Domain & Ranges

The domain of a relation (or function) is the set of the first coordinates in all the pairs of the relation.

The range of a relation is the set of the second coordinates of all pairs of the relation.

Exp 5. $E = \{(-2,1), (-1,1), (0,2), (1,3)\}$

Domain : $\{-2, -1, 0, 1\}$

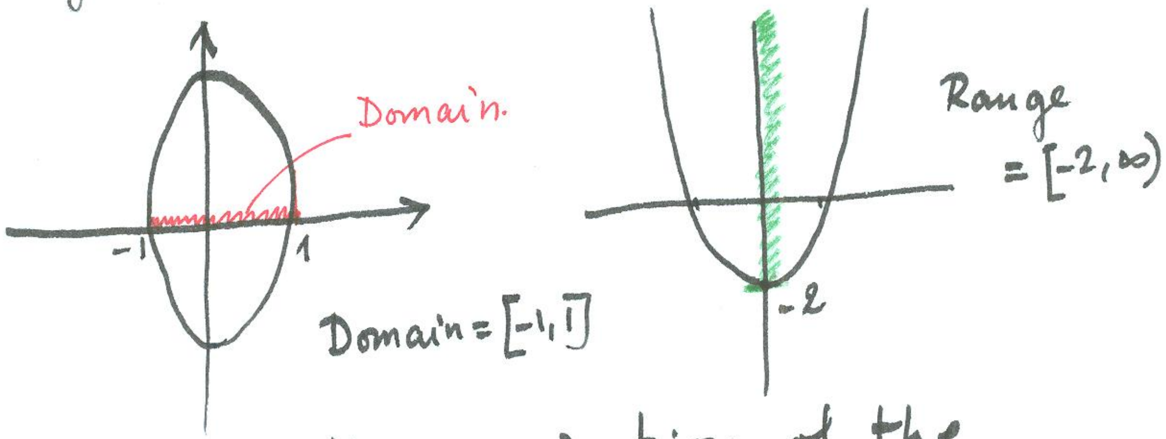
Range : $\{1, 2, 3\}$

For equations, the domain of a function is the set of numbers x for which y is defined.

The range is the set of values of y when x takes all values of the domain.

Domain & Range from the graph.

The domain of a relation or function is the projection of its graph on the x-axis



The range is the projection of the graph on the y-axis.

Finding the domain & Range from equations.

Exp6: Find the domain and range of the relations

a) $y = -6x + 4$

b) $x - y < 4$

c) $y = \sqrt{7 - 2x}$

d) $y = \dfrac{-7}{x - 5}$

e) $y^2 = x$.

f) $y = \dfrac{1}{\sqrt{x^2 - 4}}$ (domain only)

a) $D = \mathbb{R}$, Range $= \mathbb{R}$

b) $x - y < 4$ $D = \mathbb{R}$, Range $= \mathbb{R}$

~~Fo~~ any x, take $y = x - 1$ $\Rightarrow x - y = 1 < 4$

$\Rightarrow D = \mathbb{R}$

For any y, take $x = y + 1$ $(y+1) - y = 1 < 4$

$\Rightarrow$ Range $= \mathbb{R}$.

c) $y = \sqrt{7 - 2x}$

to be defined $7 - 2x \geqslant 0 \Rightarrow 7 \geqslant 2x$

$\Rightarrow \frac{7}{2} \geqslant x \Rightarrow D = (-\infty, \frac{7}{2}]$.

As $x \leq 7/2$

$7 - 2x \geqslant 0$

$\sqrt{7 - 2x} \geqslant 0 \Rightarrow$ Range: $[0, \infty)$

d) $y = \frac{-7}{x-5}$

$D = \mathbb{R} - \{5\}$.

As $x - 5$ can take any nonzero value, $\frac{-7}{x-5}$ can take any value apart from zero

Range $= \mathbb{R} - \{0\}$

Function Notation.

$$F = \{(x,y) \,/\, y = x^2+1\}$$

Instead of the above notation we use

$F(x) = x^2+1$ (Function notation)

Exp. 7 : Let $f(x) = -x^2+5x-3$, $g(x) = 2x+3$

Find and simplify each of the following.

a) $f(2)$ b) $f(q)$ c) $g(a+1)$

d) $f(a+1)$

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a) $f(2) = -2^2+5(2)-3 = 3$

b) $f(q) = -q^2+5q-3$

c) $g(a+1) = 2(a+1)+3 = 2a+5$

d) $f(a+1) = -(a+1)^2+5(a+1)-3$

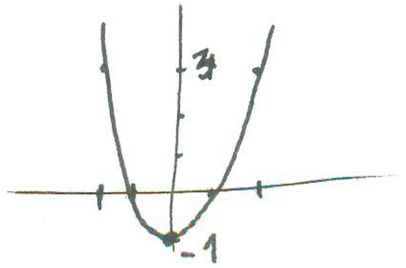
$= -(a^2+2a+1)+5a+5-3$

$= -a^2+3a+1$

Exp 8. Find $f(2)$ & $f(-1)$ for

a) $f = \{(2,5), (3,9), (-1,11), (5,3)\}$ —

b)



a) $(2,5) \in f \Rightarrow f(2) = 5$

$(-1, 11) \in f \Rightarrow f(-1) = 11$

b) $f(-1) = 0 \qquad f(2) = 3$

Exp 9. Write in function notation.

a) $f = \{(x,y) : y - 2x^2 + x = 1\}$

b) $g = \{(x,y) : x - y^3 + 3 = 0\}$

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a)
$$y - 2x^2 + x = 1$$
$$y = 2x^2 - x + 1$$
$$f(x) = 2x^2 - x + 1$$

b) $y^3 = x + 3 \Rightarrow y = \sqrt[3]{x+3}$

$$\Rightarrow g(x) = \sqrt[3]{x+3}$$

Increasing & Decreasing functions.

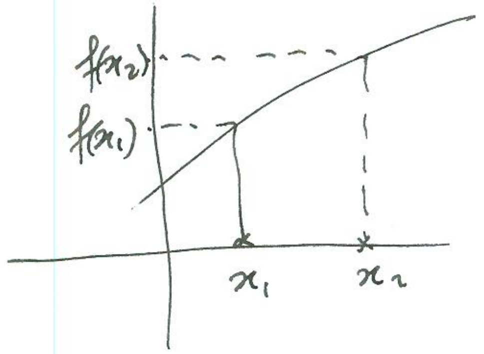
Let I is an interval, f is a function defined over I,

(a) f increasing over I if for any $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$

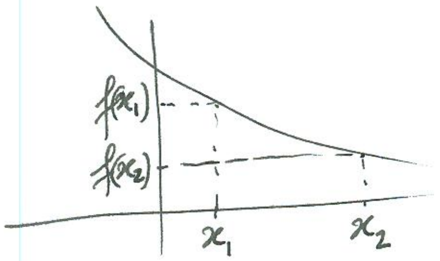
(b) f decreasing over I if for any $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$

c) f is constant on I if for any $x_1, x_2 \in I$

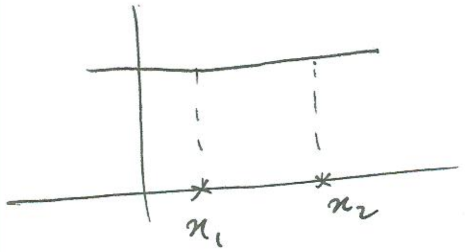
$$\Rightarrow f(x_1) = f(x_2)$$



increasing

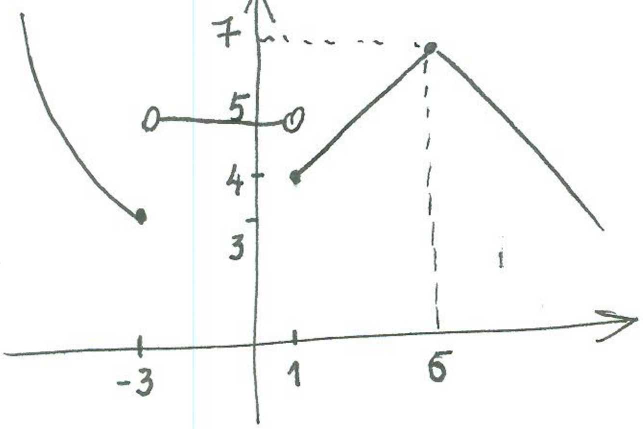


decreasing



constant

Exp Determine the intervals over which the function is decreasing, increasing, constant.



Sol:

decreasing over

$(-\infty, -3]$, $[6, \infty)$

increasing over

$[1, 6]$

Constant over $(-3, 1)$