Δx

+y

$$∆l+∆x$$

Δy

+x

Δx

To calculate the change in elastic energy we need to calculate change in length as a function of position and time.

$$dl=\sqrt{dx^{2}+dy^{2}} -dx=dx\left[1+\left(\frac{∂y}{∂x}\right)^{2}\right]^{^{1}/\_{2}}-dx$$

$ =dx\left[1+\frac{1}{2}\left(\frac{∂y}{∂x}\right)^{2}+…\right]-dx=\frac{1}{2}\left(\frac{∂y}{∂x}\right)^{2}dx$

$dl=\frac{1}{2}\left(\frac{∂y}{∂x}\right)^{2}dx$

Now, $y\left(x,t\right)= y\_{m}sin(kx-wt)$

$$then \frac{∂y}{∂x}= y\_{m}k cos\left(kx-wt\right)$$

Since the change in potential energy is equal to the applied work then

$$dU=τdl=\frac{1}{2}τ\left(\frac{∂y}{∂x}\right)^{2}dx$$

$$dU=\frac{1}{2}τy\_{m}^{2}k^{2}cos^{2}\left(kx-wt\right)dx$$

$But v=\sqrt{\frac{τ}{μ}} or τ=v^{2}μ=\frac{w^{2}}{k^{2}}μ$

$$dU=\frac{1}{2}\frac{w^{2}}{k^{2}}μy\_{m}^{2}k^{2}cos^{2}\left(kx-wt\right)dx$$

$dU=\frac{1}{2}μw^{2}y\_{m}^{2}cos^{2}\left(kx-wt\right)dx$ ……….. (1)

We can calculate the change in kinetic energy as follow:

$$dK=\frac{1}{2}dm u^{2}=\frac{1}{2}μ dx u^{2}=\frac{1}{2}μ dx \left(\frac{∂y}{∂t}\right)^{2}$$

$$\frac{∂y}{∂t}=-y\_{m}w cos⁡(kx-wt)$$

$dK=\frac{1}{2}μw^{2}y\_{m}^{2}cos^{2}\left(kx-wt\right)dx$ ……….. (2)

$therefore dK=dU and P=\frac{dK}{dt}+\frac{dU}{dt}=2\frac{dK}{dt}=μw^{2}y\_{m}^{2}cos^{2}\left(kx-wt\right)\frac{dx}{dt}$

 $P=μ v w^{2}y\_{m}^{2} cos^{2}\left(kx-wt\right)$

$$P\_{avg}=μ v w^{2}y\_{m}^{2} [cos^{2}\left(kx-wt\right)]\_{avg}$$

$$P\_{avg}=μ v w^{2}y\_{m}^{2}\left( \frac{1}{2}\right)$$

$$P\_{avg}= \frac{1}{2}μ v w^{2}y\_{m}^{2}$$