The speed of sound is $v=\sqrt{\frac{B}{ρ}}$

Imagine a wave moving along the -x-axis with a speed $v$ and we are moving with one of the pulses, which has a pressure$ p+∆p$. An element of air of width $∆x$ relative to us will move with a speed $v$ toward the +x-axis.

Let us assume that pulse is stationary and the element of air is moving toward it with speed $v$.

$$P,v$$

$$∆m$$

$$∆m$$

A

$$F\_{L}$$

$$F\_{R}$$

$$P+∆P, v+∆v$$

$$v$$

$$P+∆P, v+∆v$$

$$P,v$$

There will be a net force on the air element due to the difference in pressure.

$$F\_{net}=F\_{L}-F\_{R}= AP-A\left(P+∆P\right)=-A∆P=∆m a\_{avg}=∆m\frac{∆v}{∆t}$$

$$-A∆P=∆m\frac{∆v}{∆t}$$

$$∆m=ρAv∆t$$

$$-A∆P=ρAv∆t \frac{∆v}{∆t}=ρAv∆v$$

$$-∆P= ρv^{2}\frac{∆v}{v} ………………………. (1)$$

Now $V=A∆x=Av∆t since ∆x=v∆t$

And $∆V=A∆v∆t$

Therefore $\frac{∆V}{V}=\frac{A∆v∆t}{Av∆t}=\frac{∆v}{v}$

Substitute $\frac{∆v}{v} $from last equation in (1) we get

$$-∆P= ρv^{2}\frac{∆V}{V}$$

$$But -∆P= B\frac{∆V}{V}$$

$$therefore B= ρv^{2} or v=\sqrt{\frac{B}{ρ}}$$

If s(x,t) represent the distance of small volume element measured from its equilibrium position moving in simple harmonic motion we can express it as

$$S\left(x,t\right)=S\_{m} cos⁡(kx-wt)$$

What would be the change in pressure in medium that caused by the propagation of the wave?

Let us concentrate on a small volume element somewhere in between two pulses. The side of the element will experience different pressure according to their location therefore, they will move with different speed.

$$∆x$$

$$S$$

$$S=0$$

$$v$$

Let us call the left side of the element side 1 and the right side one side 2.

A

$$S\_{2}$$

2

$$∆x$$

$$S\_{1}$$

1

At t=0

At some other timet=0

1

2

$$∆x'$$

$$∆x^{'}=∆x+ S\_{2}-S\_{1}=∆x+∆S$$

Since$ V=A∆x then ∆V=A∆S$

$$But ∆P= -B\frac{∆V}{V} then$$

$$∆P= -B\frac{A∆S}{A∆x}=-B\frac{∆S}{∆x} or ∆P=-B\frac{∂S}{∂x} $$

The gives us

$$∆P=BkS\_{m}sin⁡(kx-wt)$$

Which can be written as

$$∆P=∆P\_{m}\sin(\left(kx-wt\right)) $$

$$ where ∆P\_{m}= BkS\_{m}=v^{2}ρkS\_{m} o r ∆P\_{m}=vρwS\_{m}$$