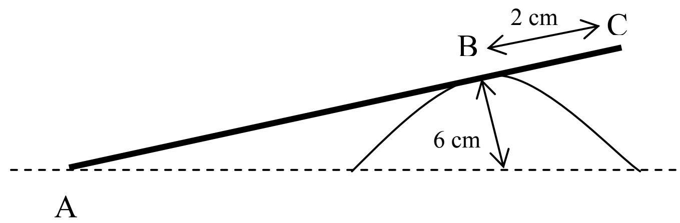
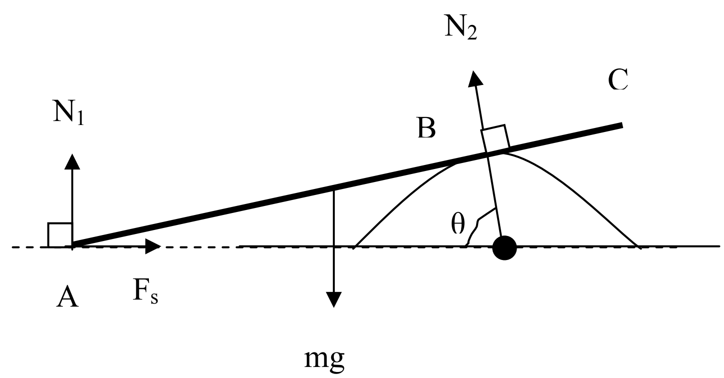
Extra Problems of Chapter 13

Problem 1: ABC is a uniform thin rod its length 10 cm, which is touching the rough floor at A and smooth half sphere shape of radius 6 cm at B (See the figure). If the rod is about to slip calculate static coefficient between the rod and the floor.



Answer:

The free force diagram is like the following:



$\sum F_x = 0$

$F_s - N_2 \cos\theta = 0$

$F_s = N_2 \cos\theta \Rightarrow F_s = \frac{3}{5} N_2$ ---------(1)

$\sum F_y = 0$

$N_2 \sin\theta + N_1 - mg = 0$

$\frac{4}{5} N_2 + N_1 = mg$ -----------(2)

Torque about point A: $\tau_z = 0$

$(N_2 \times 8) - (mg \times 4) = 0$

$N_2 = \frac{1}{2} mg$ -----------(3)

Substitute the value of N_2 form formula (3) into (1) and (2):

$F_s = \frac{3}{5} \times \frac{1}{2} mg = \frac{3}{10} mg$

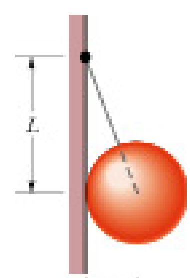
$N_1 = mg - \frac{4}{5} \times \frac{1}{2} mg = \frac{6}{10} mg$

The rod is about to slip:

$\mu = \frac{F_s}{N_1} = \frac{3}{10} mg \times \frac{10}{6 mg} = \frac{1}{2}$

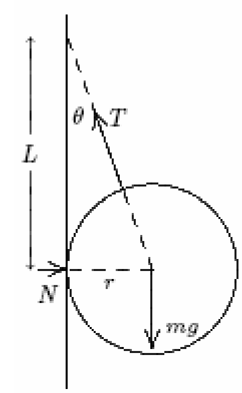
Problem 2: In the figure, a uniform sphere of mass m and radius r is held in place by a massless rope attached to a frictionless wall a distance L above the center of the sphere.

A- Find the tension in the rope **B-** The force on the sphere from the wall.



Answer:

Three forces act on the sphere: the tension force _*T* of the rope (acting along the rope), the force of the wall *N* (acting horizontally away from the wall), and the force of gravity *m_g* (acting downward). Since the sphere is in equilibrium they sum to zero. Let *θ* be the angle between the rope and the vertical. Then, the vertical component of Newton's second law is $T \cos\theta - mg = 0$. The horizontal component is $N - T \sin\theta = 0$.



A- We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L2 + r2}$ to obtain $T = mg\sqrt{L2 + r2}/L$.

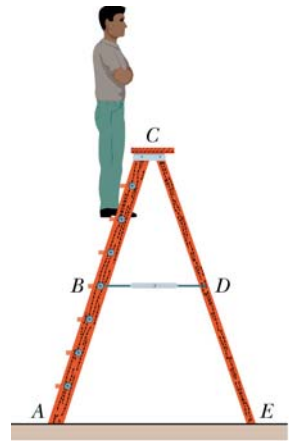
B- We solve the second equation for the normal force: $N = T \sin\theta$. Using $\sin\theta = r/\sqrt{L2 + r2}$, we obtain $N = Tr/\sqrt{L2 + r2} = (mg\sqrt{L2 + r2}/L)(r/\sqrt{L2 + r2}) = mgr/L$

Problem 3: For the stepladder shown in the figure, side AC and CE are each 2.44 m long and hinged at C. Bar BD is tie-rod 0.762 m long, halfway up. A man weighting 854 N climbs 1.8 m along the ladder. Assuming that the floor is frictionless and neglecting the mass of the ladder, find:

A- The tension in the tie-rod

B- The magnitude of the force on the ladder from the floor A and C.

Hint: It will help to isolate parts of the ladder in applying the equilibrium conditions.



Answer: The diagrams to the right show the forces on the two sides of the ladder, separated. FA and FE are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), Fh is the horizontal component of the force exerted by one side of the ladder on the other, and Fv is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction.

Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $Fv + FA - W = 0$. The horizontal components must sum to zero: $T - Fh = 0$. The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then $FAL\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0$. Here we recognize that the man is one-fourth the length of the ladder side from the top and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $FE - Fv = 0$, $Fh - T = 0$, and $FEL\cos\theta - T(L/2)\sin\theta = 0$.

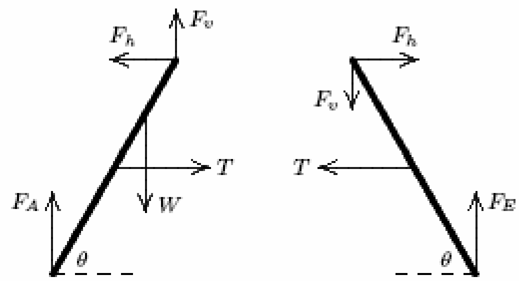
There are 5 different equations:

$Fv + FA - W = 0,$

$T - Fh = 0$

$FAL\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0$

$FE - Fv = 0$



$FEL\cos\theta - T(L/2)\sin\theta = 0$.

The unknown quantities are FA, FE, Fv, Fh, and T.

A- First we solve for T by systematically eliminating the other unknowns.

The first equation gives

$FA = W - Fv$ and the fourth gives $Fv = FE$. We use these to substitute into the remaining three equations to obtain

$T - Fh = 0$

$WL\cos\theta - FEL\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0$

$FEL\cos\theta - T(L/2)\sin\theta = 0$.

The last of these gives $FE = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is $T = (3W/4\tan\theta)$.

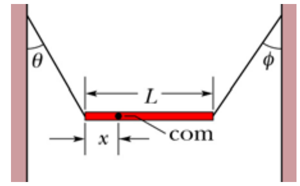
To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half The tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of (1.22 m)2 − (0.381 m)2 = 1.16 m. This means $\tan\theta = (1.16\text{ m})/(0.381\text{ m}) = 3.04$. Thus, $T = 3(854\text{ N})/4(3.04) = 211$ N .

B- We now solve for FA. Since $Fv = FE$ and $FE = T\sin\theta/2\cos\theta$, $Fv = 3W/8$. We substitute this into $Fv + FA - W = 0$ and solve for FA. We find

$FA = W - Fv = W - 3W/8 = 5W/8 = 5(884\text{ N})/8 = 534$ N .

We have already obtained an expression for FE: $FE = 3W/8$. Evaluating it, we get $FE = 320$ N.

<u>Problem 4</u>: A nonuniform bar is suspended at rest in a horizontal position by two massless cords as shown in the figure. One cord makes the angle θ = 36.9° with the vertical; the other makes the angle ϕ = 53.1° with the vertical. If the length L of the bar is 6.1 m, compute the distance x from the left-hand end of the bar to its center of mass.



<u>Answer</u>: The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let *Tl* be the
tension force of the left-hand cord, *Tr* be the tension force of the right-hand cord, and *m* be the mass of
the bar. The equations for equilibrium are:
vertical force components $Tl \cos\theta + Tr \cos\varphi - mg = 0$
horizontal force components $-Tl \sin\theta + Tr \sin\varphi = 0$
torques $mgx - TrL\cos\varphi = 0$.
The origin was chosen to be at the left end of the bar for purposes of calculating the torque.
The unknown quantities are *Tl*, *Tr*, and *x*. We want to eliminate *Tl* and *Tr*, then solve for *x*. The second equation yields $Tl = Tr \sin\varphi / \sin\theta$ and when this is substituted into the first and solved for *Tr* the result is $Tr = mg \sin\theta / (\sin\varphi \cos\theta + \cos\varphi \sin\theta)$. This expression is substituted into the third equation and the result is solved for *x*:
$x = L \sin\theta \cos\varphi / \sin\varphi \cos\theta + \cos\varphi \sin\theta = L \sin\theta \cos\varphi / \sin(\theta + \varphi)$.
The last form was obtained using the trigonometric identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$. For the special case of this problem $\theta + \varphi = 90^\circ$ and $\sin(\theta + \varphi) = 1$. Thus,
$x = L \sin\theta \cos\varphi = (6.10\text{m}) \sin 36.9^\circ \cos 53.1^\circ = 2.20\text{m}$.