— 10.1 Parabolas —

Objectives:
- Vertical & horizontal parabolas
- Geometric Definition, Equations, Exercises.

Quadratic Functions & Parabolas.

We call the graph of $f(x) = ax^2 + bx + c = a(x-h)^2 + k$

a parabola with vertex $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

& axis of symmetry $x = h = -\frac{b}{2a}$.

Horizontal Parabolas.

The graph of $x = ay^2 + by + c = a(y-k)^2 + h$

is also a parabola with vertex (h, k) &

axis of symmetry $y = k$.

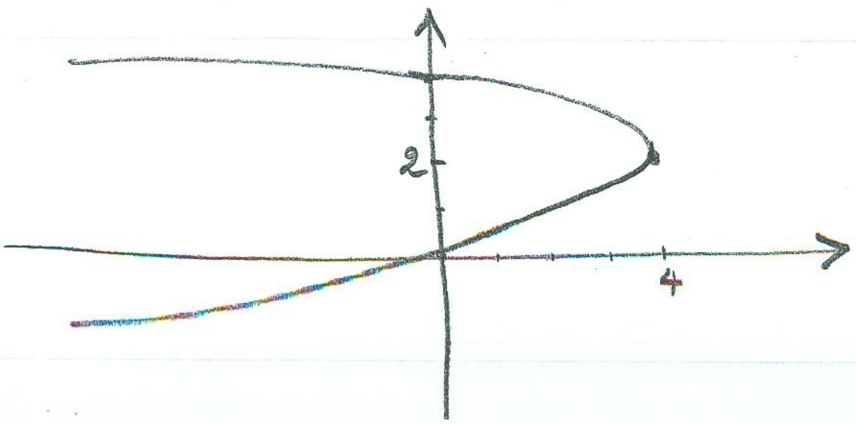
The parabola opens left if $a < 0$

" " right if $a > 0$

Exp1. Sketch the graph of $(x-4) = -(y-2)^2$

$V = (4, 2)$ $a = -1 < 0 \Rightarrow$ opens left

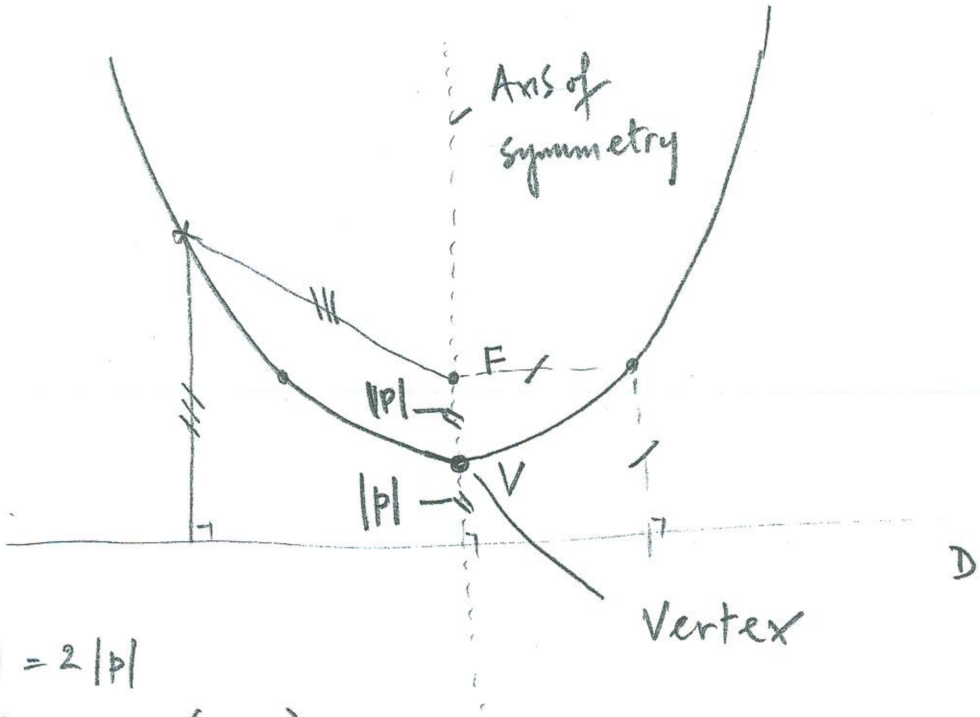
If $x = 0$ $-4 = -(y-2)^2 \Rightarrow y = 0, 4$



Geometric Definition.

Given a line D & a point F (not on D). The parabola of directrix D & focus F, is the set of points that are at the same distance from F & D.

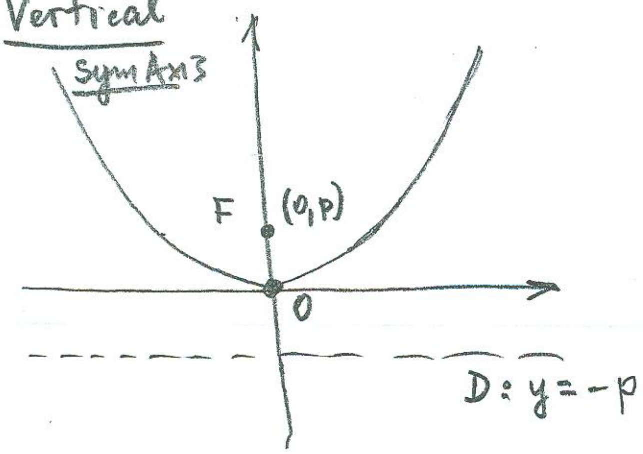
$$\mathcal{P}(F, D) = \{P \,/\, d(P,F) = d(P,D)\}$$



We set $d(F, D) = 2|p|$

$$\Rightarrow \quad |p| = d(V,F) = d(D,F)$$

Equation of parabola of center (0,0).

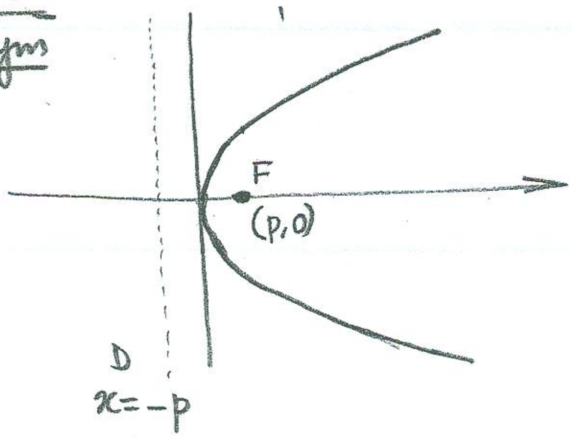


$$\boxed{x^2 = 4py}$$

If $p > 0$ opens up

If $p < 0$ opens down.

<u>Horizontal</u>
<u>Axis of sym</u>



$y^2 = 4px$

If $p>0$ opens to right.
If $p<0$ opens to left.

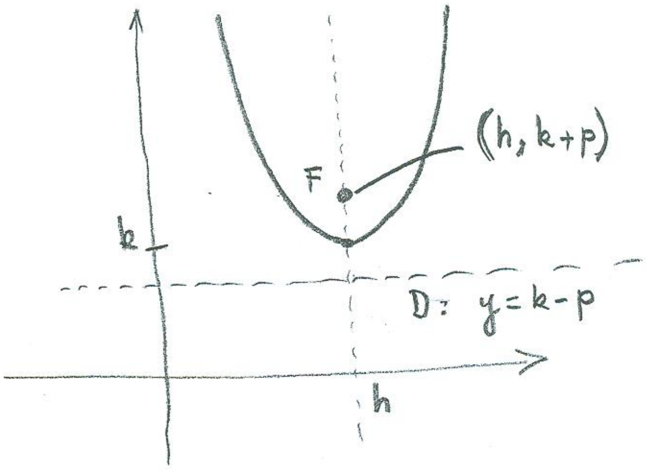
<u>Exp</u>. Find the focus, the directrix & the vertex of

a) $x^2 = 8y$ b) $y^2 = -28x$

<u>Parabolas of Center (h, k)</u>.

<u>Axis of Sym</u>: Vertical

Equation : $\boxed{(x-h)^2 = 4p(y-k)}$



Focus : $(h, k+p)$

Directrix : $y = k-p$

Equation of Axis of Sym : $\boxed{x=h}$

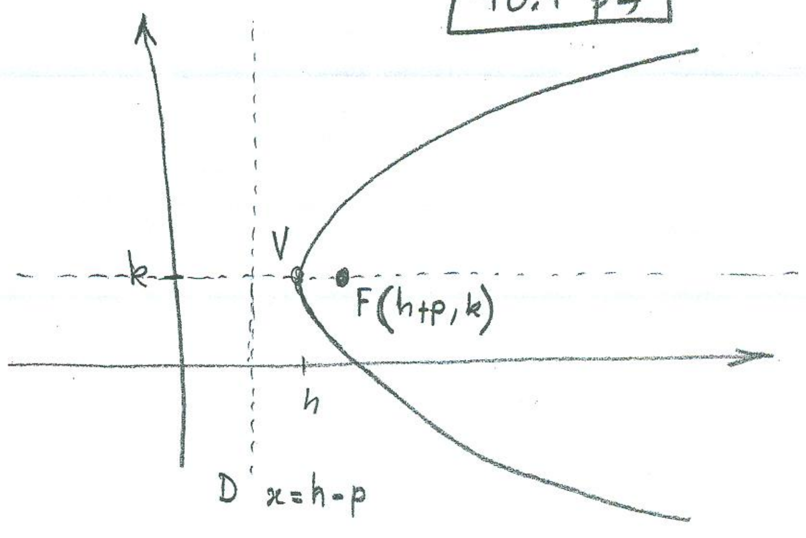
Axis of Sym: Horizontal

Eqn. $(y-k)^2 = 4p(x-h)$

Focus: $(h+p, k)$

Directrix: $x = h-p$

Eqn of Axis of Sym: $y = k$



Properties.

1) $\underbrace{F(x, k), V(h, k)}_{\text{same } 2^{nd} \text{ coord}} \Rightarrow$ Horizontal Parabola $\Rightarrow (y-k)^2$.

2) $\underbrace{F(h, y), V(h, k)}_{\text{same } 1^{st} \text{ coord}} \Rightarrow$ Vertical Parabola $\Rightarrow (x-h)^2$.

3) Directrix $y = k-p$ or hori $\Rightarrow$ Vert Parabola $\Rightarrow (x-h)^2$

Directrix $x = h-p$ or Vert $\Rightarrow$ Horizontal Parabola $\Rightarrow (y-k)^2$

Exercise 1. Find the vertex, focus, directrix & graph of

$y^2 - 2y + 4x + 9 = 0$

$y^2 - 2y = -4x - 9$

$y^2 - 2y + 1 = -4x - 9 + 1 = -4x - 8$

$(y-1)^2 = -4(x+2)$

$(y-k)^2 = 4p(x-h)$

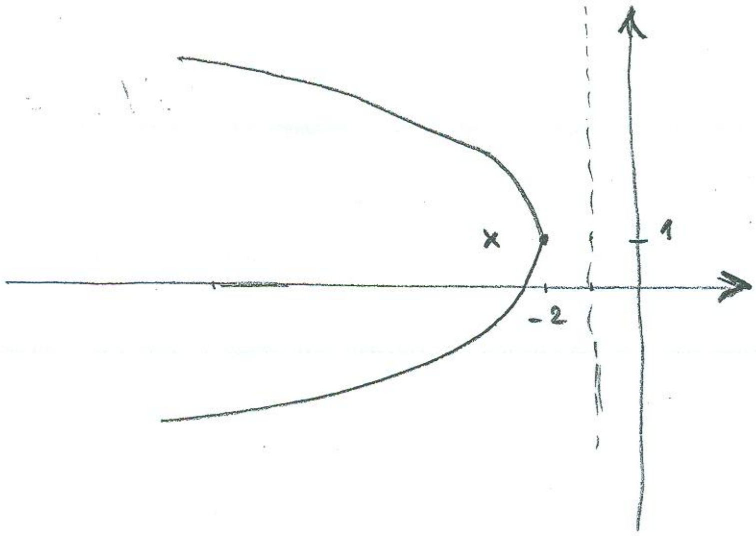
$\Rightarrow h = -2, k = 1$

$p = -1$

$V = (-2, 1)$

$(y-1)^2 \Rightarrow$ Horizontal Parabola

$p = -1 < 0 \Rightarrow$ to left



$F(h+p, k) = (-2-1, 1)$
$= (-3, 1)$

Directrix

$x = h - p$
$= -2 + 1 = -1$

$\boxed{x = -1}$

Exp 2. A parabola with focus $F(3,2)$ & directrix $x = -1$. Find its equation,

D: $x = -1 = h - p \Rightarrow F(h+p, k) = (3, 2) \Rightarrow \boxed{k = 2}$

$h + p = 3$
$h - p = -1$

$2h = 2 \Rightarrow \boxed{h = 1}$

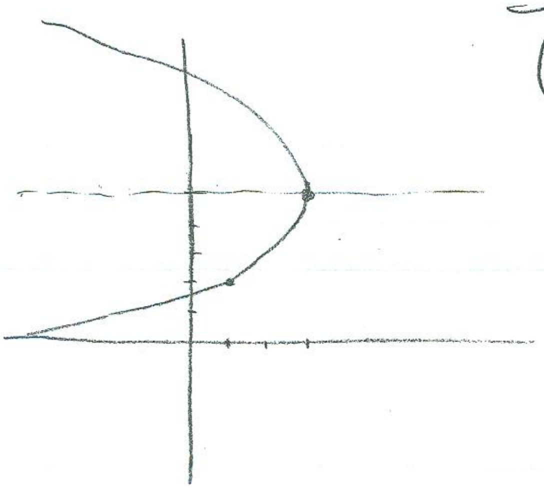
$h + p = 3 \Rightarrow \boxed{p = 2}$

$V = (h, k) = (1, 2)$, $F = (3, 2)$

$\Rightarrow (y-k)^2 = 4p(x-h)$

$\boxed{(y-2)^2 = 8(x-1)}$

Exp 3. A parabola with vertex $V = (3, -5)$ with axis of sym parallel to x-axis & passes through $(-2, 2)$. Find equation.



$(y+5)^2 = 4p(x-3)$

$(-2, 2) \in \mathcal{P}$ — $(-2, 2)$ satisfies equation.

$(2+5)^2 = 4p(-2-3)$

$49 = 20p \Rightarrow p = \frac{49}{20}$

$(y+5)^2 = 4\left(\frac{49}{20}\right)(x-3)$

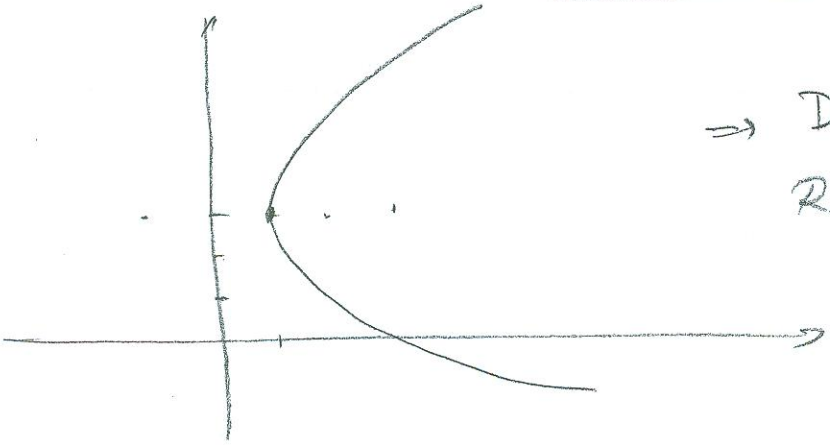
$\boxed{(y+5)^2 = \frac{49}{5}(x-3)}$

Ex Find the equation of the parabola with vertex $V(1,3)$, $F(-1,3)$.
Find the domain & range of this parabola.

$V=(1,3)$, $F(-1,3)$ same y-coord $\Rightarrow (y-3)^2 = 4p(x-1)$

$\Rightarrow F(h-p, k) = (1-p, 3) = (-1, 3) \Rightarrow 1-p=-1$

$\Rightarrow \boxed{p=2} \Rightarrow \boxed{(y-3)^2 = 8(x-1)}$



$\Rightarrow D = [1, \infty)$

Range: $(-\infty, \infty)$

Domain & Range.

		Domain	Range
$(x-h)^2 = 4p(y-k)$	$p>0$	$(-\infty, \infty)$	$[k, \infty)$
	$p<0$	$(-\infty, \infty)$	$(-\infty, k]$
$(y-k)^2 = 4p(x-h)$	$p>0$	$[h, \infty)$	$(-\infty, \infty)$
	$p<0$	$(-\infty, h]$	$(-\infty, \infty)$