— 2.8. Function Operations & Composition —

Objectives.

- To define the basic operations on functions.
- To define the composition of functions & its domain.

Basic Operations.

Addition. $(f+g)(x) = f(x) + g(x)$

$$D(f+g) = D(f) \cap D(g)$$

Multiplication. $(f \cdot g)(x) = f(x) \cdot g(x)$

$$D(fg) = D(f) \cap D(g)$$

Subtraction. $(f-g)(x) = f(x) - g(x)$

$$D(f-g) = D(f) \cap D(g)$$

Division. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$D(f/g) = D(f) \cap D(g) - \{x / g(x) = 0\}$$

Exp 1 $f(x) = x^2 + 1$, $g(x) = \sqrt{x+1}$, $h(x) = \frac{x+4}{2x-4}$

a) Find $(f+g)(1)$, $(f-h)(4)$, $(f \cdot g)(3)$,

$\frac{f}{g}(1)$

b) Find the domains of $(f+g)$, $(f \cdot h)$

$\frac{g}{h}$.

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a) $(f+g)(1) = ((1)^2+1) + \sqrt{1+1} = 2+\sqrt{2}$

$(f-h)(4) = f(4) - h(4) = 17 + \frac{8}{4} = 17+2 = 19$

b) $\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \mathbb{R} \cap [-1, \infty) = [-1, \infty)$

$\mathcal{D}(f \cdot h) = \mathcal{D}(f) \cap \mathcal{D}(h) = \mathbb{R} \cap (\mathbb{R} - \{2\}) = \mathbb{R} - \{2\}$

$\mathcal{D}(g/h) = \mathcal{D}(g) \cap \mathcal{D}(h) - \{x / h(x) = 0\}$

$= [-1, \infty) \cap (\mathbb{R} - \{2\}) - \{-4\}$

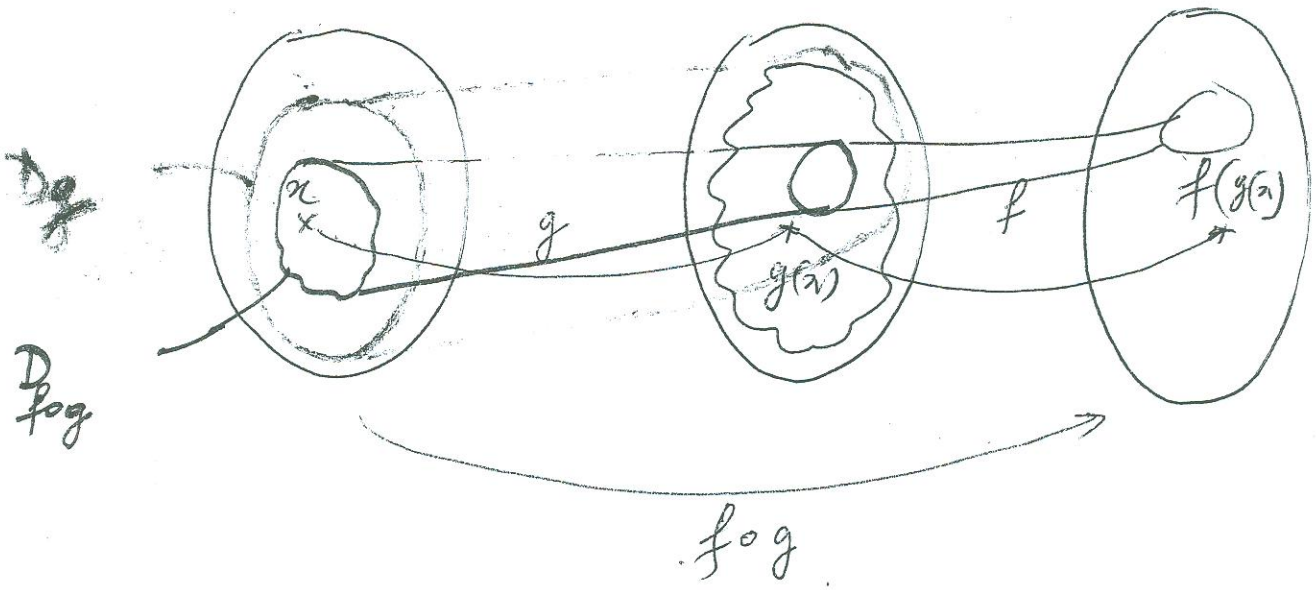
$= [-1, 2) \cup (2, \infty)$

Composition

$$(f \circ g)(x) = f(g(x))$$

In general

$$f \circ g \neq g \circ f$$



$$D(f \circ g) = \{ x \in D(g) \ / \ g(x) \in D_f \}$$

$$\text{or} = D_g \cap \{ x / g(x) \in D_f \}$$

Exp. For the following couples of functions $f \circ g$ Compute $f \circ g$, $g \circ f$ & their domains.

a) $f(x) = \sqrt{x-2}$, $g(x) = 2x$

b) $f(x) = \sqrt{x+4}$, $g(x) = -\frac{2}{x}$

c) $f(x) = \frac{1}{x+4}$ $g(x) = \frac{1}{x}$

a) $(f \circ g)(x) = f(g(x)) = f(2x) = \sqrt{2x-2}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-2}) = 2\sqrt{x-2}$

$\mathcal{D}_{f\circ g} = \{x \in \mathcal{D}_g \;/\; g(x) \in \mathcal{D}_f\} = \{x \in \mathbb{R} \;/\; 2x \in [2,\infty)\}$

$x \in [1,\infty)$

$= [1,\infty)$

$\mathcal{D}_{g\circ f} = \{x \in \mathcal{D}_f \;/\; g(x) \in \mathcal{D}_g\} = [2,\infty)$

$2x \in \mathbb{R}$

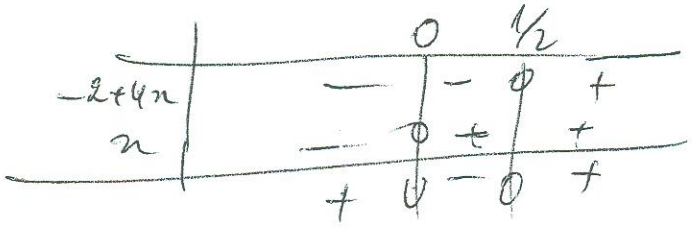
b) $\mathcal{D}_f = [-4,\infty)$, $\mathcal{D}_g = \mathbb{R} - \{0\}$

$(f \circ g)(x) = f(g(x)) = f(-\frac{2}{x}) = \sqrt{-\frac{2}{x}+4} = \sqrt{\frac{-2+4x}{x}}$

$\mathcal{D}_{f\circ g} = D_g \cap \{x \;/\; g(x) \in D_f\}$

$= \mathbb{R} - \{0\} \cap \{x \;/\; -\frac{2}{x} \in [-4,\infty)\}$

$-\frac{2}{x} \geq -4 \iff -\frac{2+4x}{x} \geq 0$



$\Rightarrow \mathcal{D} = (0, \frac{1}{2}]$

Exp. Find f & g such that $h(x) = (f \circ g)(x)$

a) $h(x) = (11x^2 + 12x)^2$

b) $h(x) = \sqrt{x^2 - 1}$

c) $h(x) = \sqrt{3x} - 2$

d) $h(x) = x^6 - 2x^3 - 2$

e) $h(x) = \dfrac{-1}{\sqrt[3]{4x - 2}}$

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d) Take $g(x) = x^3$, $f(x) = x^2 - 2x - 2$

$(f \circ g)(x) = x^6 - 2x^3 - 2$

e) $h(x) = \dfrac{-1}{\sqrt[3]{4x - 2}}$

take $g = \sqrt[3]{4x - 2}$ & $f(x) = \dfrac{1}{x}$

$f(x) = \dfrac{1}{\sqrt[3]{x}}$

or take $g(x) = 4x - 2$

c) $h(x) = \sqrt{3x} - 2$

$f(x) = x - 2$ $\quad g(x) = \sqrt{3x}$.