— 2.7. Graphing Techniques —

Stretching & Shrinking.

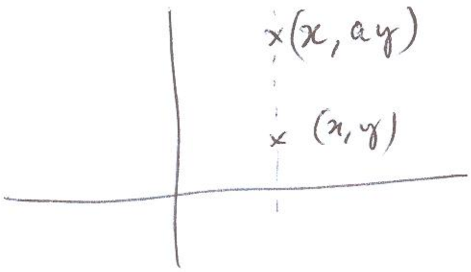
We have two types, horizontal & Vertical.

Vertical Stretching / Shrinking.

This is an "operation" which takes a point of the plane to another point.

Let $a > 0$,

If $a > 1$, Then transforming a pt (x, y) to (x, ay) is a vertical stretching by a.



If $0 < a < 1$ it is called a vertical shrinking by a.

Exp 1/ i) If $a = 2$

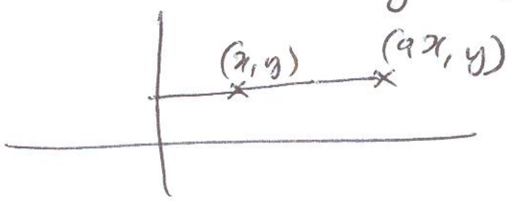
The point $(1, 4)$ goes to $(1, 8)$ by the vertical stretching by 2.

ii) If $a = \frac{1}{3}$, $(1, 4)$ goes to $(1, \frac{4}{3})$

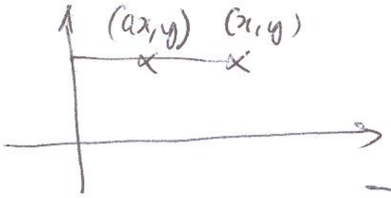
The Horizontal Stretching or Shrinking by a is

$(x, y) \longrightarrow (ax, y)$

If $a > 1$ we call it horizontal stretching by a



If $0 < a < 1$, we call it horizontal shrinking by a.



Question. If the graph of a function is Stretched or Shrunk horizontally or vertically, how does the equation change?

The graph of $y = f(x)$ after	The new eq^n is
a vertical Stretching/Shrink by a	$y = af(x)$
A horizontal Stretching or Shrinking by a	$y = f\left(\frac{x}{a}\right)$

Exp2. The graph of $y = 2x^2 - 3x + 2$ is

a) Stretched vertically by 3

b) Shrunk vertically by $\frac{1}{2}$

c) Stretched horizontally by 5

d) Shrunk horizontally by $\frac{1}{3}$

Find the equation of the new graph for each case.

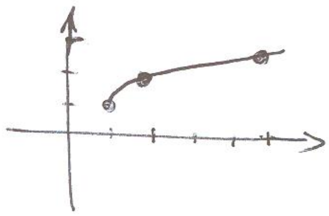
a) $y = 3(2x^2 - 3x + 2) = 6x^2 - 9x + 6$

b) $y = \frac{1}{2}(2x^2 - 3x + 2) = x^2 - \frac{3}{2}x + 1$

c) $y = 2\left(\frac{x}{5}\right)^2 - 3\left(\frac{x}{5}\right) + 2 = 2\frac{x^2}{25} - \frac{3}{5}x + 2$

d) $y = 2(3x)^2 - 3(3x) + 2 = 18x^2 - 9x + 2$

Exp3. Using the graph of $= \sqrt{x-1} + 1$



Graph a) $y = 3\sqrt{x-1} + 3$

b) $y = \sqrt{\frac{x}{2} - 1} + 1$

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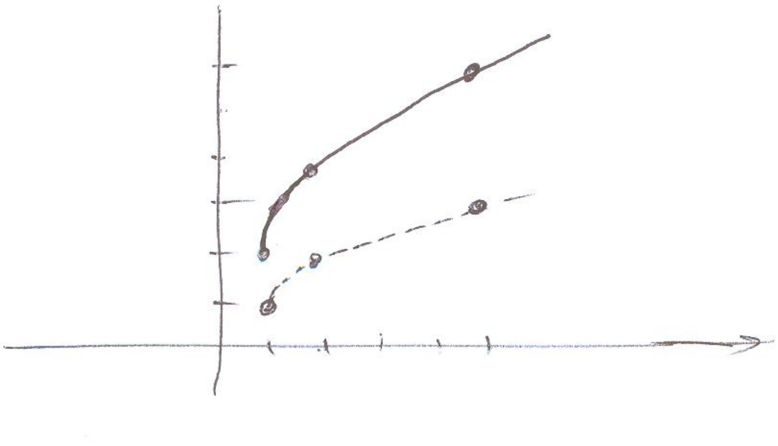
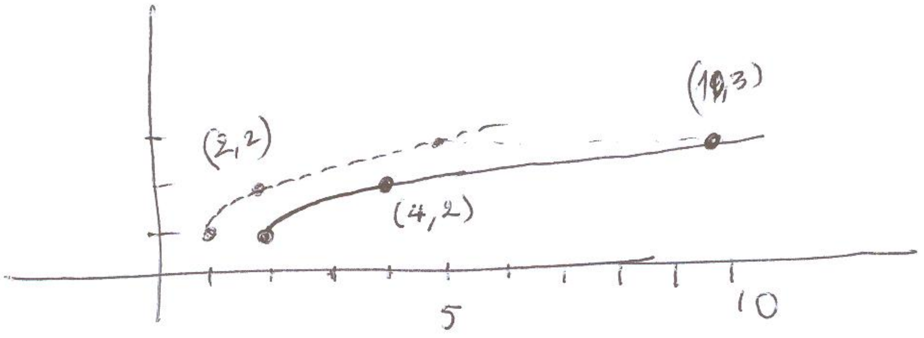
a) $y = 2f(x) \Rightarrow$ Vertical Streching by 2



b) $y = \sqrt{\frac{x}{2} - 1} + 1 = f\left(\frac{x}{2}\right)$

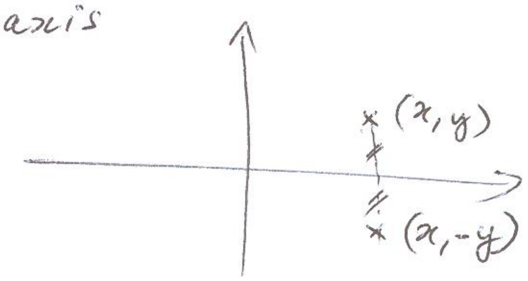
Horizontal stretching by $a = 2$



Reflections

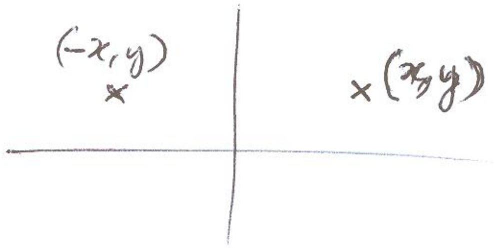
1) Reflection with respect to x-axis

$(x, y) \longrightarrow (x, -y)$



2) Reflection w.r.t y-axis

$(x, y) \longrightarrow (-x, y)$



The graph of $y = f(x)$ after	The new equation
a reflection w.r.t x-axis	$y = -f(x)$
a reflection w.r.t y-axis	$y = f(-x)$

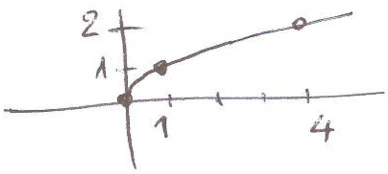
Exp 4. a) Use the graph of $\sqrt{x}$, to graph $y = -\sqrt{x}$

& $h(x) = \sqrt{-x}$.

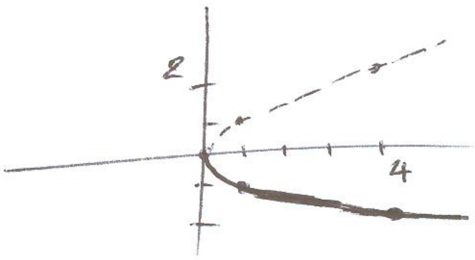
b) Graph $f(x) = -x^3$, $g(x) = \sqrt{(-x)+1} + 1$

a)

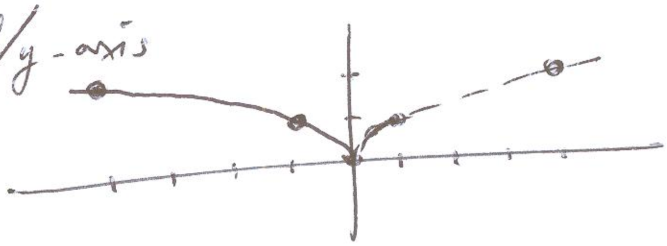
$f(x) = \sqrt{x}$



$y = -\sqrt{x} = -f(x)$ Ref/x-axis



$h(x) = \sqrt{-x} = f(-x) \longrightarrow$ Ref/y-axis

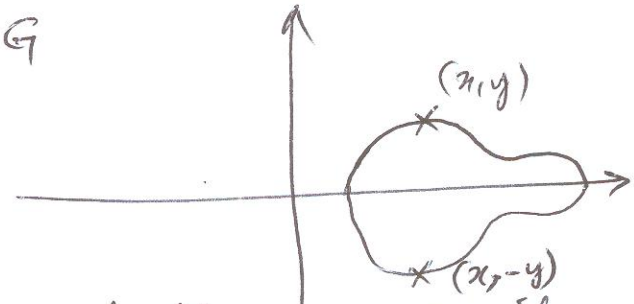


b)

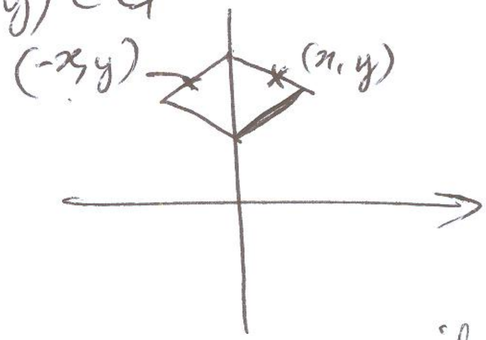
Symmetry.

Symmetry is a property of graphs. We have 3 types.

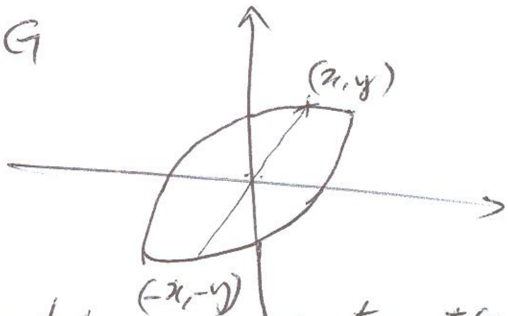
1) A graph G is symmetric w.r.t the x-axis if for any $(x,y) \in G \Rightarrow (x,-y) \in G$



2) A graph G is symmetric w.r.t the y-axis if for any $(x,y) \in G \Rightarrow (-x,y) \in G$



3) A graph G is symmetric w.r.t the origin if for any $(x,y) \in G \Rightarrow (-x,-y) \in G$



Criteria for symmetry.

G is defined by an equation. $E(x,y)=0$

1) G is symmetric w.r.t x-axis if the eq^n is unchanged if (y) is replaced by $(-y)$

2) G is symmetric w.r.t y-axis if the eq^n is unchanged if (x) is replaced by $(-x)$

3) G is symmetric w.r.t the origin if the eq^n is unchanged if x, y are replaced by $(-x), (-y)$.

Rules. If the eq^n has

1) Only x^{2n}, $|x|$ in the terms of x $\Rightarrow$ sym / y-axis
2) Only y^{2n}, $|y|$ in y-terms $\Rightarrow$ sym / x-axis
3) Only x^{2n}, $|x|$, y^{2n}, $|y|$ $\Rightarrow$ sym / y-axis

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Exp 5. Determine with respect of what the graphs of the following equations are symmetric.

a) $y = x^2$, b) $y = x^3$, c) $x^2 + y^2 = 4$

d) $|y - x| = 0$

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a) $y \to -y$ $\quad -y = x^2$ Not same eq^n $\Rightarrow$ No sym / x-axis

$x \to -x$ $\quad y = (-x)^2 \Leftrightarrow y = x^2 \Rightarrow$ Sym / y-axis

$x \to -x, y \to -y$ $\quad -y = (-x)^2 \Leftrightarrow -y = x^2$ not same

$\Rightarrow$ Not sym / origin

b) $y = x^3$ sym / origin only

Even & Odd Functions:

1) A f^n f is even f^n if $f(-x) = f(x)$ for any x.
2) A f^n f is odd function if $f(-x) = -f(x)$ for any x.

Properties:

1) x^2, $|x|$, x^{2n}, constant are even f^{ns}
2) x^3, x^{2n+1} are odd f^n

Operations on even & odd functions.

e: even f^n, o: odd f^n Then

1) $e \pm e = e$, $o \pm o = o$ (but $o \pm o$ should not be 0)
2) $e \pm o$ ($o \neq 0$) neither odd not even
3) $e . e = e$, $\frac{e}{e} = e$

   $o . o = e$, $\frac{o}{o} = e$

   $e o = o$, $\frac{e}{o} = e$

Geometric Properties

- The graph of an even function is symmetric with respect to the y-axis.
- The graph of an odd function is symmetric with respect to the origin.

Exp. Determine whether the following functions are even, odd or neither.

a) $f(x) = 4x^4 + 3x^2 - 2$

b) $g(x) = \dfrac{x^3 - 2x}{4 + x^2}$ c) $h(x) = 4x^4 + 2x^3$

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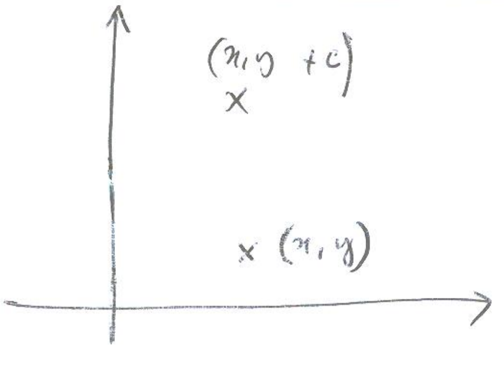
a) $f(x) = \underbrace{4\,x^4}_{e} + \underbrace{3\,x^2}_{e} - \underbrace{2}_{e}$ $\longrightarrow$ even

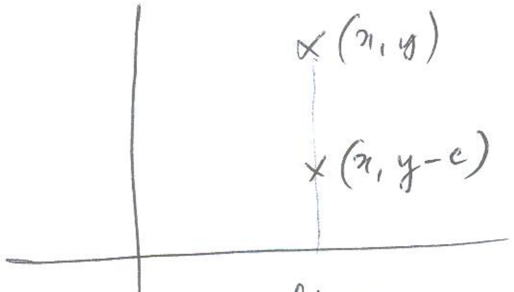
c) $h(x) = \underbrace{4x^4}_{e} + \underbrace{2x^3}_{o}$ $\longrightarrow$ neither even nor odd.

Translations

We have two types horizontal & vertical.

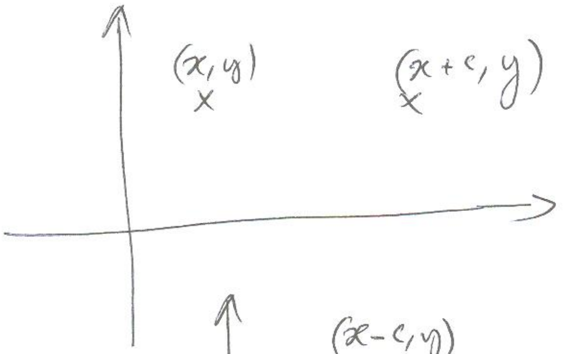
Vertical Translation or shift by $c > 0$ up



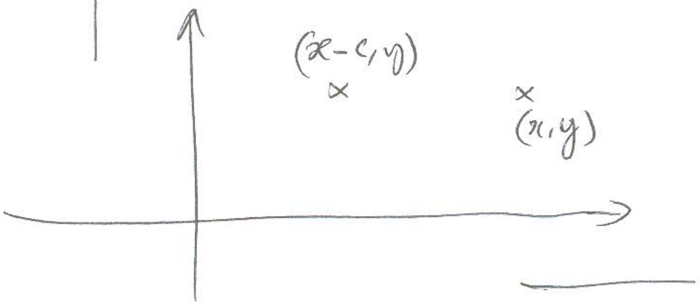


Vert Shift by c units down.

Horizontal Translation



Shift by c units to the right



Shift by c units to the left.

Rules for translations of graphs.

	The graph of	After	becomes
functions	$y = f(x)$	Shift by c up	$y = f(x) + c$
	$y = f(x)$	" " " down	$y = f(x) - c$
	$y = f(x)$	" " c to right	$y = f(x-c)$
	$y = f(x)$	" " c to left	$y = f(x+c)$
Equations	$E(x,y) = 0$	Shift by c up	$E(x, y-c) = 0$
	"	" " c down	$E(x, y+c) = 0$
	"	Shift by c to right	$E(x-c, y) = 0$
	"	Shift by c to left	$E(x, y-c) = 0$

<u>Exp</u>. Use the graphs of basic functions to graph

a) $y = (x+3)^2$ b) $y = |x| - 1$

c) $y = |x+3| + 2$ d) $y = \sqrt{x-3}$

<u>Exp</u>. Graph

$y = -|x+3| + 1$ 2) $f(x) = 2(x-2)^2 - 4$

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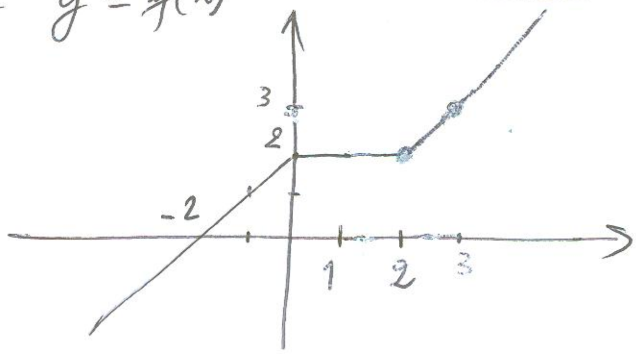
Exp. Given the graph of $y = f(x)$



Graph

a) $y = g(x-2) + 1$

b) $y = -g(x) + 2$

c) $y = g(2x+3)$

d) $y = -3\,g(-x+3) + 1.$

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a) $y = g(x) \xrightarrow[\text{2 right}]{} y = g(x-2) \xrightarrow{\text{1 up}} y = g(x-2) + 1$

c) $y = g(2x+3) = g\left(2\left(x + \frac{3}{2}\right)\right)$

$y = g(x) \xrightarrow[\text{H. Shrinking by } 1/2]{} y = g(2x) \xrightarrow[\text{Shift by } 3/2 \text{ to left.}]{} y = g\left(2\left(x+\frac{3}{2}\right)\right)$

d) $y = g(x) \xrightarrow[\text{3 left}]{} g(x+3) \xrightarrow[\text{Ref/y}]{} g(-x+3) \xrightarrow[\text{by 3}]{\text{Vert stret}} 3g(-x+3)$

$\downarrow$ Ref/x-axis

$y = -3g(-x+3)+1 \xleftarrow[\text{1 up}]{} -3g(-x+3)$