— 10.3 Hyperbolas —

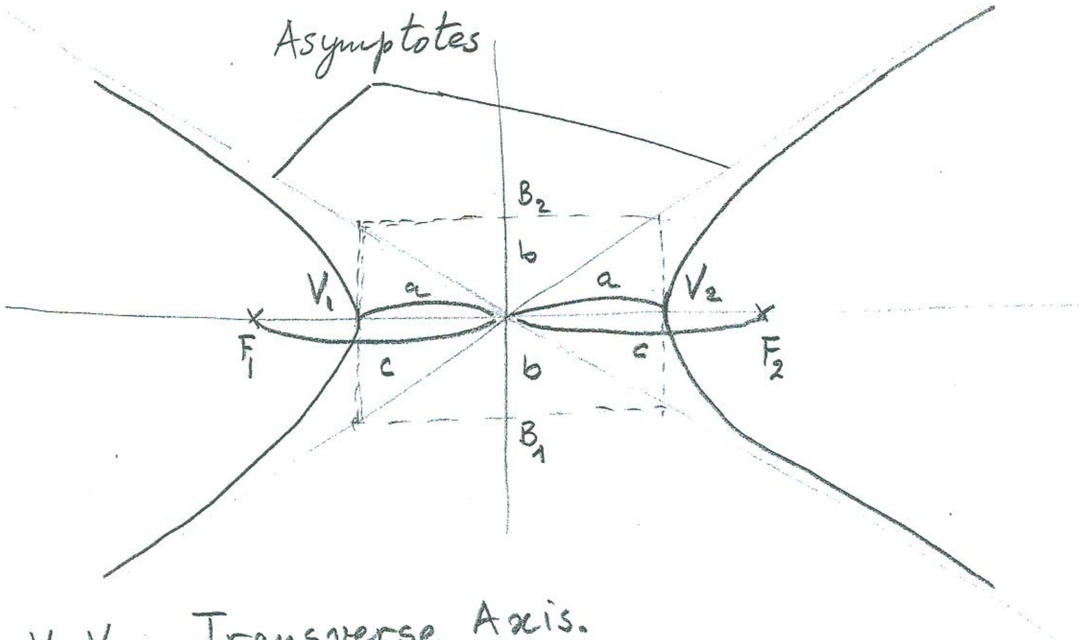
Objectives

Defining hyperbolas geometrically,
Giving their equations, foci, Vertices, axes.

Geometric Definition.

Given 2 points F_1, F_2 & $a > 0$. The hyperbola of foci F_1, F_2 and distance $2a$ is the set of points P whos difference of the distances to the foci is $2a$.

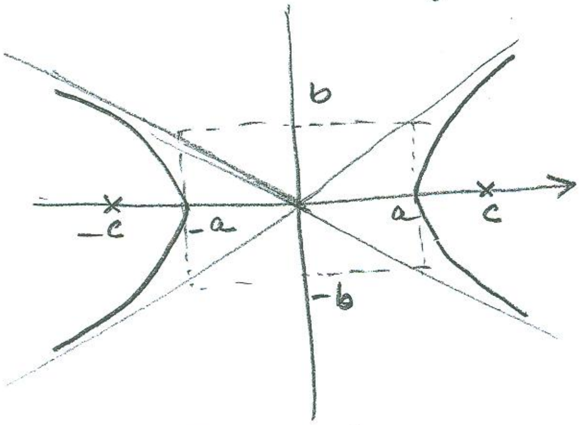
$$\mathcal{H}(F_1, F_2, 2a) = \{ P \,/\, |d(P,F_1) - d(P,F_2)| = 2a \}$$



$V_1 V_2$: Transverse Axis.

$B_1 B_2$: Conjugate Axis.

Equation of hyperbolas with Center (0,0).



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

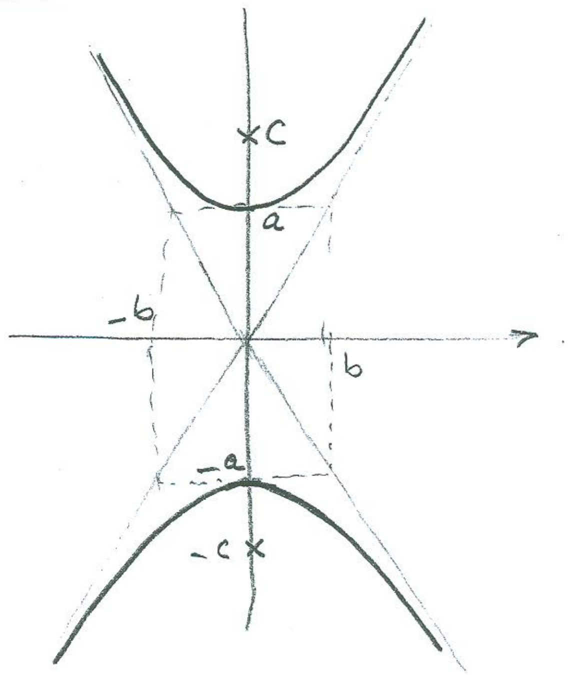
(Transverse Axis is horizontal)

Asymptotes

$$y = \pm \frac{b}{a} x$$

$$\boxed{a^2 + b^2 = c^2}$$

Vertical Transverse Axis



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Asymptotes $y = \pm \frac{a}{b} x$

$$a^2 + b^2 = c^2$$

hyperbolas with Center (h, k).

Trans Axis	Horizontal	Vertical
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Asymptotes	$y - k = \pm \frac{b}{a}(x-h)$	$y - k = \pm \frac{a}{b}(x-h)$
Vertices.	$V = (h \pm a, k)$	$V = (h, k \pm a)$
Foci	$F = (h \pm c, k)$	$F = (h, k \pm c)$
	$c = \sqrt{a^2+b^2}$	$c = \sqrt{a^2+b^2}$

Eccentricity $e = \frac{c}{a}$

- $e > 1$
- $e \simeq 1 \Rightarrow$ flat

Properties.

1) $\frac{(x-h)^2}{K} - \frac{(y-k)^2}{L} = 1$ & $K, L > 0$

$\Rightarrow a^2 = K, \quad b^2 = L$

2) $A(x-h)^2 - B(y-k)^2 = E$, $A, B > 0$ & $E \neq 0$

$\Rightarrow$ hyperbola.

3) $A(x-h)^2 - B(y-k)^2 = 0$

gives 2 line $(y-k) = \pm \frac{b}{a}(x-h)$

Exp 1. A hyperbola has eq^n $4x^2 - 9y^2 - 16x + 54y - 29 = 0$

Find its center, foci, vertices, asymptotes, graph

Exp2. Find eq^n of hyperbola with $e = 3$ $\boxed{10.3 \text{ p } 4}$

& Foci $(-2, 5)$, $(-2, -3)$.

Exp 3. $V = (5, -2), (1, -2)$

Asymp $y = -2 \pm \frac{3}{2}(x-3)$

Find eq^n.

Exp 4. Center $(9, -7)$, Focus $(9, -17)$, Vertex $= (9, -13)$

Find eq^n.