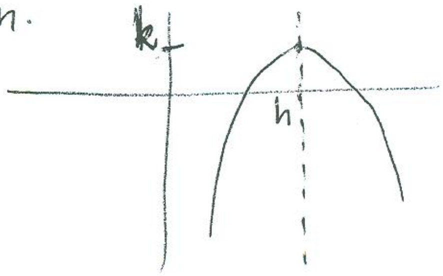
$$\boxed{f(x) = a(x-h)^2 + k}$$

Its graph is called a parabola with vertex (h, k)

& if $a > 0$ it opens up



& if $a < 0$ it opens down.



The line $x = h$ is an axis of symmetry.

Exp 1. Write in Standard Form & Sketch the graph of

a) $f(x) = 2x^2 - 12x + 23$

b) $g(x) = -3x^2 - 6x + 4$

---

a) $f(x) = 2(x^2 - 6x) + 23$

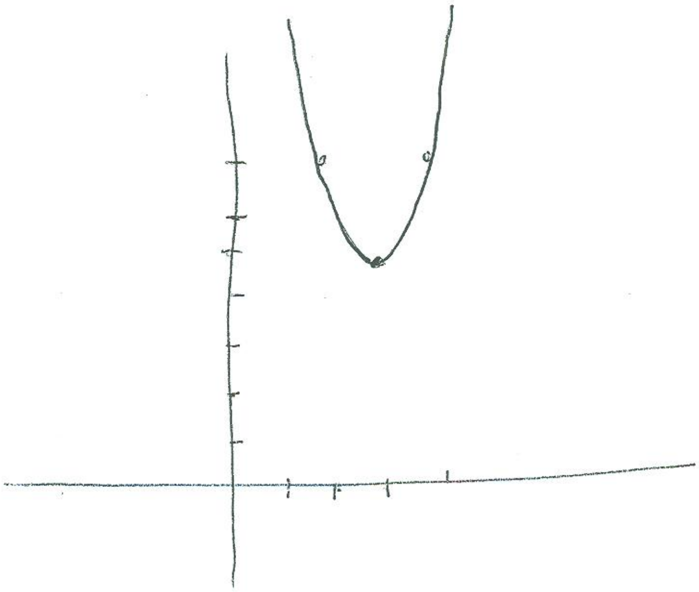
$= 2(x^2 - 6x + 3^2) + 23 - 18$

$= 2(x-3)^2 + 5$

$a = 2 > 0$ opens up

Vertex: $(3,5)$

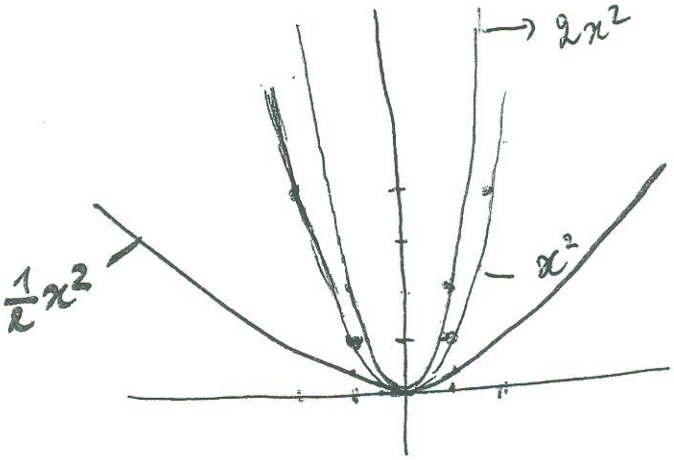
x	y
2	7
3	5
4	7



b) do it yourself.

Effect of the coefficient a.

a plays a role "similar" to the "slope".



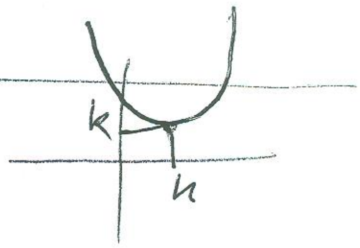
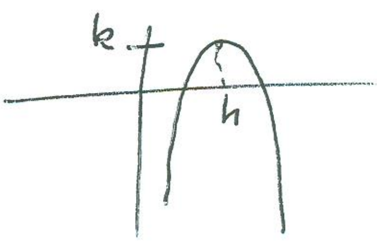
Maximum, Minimum Value & Range

Let $f(x) = a(x-h)^2 + k$, the minimum or maximum value of f occurs at $x = h$,

Moreover,

If $a > 0$, the minimum value is $f(h) = k$

If $a < 0$, f has no minimum value, & its maximum value $f(h) = k$

	Max	Min	Range	
$a > 0$	None	$f(h) = k$	$[k, \infty)$	
$a < 0$	$f(h) = k$	None	$(-\infty, k]$	

Formulas for the vertex, max, min

$$f(x) = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - \frac{b^2}{4a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

$$\boxed{h = -\frac{b}{2a}}$$

$$\boxed{k = f(h) = \frac{4ac - b^2}{4a}}$$

Exp. Find the vertex, max, min, range,

a) $f(x) = x^2 + 6x + 5$

b) $g(x) = 2x^2 - 4x + 5$

---

b) $a = 2$, $b = -4$, $c = 5$

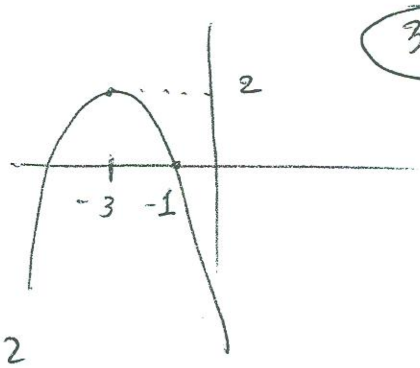
$$h = -\frac{b}{2a} = -\frac{-4}{2(2)} = 1$$

$$k = f(h) = f(1) = 2(1)^2 - 4(1) + 5 = 3$$

$a > 0 \Rightarrow \min = k = 3$.

range : $[3, \infty)$.

Exp. Find the equation of



$$f(x) = a\,(x-(-3))^2 + 2 = a(x+3)^2 + 2$$

$$0 = a(-1+3)^2 + 2 = 4a + 2$$

$$\Rightarrow a = \frac{-2}{4} = -\tfrac{1}{2}$$

$$\Rightarrow \boxed{f(x) = -\tfrac{1}{2}(x+3)^2 + 2}$$

Exp. Find two nbrs whose sum is 32 and whose product is the maximum possible.

Exp. Exp 56 p 296 or Ex 58 p 297.