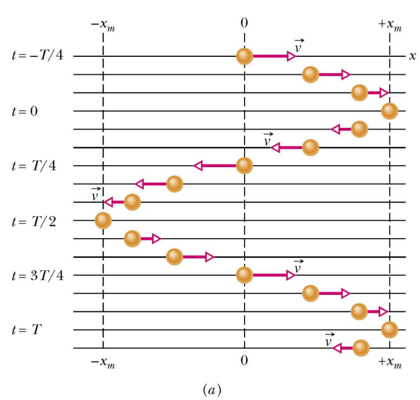
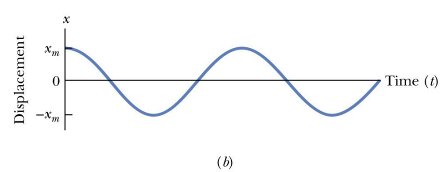
1. <u>Oscillations</u>

- You may not know it, but every atom/molecule in your body is oscillating.
- For any system, there's at least one state that the system is of the lowest potential energy. This is a point of stable equilibrium, or the bottom of the valley in a potential vs. position curve.
- If the system is of a small displacement from the point, it'll experience a restoring force, pointing to the bottom of the potential curve. The force accelerates the system so it'll swing across the equilibrium point to the other side, and the restoring force will reverse as well. Thus, it'll *oscillate* around the point of equilibrium.

2. <u>Simple Harmonic Motion</u>

- The period T of an oscillation is the time taken for the oscillating system to repeat itself, or, to complete one oscillation. For example, the time for a swinging pendulum starting from one extreme point to the come to the same point. Same position, velocity, and acceleration. T is in second.
- The frequency *f* of an oscillation is the number of complete oscillations per unit time. Clearly: $\boxed{f = \frac{1}{T}}$
- The unit for frequency is *hertz*. 1 Hz (*hertz*) = 1/s = s^{-1}
- Any motion that repeat itself at regular intervals is called *periodic motion* or *harmonic motion*. In this chapter we will be dealing with a special case of periodic motion called *simple harmonic motion*, a motion that repeats itself in a particular way as shown in the figure below:





For such motion the *displacement* *x* from the origin is given as a function of time by:

$$\boxed{x(t) = x_m \cos(\omega t + \phi)}$$

where x_m, ω and ϕ are constants.

$$\overbrace{x(t)}^{\textit{displacement}\text{ at time } t} = \underbrace{x_m}_{\textit{amplitude}} \cos\overbrace{(\underbrace{\omega}_{\textit{angular frequency}}\, \underbrace{t}_{\textit{time}} + \underbrace{\phi}_{\textit{phase constant (phase angle)}})}^{\textit{phase}}$$

<u>Interpreting ω (what is ω?)</u>

- Angular frequency ω is related to the frequency f and hence to the period T.

$$\begin{aligned} x(t+T) &= x_m \cos(\omega(t+T)+\phi) \\ &= x_m \cos(\omega T + \omega t + \phi) \end{aligned}$$

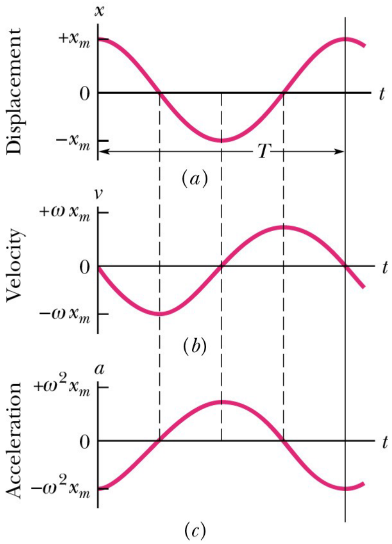
- But we know that $x(t) = x(t+T)$, this is true if $\omega T = 2\pi$, because: $\cos(2\pi + \omega t + \phi) = \cos(\omega t + \phi)$. Therefore:

$$\boxed{\omega = \frac{2\pi}{T}}$$

The Velocity of SHM

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \varphi)\right]$$

$$\Rightarrow \quad \boxed{v(t) = -v_m \sin(\omega t + \varphi)} \qquad \boxed{v_m = \omega x_m}$$



The Acceleration of SHM

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$$\Rightarrow \quad \boxed{a(t) = -a_m \cos(\omega t + \phi)} \qquad \boxed{a_m = \omega^2 x_m}$$

$$\downarrow$$

$$\boxed{a(t) = -\omega^2 x(t)}$$

- In SHM, the acceleration is proportional to the displacement but opposite in sign, and the two quantities are related by the square of the angular frequency.

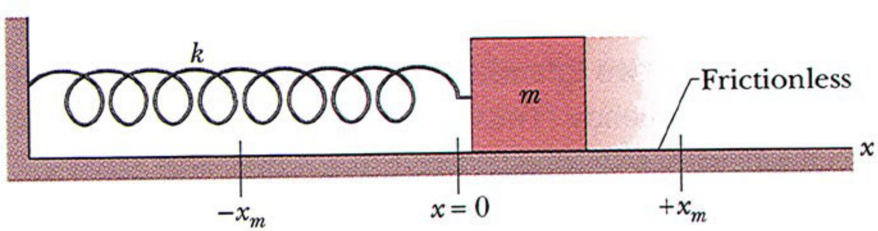
3. <u>The Force Law for Simple Harmonic Motion</u>

$$F = ma = m(-\omega^2 x) = -(m\omega^2)x$$

This result – a restoring force that is proportional to the displacement but opposite in sign – is familiar. It is Hooke's law, $\boxed{F = -kx}$ for a spring, the spring constant being $\boxed{k = m\omega^2}$

- Simple harmonic motion is the motion executed by a particle of mass *m* subject to a force that is proportional to the displacement of the particle but opposite in sign.

<u>A Linear Simple Harmonic Oscillator – The Block-Spring System</u>



$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$ (angular frequency)

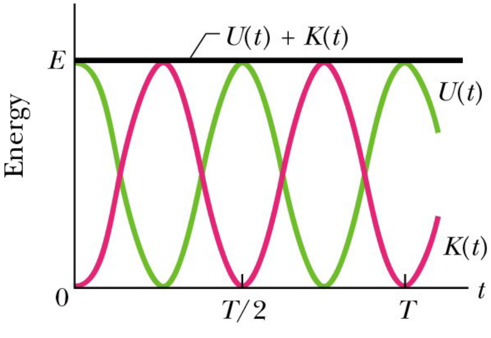
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$ (period)

4. Energy in Simple Harmonic Motion

$$\boxed{U(t)=\frac{1}{2}kx^2}=\frac{1}{2}kx_m^2\cos^2(\omega t+\phi)$$

$$\boxed{K(t)=\frac{1}{2}mv^2}=\frac{1}{2}m\omega^2x_m^2\sin^2(\omega t+\phi)$$

$$=\frac{1}{2}kx_m^2\sin^2(\omega t+\phi)\qquad(\because k=m\omega)$$

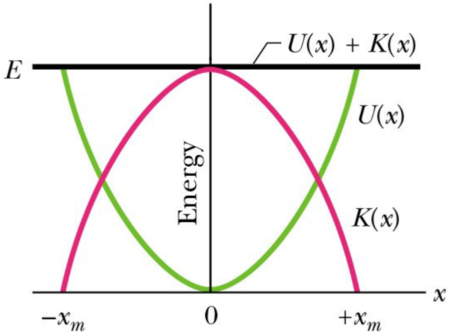


(a)

$$E=U+K=\frac{1}{2}kx_m^2\cos^2(\omega t+\phi)+\frac{1}{2}kx_m^2\sin^2(\omega t+\phi)$$

Using the fact: $\cos^2(\omega t+\phi)+\sin^2(\omega t+\phi)=1$

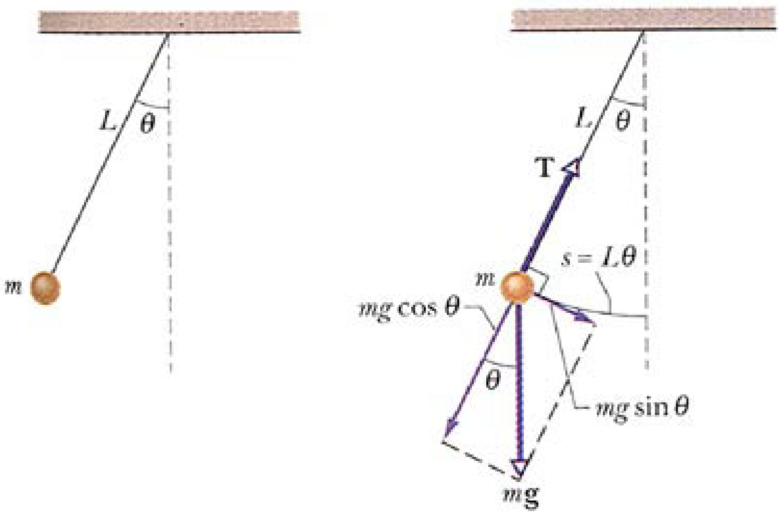
$$\Rightarrow\qquad\boxed{E=U+K=\frac{1}{2}k\,x_m^2}$$



(b)

6. Pendulums

Simple Pendulum



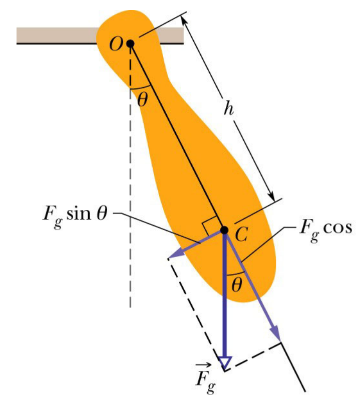
$$F = -mg\sin\theta \cong -mg\,\theta \cong -mg\frac{s}{L} = -\left(\frac{mg}{L}\right)s$$

$$\Rightarrow \quad k = m\omega^2 = \frac{mg}{L}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\cancel{m}}{\cancel{m}g/L}}$$

$$\Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Physical Pendulum



$$\tau = -(mg\sin\theta)\,h \cong -(mgh)\,\theta$$

$$\Rightarrow \quad k = m\omega^2 = mgh$$

$$T = 2\pi\sqrt{\frac{I_o}{k}}$$ note m is replaced by I_o

$$\Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{I_o}{mgh}}}$$