Exam - 022 solutions

~~Sp 3-7~~ Q1 → Q9

(Q1) $\left|\frac{1}{2}(\hat{i}+\hat{j})\right| = \frac{1}{2}\sqrt{1^2+1^2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

so not a unit vector ✓

$\left|\frac{\vec{a}}{a}\right| = \frac{1}{a}|\vec{a}| = \frac{a}{a} = 1$ unit vector

$\hat{j}\wedge\hat{i} = \hat{k}$ unit vector

$\left|\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})\right| = \frac{1}{\sqrt{3}}\sqrt{1^2+1^2+1^2} = 1$ unit vector

$|(0.6\,\hat{i}+0.8\,\hat{j})| = \sqrt{0.6^2+0.8^2} = 1$ unit vector.

(Q2) $\vec{A}\cdot\vec{B} = AB\cos\phi$

$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB}$

$\vec{A}\cdot\vec{B} = (\hat{i}-2\hat{j}+2\hat{k})\cdot(-2\hat{i}+\hat{j}+2\hat{k})$

$= \hat{i}\cdot(-2\hat{i}+\hat{j}+2\hat{k})$
$+2\hat{j}\cdot(-2\hat{i}+\hat{j}+2\hat{k})$
$+2\hat{k}\cdot(-2\hat{i}+\hat{j}+2\hat{k})$

$= (1)(-2) + 0 + 0$

$+ 0 + (-2)(1) + 0$

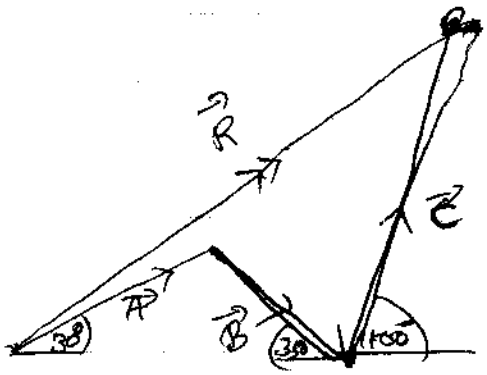
$+ 0 + 0 + (2)(2)$

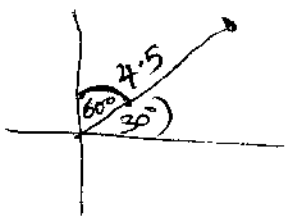
$= (1)(-2) + (-2)(1) + (2)(2)$

$\vec{A} \cdot \vec{B} = -2 - 2 + 4 = 0$

$\Rightarrow \cos\phi = 0$ $\quad \boxed{\phi = 90^\circ}$

(Q3)





$\vec{A} = \langle 4.5\cos 30^\circ, 4.5\sin 30^\circ \rangle$

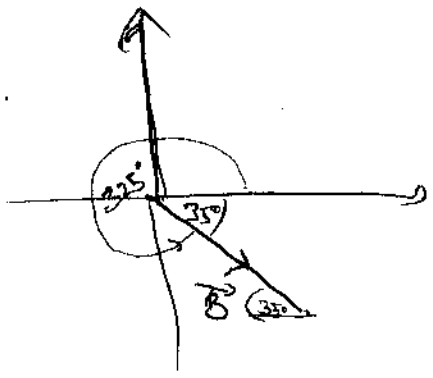
$= \langle 3.90, 2.25 \rangle$

or

$\vec{A} = \langle 4.5\cos 30^\circ, 4.5\cos 60^\circ \rangle$

$= \langle 3.90, 2.25 \rangle$

$\vec{B} =$



$$\vec{B} = \langle 4.1 \cos 325^\circ,\ 4.1 \sin 325^\circ \rangle \equiv \langle 3.36, -2.35 \rangle$$

or $$\equiv \langle 4.1 \cos(-35^\circ),\ 4.1 \sin(-35^\circ) \rangle \equiv \langle 3.36, -2.35 \rangle$$

or $$\equiv \langle 4.1 \cos 35,\ -4.1 \sin 35^\circ \rangle \equiv \langle 3.36, -2.35 \rangle$$

$$\vec{C} \equiv \langle 6.0 \cos 100,\ 6.0 \sin 100 \rangle \equiv \langle -1.04, 5.91 \rangle$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$= (3.90 + 3.36 - 1.04)\,\hat{i} + (2.25 - 2.35 + 5.91)\,\hat{j}$$

$$= 6.22\,\hat{i} + 5.81\,\hat{j}$$

$$\boxed{\vec{R} = 6.2\,\hat{i} + 5.8\,\hat{j}}$$

(Q4) $1\ \text{Å} = 0.1\ \text{nm}$

~~$1\ \text{nm} = 1\ \text{nm} \left[\frac{10^{+9}\ \text{m}}{1\ \text{m}}\right]$~~

$$1\ \text{nm} = 1\ \text{nm} \left[\frac{10^{-3}\ \text{m}}{1\ \text{mm}}\right] \times \left[\frac{10^{+9}\ \text{mm}}{1\ \text{m}}\right] \times \left[\frac{1\ \text{Å}}{0.1\ \text{mm}}\right]$$

$$= \frac{1 \times 10^{-3} \times 10^{+9} \times 1\ \text{Å}}{10^{-1}}$$

$$1\ \text{nm} = 10^{7}\ \text{Å}$$

(Q5) $c = \frac{d}{t}$ constant velocity c

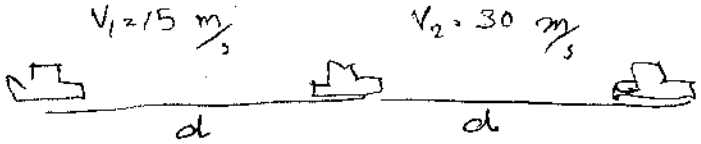
$$d = (c)t = 3.0 \times 10^{8} \left(\frac{\text{m}}{\text{s}}\right) \times 8.4\ (\text{min}) \left[\frac{60\ \text{s}}{1\ \text{min}}\right]$$

$$= 3.0 \times 10^{8} \times 8.4 \times 60 \quad (\text{m})$$

$$= 1.50 \times 10^{11}\ \text{m}$$

$$= 1.50 \times 10^{8}\ \text{km}$$

(26)



$$\bar{V} = \frac{2d}{t_1 + t_2}$$

$$t_1 = \frac{d}{V_1} \qquad t_2 = \frac{d}{V_2}$$

$$\bar{V} = \frac{2d}{d/V_1 + d/V_2} = \frac{2d}{d\left(\frac{1}{V_1} + \frac{1}{V_2}\right)}$$

$$= \frac{2V_1V_2}{V_1 + V_2}$$

$$= \frac{2(15)(30)}{15+30} = \frac{2(15)(30)}{45} = 20 \text{ m/s}$$

(97) $x = 3t - t^3$

$v = 3 - 3t^2$

$a = -6t$ (a is not constant)

V is when v is zero

$0 = 3 - 3t^2$

$t = 1\ s$

$a = -6(1) \Rightarrow a(t = 1.0) = -6(1)$

$a(1) = -6.0\ m/s^2$

(98)

$V_0 = 10\ \frac{cm}{s}$ $x_0 = 10\ cm$ a? $(t = 4.0\ s)$ V? $x = -14\ cm$

$x - x_0$	v	t	a	V_0
$(-14 - 10)\ cm$	?	4.0 s	?	$10\ \frac{cm}{s}$

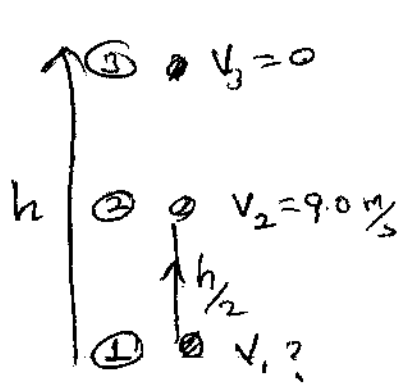
$x - x_0 = v_0 t + \frac{1}{2} a t^2$

$-24 = (10)(4) + \frac{1}{2} a (4)^2$

$\frac{(-24 - 40)}{16} = a = \frac{(-8)(2)}{1}$

$a = -8.0\ \frac{cm}{s^2}$

(99)



We can take 3 combination of initial & final:

initial	final	listing 5 quantities					Number of unknowns
		Δy	v_0	v	a	t	
① →	②	$h/2$?	v_1 ?	$v_2 = 9$	-9.8	?	3
① →	③	h ?	v_1 ?	$v_3 = 0$	-9.8	?	3
② →	③	$h/2$?	$v_2 = 9$	$v_3 = 0$	-9.8	?	only 2 ✓

(i) Only the last choice of initial & final have only 2 unknowns. So can be solved for Δy & t

(ii) Δy there is equal to $(h/2)$!

② → ③ $\quad v^2 = v_0^2 + 2a\Delta y$

$0 = (9.0)^2 + 2(-9.8)(h/2)$

$\boxed{h = (9.0)^2/9.8 = 8.3 \text{ m}}$