Chapter 5.1

Page 1:

Alright everyone, welcome. We are now embarking on Chapter 5.1, which delves into the "Fundamentals of Lasers." This section, prepared by Distinguished Professor Dr. M A Gondal for the Physics 608 Laser Spectroscopy course, will lay the foundational principles that govern how lasers operate.

Understanding these fundamentals is absolutely crucial before we can explore their sophisticated applications in spectroscopy. We'll be covering the essential ingredients of a laser, the conditions required for light amplification, the role of the optical resonator, and the basic equations that describe laser behavior. So, let's begin our journey into the fascinating world of lasers.

Page 2:

Now, let's start with a crucial question: "Lasers in Spectroscopy — Why Do They Matter?" Why have lasers become such indispensable tools in the field of spectroscopy, revolutionizing it in many ways? This page outlines several key reasons, and I want to elaborate on each because their importance cannot be overstated.

First, lasers "Provide monochromatic radiation with linewidths far below 10^{-6} nanometers." Let's break this down. "Monochromatic radiation" means light of essentially a single wavelength, or a very narrow range of wavelengths. The "linewidth" quantifies this narrowness – how spread out in wavelength or frequency the laser light is. A linewidth far below 10^{-6} nanometers, which is just one picometer, is exceptionally narrow. To put this in perspective, conventional light sources, like lamps, emit light over a very broad range of wavelengths. For example, a typical deuterium lamp used in UV spectroscopy might have a useful output over tens or hundreds of nanometers. Even so-called "line sources" like hollow cathode lamps have linewidths orders of magnitude broader than what a laser can achieve.

Why is this extreme monochromaticity so vital for spectroscopy? Spectroscopy is all about studying the interaction of light with matter to probe energy levels, which are often very sharply defined. If your light source is broad, it's like trying to measure a very fine detail with a thick, blunt ruler. You simply can't resolve the fine features of an atomic or molecular spectrum. The narrow linewidth of a laser allows us to selectively excite or probe extremely specific transitions, resolving closely spaced energy levels and observing subtle spectral details that would be completely smeared out with a conventional source. This high spectral resolution is a hallmark of laser spectroscopy.

Second, lasers "Deliver extremely high spectral radiance (power per unit area per unit solid angle per unit bandwidth) exceeding that of conventional lamps by greater than 10^{10}." "Spectral radiance" is a critical figure of merit for a light source. Let's unpack its components:

• Power: The total optical power emitted by the source. • Per unit area: The power emitted from a specific area of the source. • Per unit solid angle: The power emitted into a specific cone of directions. • Per unit bandwidth: The power emitted within a specific narrow range of wavelengths or frequencies.

So, spectral radiance tells us how much power we can get, from a small spot, going in a specific direction, within a very narrow spectral interval. The fact that lasers exceed conventional lamps in spectral radiance by a factor of more than ten billion – that's 10^{10} – is astounding!

What does this mean practically for spectroscopy?

• High sensitivity: Because so much light can be concentrated into the specific wavelength and direction interacting with your sample, you can detect very weak signals or very small amounts of material. • Nonlinear spectroscopy: Many advanced spectroscopic techniques, which we will discuss later, rely on nonlinear optical effects. These effects typically require very high light intensities to become significant. The high spectral radiance of lasers provides these necessary high intensities, opening up entirely new avenues of spectroscopic investigation that are impossible with conventional sources.

Third, lasers "Supply excellent spatial coherence, which leads to diffraction-limited beams, enabling long interaction lengths and tight focusing." "Spatial coherence" means that the phase of the light wave is well-defined and consistent across the wavefront of the beam. Imagine ripples on a pond; if they are all perfectly ordered and in step, that's like spatial coherence. A consequence of high spatial coherence is that laser beams can be "diffraction-limited." This means the beam diverges (spreads out) only by the minimum amount dictated by the laws of diffraction, and it can be focused down to a very small spot size, also limited only by diffraction.

What are the benefits for spectroscopy?

• Long interaction lengths: Because a laser beam can remain highly collimated (parallel) over long distances, you can pass it through a long path length of your sample. For absorption spectroscopy, according to the Beer-Lambert law, the absorbance is proportional to the path length. So, long interaction lengths dramatically increase sensitivity for detecting weakly absorbing species. • Tight focusing: The ability to focus a laser beam to a very small spot (micrometers or even nanometers) creates extremely high local intensities. This is vital for nonlinear spectroscopy, as mentioned, and also for techniques like laser scanning microscopy or for probing very small sample volumes.

These three properties – extreme monochromaticity, enormous spectral radiance, and excellent spatial coherence – are the primary reasons why lasers have transformed spectroscopy.

Page 3:

Continuing with "Lasers in Spectroscopy — Why They Matter," we have two more critical advantages. The fourth key property is that lasers "Allow precise control of frequency and phase." This capability is "essential for Doppler-free, saturation, and heterodyne techniques." Let's delve into this. "Precise control of frequency" means we can tune the laser's emission wavelength with very high accuracy and stability, often to within kilohertz or even hertz of a desired frequency. "Precise control of phase" means we can manage the phase of the electromagnetic wave, which is crucial for interferometric measurements. Why is this control so important for these advanced techniques?

- Doppler-free spectroscopy: In a gas, atoms or molecules are moving randomly. Due to the Doppler effect, each particle "sees" a slightly different light frequency depending on its velocity relative to the beam. This leads to Doppler broadening of spectral lines, often obscuring fine details. Doppler-free techniques, like saturation spectroscopy or two-photon spectroscopy, cleverly use the properties of lasers (like counter-propagating beams of precisely the same frequency) to interrogate only those atoms or molecules with a specific velocity component (usually zero velocity along the beam axis), thereby eliminating Doppler broadening and revealing the true, natural linewidth of the transition. This requires exquisite frequency control.

- Saturation spectroscopy: This is a common Doppler-free technique. A strong "pump" beam, tuned to an atomic or molecular transition, selectively excites (saturates) particles in a narrow velocity group. A weaker "probe" beam, often from the same laser or another precisely controlled laser, then measures the absorption. The precise frequency control of both beams is paramount.

- Heterodyne techniques: Heterodyne detection involves mixing the signal light (e.g., light transmitted through or scattered by a sample) with a strong, stable reference laser beam, called a local oscillator, on a detector. The detector then produces a beat signal at the difference frequency. This technique allows for extremely sensitive detection of weak signals and measurement of both amplitude and phase of the light. The phase stability and precise frequency control of the local oscillator laser are absolutely critical for successful heterodyne detection.

The fifth major advantage is that lasers "Offer tunability (via dye, Ti:sapphire, OPO, etc.)." This tunability allows "access to virtually any atomic / molecular transition when combined with nonlinear optics." "Tunability" means we can change the output wavelength of the laser, ideally over a broad range. While some lasers operate at fixed frequencies (like the He-Ne laser at 632.8 nanometers), many important laser types are tunable:

- Dye lasers: These use organic dyes as the gain medium and can be tuned over tens to hundreds of nanometers, typically in the visible and near-UV/IR spectral regions, by changing the dye or tuning elements within the cavity.

- Titanium-sapphire lasers (often written Ti:Al_2O_3, pronounced Ti-sapphire): These are solid-state lasers with a very broad tuning range, typically from about 650 nanometers to 1100 nanometers in the near-infrared. They are workhorses for many applications.

- Optical Parametric Oscillators (OPOs): These are devices based on nonlinear optical crystals that can convert a fixed-frequency pump laser into two tunable output beams (signal and idler) whose frequencies sum to the pump frequency. OPOs can provide tunable output over very wide ranges, from the UV to the mid-infrared.

The ability to tune the laser wavelength is incredibly powerful because different atoms and molecules have their characteristic transitions at different wavelengths. A tunable laser allows a spectroscopist to "dial in" to the specific transition of interest. Furthermore, "when combined with nonlinear optics," the wavelength coverage can be extended even further. Nonlinear optical processes like:

- Frequency doubling (Second Harmonic Generation, SHG): Converts light to twice its frequency (half its wavelength). For example, green laser pointers are often infrared lasers whose output is frequency-doubled.

- Sum Frequency Generation (SFG) and Difference Frequency Generation (DFG): Mix two laser beams in a nonlinear crystal to produce output at the sum or difference of their frequencies.

By using these nonlinear techniques with tunable lasers, spectroscopists can generate coherent radiation at virtually any wavelength needed to probe a vast array of atomic and molecular transitions, from the deep UV to the far-infrared. This "access to virtually any transition" has truly opened up the entire electromagnetic spectrum for high-resolution spectroscopic study.

Page 4:

This page provides a wonderful visual summary of the key laser properties we've just discussed, comparing lasers to conventional lamp sources and illustrating the benefits.

Let's first look at the top graph, titled "Laser vs. Lamp Spectrum (Monochromaticity & High Radiance)." The vertical axis represents "Intensity" in arbitrary units, and the horizontal axis is "Wavelength" in nanometers, ranging from 400 nanometers (blue light) to 700 nanometers (red light), covering the visible spectrum.

We see two distinct spectral profiles: * The "Lamp Spectrum" is shown as a broad, gentle blue curve, peaking around 550 nanometers but spread out over the entire visible range. This illustrates the wide range of wavelengths emitted by a typical broadband lamp. *In stark contrast, the "Laser Line" is depicted as an extremely sharp, tall red spike, also centered around* 550 nanometers in this example. The key here is its incredible narrowness. An annotation states "$\Delta\lambda < 10^{-6}$ nm" – that is, "Delta lambda is less than 10^{-6} nanometers" – emphasizing the picometer-level or better linewidth we discussed. While the height of the laser line here is shown as approximately five times the peak of the lamp spectrum, this is purely illustrative for visibility; the true distinction in spectral radiance* is far more dramatic. As another annotation points out: "Spectral Radiance: Laser is greater than 10^{10} times that of a Lamp." This underscores the immense concentration of power within that tiny spectral bandwidth for a laser.

To the right of this graph, a list summarizes these "Key Laser Properties": * "Monochromatic ($\Delta\lambda \ll 10^{-6}$ nm)" – Delta lambda is much, much less than 10^{-6} nanometers. We see this in the narrow red spike. * "High Spectral Radiance ($> 10^{10}$ × conventional)" – Greater than 10^{10} times conventional sources. This is represented by the concept of packing so much energy into that narrow spike. * "Spatially Coherent (Diffraction-limited beams)" – We'll see this in the next diagram. * "Precisely Controllable (Frequency/Phase)" – Essential for advanced techniques. * "Tunable (Broad Wavelength Access)" – Illustrated in the bottom right diagram.

Now, let's move to the bottom left, under "Spatial Coherence & Long Interaction." We see a schematic: a "Laser Source" emits a beam that is depicted as highly collimated (parallel rays) with an initial diameter $d \approx 1\,\text{mm}$ (d is approximately 1 millimeter). This beam passes through an "Absorption Cell" containing the sample. The beam remains well-collimated over a "Long Interaction Length (e.g., $L = 1 - 100\,\text{cm}$)" – L equals 1 to 100 centimeters. An annotation highlights this as an "Excellent Spatial Coherence → Highly Collimated Beam." This ability to maintain a narrow, directed beam over significant distances is a direct result of spatial coherence and is crucial for sensitive measurements, particularly in absorption spectroscopy.

Finally, at the bottom right, we have "Frequency Tunability." This is illustrated with a "Frequency Tuning Knob" superimposed on a dial that spans spectral regions: "UV" (ultraviolet), "VIS" (visible), "NIR" (near-infrared), and "IR" (infrared), indicating a "Wide Tunable Range." Below this, text explains: "Access to virtually any transition (with NLO)" – NLO stands for NonLinear Optics – "e.g., Dye, Ti:Sapphire, OPO." This visually reinforces the idea that by choosing the right type of laser (like Dye, Titanium-Sapphire, or an Optical Parametric Oscillator) and potentially employing nonlinear optical techniques, we can generate laser light across a vast portion of the electromagnetic spectrum.

So, this page beautifully encapsulates why lasers are so powerful for spectroscopy: they provide light that is incredibly pure in color, intensely bright within that color, highly directional, precisely controllable, and widely tunable.

Page 5:

Now we're moving on to "Slide 2: Minimum Ingredients of Any Laser." What are the fundamental components and conditions required to build a device that can produce this special kind of light we call laser light? There are essentially three core ingredients, plus the resonator which we'll detail shortly.

The first, and perhaps most central, ingredient is the "Active medium." This is defined as a "material that can amplify light." The active medium, also sometimes called the gain medium, is the heart of the laser. It's a collection of atoms, ions, molecules, or specific centers in a solid-state material that can be energized, or "excited," to a state where they can release that energy in the form of light through a process called stimulated emission. This stimulated emission is what leads to light amplification. The choice of active medium determines the range of wavelengths the laser can produce and many of its other characteristics. Examples range from gases like helium and neon in a He-Ne laser, to crystals like Nd:YAG (Neodymium-doped Yttrium Aluminum Garnet), to semiconductors in diode lasers, or liquids like organic dyes in dye lasers.

The second point elaborates on a property of this active medium: it "Contains discrete energy levels $E_\mathrm{i}, E_\mathrm{k}$ of atoms, ions, molecules, or solid-state centers." (E_i, E_k). Laser action relies on transitions between well-defined, quantized energy levels within the constituents of the active medium. When we talk about E_i and E_k, we're typically referring to a lower energy level (E_i) and an upper energy level (E_k) that are involved in the lasing transition. The energy difference between these levels, $E_k - E_i$, dictates the energy, and thus the frequency (or wavelength), of the photons that will be emitted, according to Planck's relation $E = h\nu$ (Energy equals Planck's constant times frequency). Without these discrete energy levels, we wouldn't have the specific, well-defined wavelengths characteristic of lasers.

The third essential ingredient is an "Energy pump." This is an "external energy source that drives population inversion." Now, this is critical. Under normal thermal equilibrium conditions, lower energy levels are always more populated than higher energy levels, following the Boltzmann distribution. If light of the transition frequency passes through such a medium, it will be absorbed more than it is amplified. To achieve light amplification (gain), we need to reverse this situation for the specific pair of energy levels E_i and E_k. We need more atoms (or molecules, etc.) in the upper energy level E_k than in the lower energy level E_i. This non-equilibrium condition is called "population inversion." The energy pump is the mechanism by which we supply energy to the active medium to create and maintain this population inversion. The pump "lifts" atoms from lower energy states to the upper lasing level E_k (or to levels that rapidly feed E_k).

The fourth point provides "Examples" of such energy pumps:

- "Electrical discharge": This is common in gas lasers. An electric current is passed through the gas, and collisions between electrons and gas atoms/molecules excite them to higher energy levels. Think of a neon sign – that's an electrical discharge exciting neon atoms, though not necessarily creating a population inversion for lasing in that simple case. In lasers like He-Ne or CO_2 (carbon dioxide) lasers, the discharge parameters are carefully controlled to achieve inversion.

- "Optical flashlamp": Intense flashes of light from a lamp (like a xenon flashlamp) can be used to pump solid-state lasers (like ruby or Nd:YAG) or dye lasers. The atoms in the gain medium absorb the pump light and are excited.

- "Chemical reaction": In some lasers, known as chemical lasers (e.g., HF, or hydrogen fluoride lasers), the energy released during an exothermic chemical reaction directly populates the upper laser level, creating population inversion.

- "Another laser": It's very common to use one laser to pump another. For example, high-power diode lasers are often used to pump solid-state lasers like Nd:YAG or Ti:sapphire lasers. This offers high efficiency and selectivity in pumping.

So, to summarize this slide, we need an active medium with discrete energy levels, and an energy pump to create a population inversion in that medium. These are the prerequisites for light amplification.

Page 6:

Slide 2: Minimum Ingredients of Any Laser

Continuing with "Slide 2: Minimum Ingredients of Any Laser," we now come to the crucial component that turns a light amplifier into an oscillator, that is, a laser.

This is the "Optical resonator." It's "typically two or more mirrors forming a Fabry-Pérot cavity of length d."

An optical resonator, or optical cavity, is an arrangement of mirrors that confines light, allowing it to make multiple passes through the active medium. The simplest and most common type is the Fabry-Pérot resonator, which consists of two mirrors aligned parallel to each other, separated by a distance d, with the active medium placed between them. One mirror is usually a high reflector (close to 100% reflectivity), and the other is a partial reflector (the output coupler), which allows a fraction of the light to escape as the laser beam. While two mirrors are typical, more complex resonator designs can involve more optical elements.

The next bullet point explains the vital roles of this optical resonator: It "Provides feedback, selects longitudinal and transverse modes, and increases photon dwell time." Let's break these functions down:

* "Provides feedback": This is the most fundamental role. As photons are generated in the active medium via stimulated emission, they travel towards the mirrors. The mirrors reflect these photons back into the active medium. These reflected photons can then stimulate further emission, leading to an avalanche effect and a buildup of light intensity. This positive feedback is what transforms an amplifier (which would just boost an incoming signal once) into an oscillator (which generates its own sustained output).

* "Selects longitudinal and transverse modes": A resonator doesn't just amplify any light; it selectively amplifies light that "fits" certain conditions. * Longitudinal modes refer to specific resonant frequencies (or wavelengths) that can form stable standing waves within the cavity. Only light at these frequencies will constructively interfere and build up in intensity. The spacing of these modes depends on the cavity length d. * Transverse modes (like TEM_{00}, TEM_{01}, etc. – Transverse ElectroMagnetic modes) describe the spatial intensity pattern of the beam in the plane perpendicular to its propagation. The curvature of the mirrors and any apertures within the cavity determine which transverse modes are stable and have low loss. Often, lasers are designed to operate in the fundamental TEM_{00} mode, which has a desirable Gaussian beam profile.

* "Increases photon dwell time": By reflecting photons back and forth, the resonator significantly increases the time that photons spend interacting

with the active medium (the "dwell time"). This allows for a much greater cumulative amplification. Even if the gain per pass through the active medium is small, many passes can lead to substantial overall gain, sufficient to overcome losses and sustain laser oscillation.

So, the active medium provides the potential for amplification, the pump creates the necessary population inversion, and the optical resonator provides the feedback and mode selection that channels this amplified energy into a coherent, directional laser beam.

Page 7:

This page, still part of "Slide 2: Minimum Ingredients of Any Laser," presents a "Schematic Cross-Section of a Fabry-Pérot Laser," which beautifully visualizes the components we've just discussed.

Let's examine the diagram. We see two parallel mirrors, labeled R, which stands for reflectivity. These mirrors define the "Optical Resonator." The distance between these mirrors is labeled d, representing the "Cavity Length."

Positioned between these mirrors is a light blue rectangular block labeled "Active Medium." The length of this active medium is denoted by L (capital L), labeled as "Active Medium Length." Notice that in this schematic, $L < d$, meaning the active medium does not necessarily fill the entire cavity, which is often the case in real lasers.

Orange arrows, labeled "Energy Pump," are shown impinging on the active medium from the top and bottom. This represents the external energy source that, as we discussed, creates the population inversion within the active medium.

A dashed line runs horizontally through the center of the cavity and the active medium, representing the optical axis.

From the right-hand mirror, a thick red arrow labeled "Laser Output" is shown emerging along the z-axis (indicated by a small 'z'). This signifies that the right-hand mirror is partially transmissive, acting as the output coupler, allowing a portion of the amplified light circulating within the cavity to escape as the usable laser beam. The left-hand mirror would typically be a high reflector.

This simple diagram elegantly brings together all the minimum ingredients:

1. The "Active Medium" (length L), which is the material capable of amplifying light. 2. The "Energy Pump," which energizes the active medium to achieve population inversion. 3. The "Optical Resonator" (length d), formed by the two mirrors, which provides feedback, allows for multiple passes through the active medium, and helps select the lasing modes.

The interplay of these components leads to the generation of the "Laser Output." This schematic is fundamental to understanding the basic structure of most lasers.

Page 8:

Slide 3: The Active Medium — Energy Levels and Gain

Now we turn to "Slide 3: The Active Medium — Energy Levels and Gain." Having established the need for an active medium, let's delve deeper into how it works, focusing on the energy level populations and the condition for achieving gain.

The first point addresses the situation in "Thermal equilibrium: population follows Boltzmann law."

Under normal conditions, without any external pumping, the distribution of atoms or molecules among their various energy levels is governed by thermal equilibrium and is described by the Boltzmann law. The equation presented is:

$$N_{\text{eq}}(E) = N_0 \exp\left(-\frac{E}{k_{\text{B}}T}\right)$$

Let's read this out carefully: "N sub e q of capital E, equals N sub zero, times the exponential of, minus capital E divided by the product of k sub B and capital T."

Now, let's deconstruct each term:

- $N_{\text{eq}}(E)$ (N sub e q of capital E): This is the population density (number of atoms or molecules per unit volume) in an energy level E, when the system is in thermal equilibrium. Its units would typically be something like inverse cubic centimeters (cm^{-3}).

- N_0 (N sub zero): This represents the population density of the ground state (the lowest energy level). It also has units of inverse cubic centimeters.

- $\exp$: This is the exponential function.

- E (capital E): This is the energy of the specific level whose population we are considering, relative to the ground state energy (which is often taken as zero). Its units are Joules.

- k_{B} (k sub B): This is the Boltzmann constant, a fundamental physical constant. Its value is given on the slide.

- T (capital T): This is the absolute temperature of the medium, in Kelvin.

The Boltzmann law tells us that at any finite temperature, higher energy levels are exponentially less populated than lower energy levels. For laser action, we need to overcome this natural tendency.

The second bullet point clarifies the constants used:

- N_0 (N sub zero): ground-state population density $\(\text{cm}^{-3}$\)

- k_{B} (k sub B): Boltzmann constant, which is 1.38×10^{-23} J K^{-1}

- T (capital T): absolute temperature $[K]$

The third, and most crucial point on this slide, is the "Population inversion condition." For an active medium to provide gain (i.e., to amplify light), we must create a situation where there are more atoms (or molecules) in the upper lasing energy level than in the lower lasing energy level, when accounting for the degeneracies of these levels. This is population inversion. The condition is given by the inequality:

$$N_\mathrm{k} > \left(\frac{g_\mathrm{k}}{g_\mathrm{i}}\right) N_\mathrm{i}$$

Let's read this: "N sub k, is greater than, the ratio of g sub k over g sub i, all times N sub i."

Let's deconstruct this:

- N_k (N sub k): This is the population density of the upper lasing level (energy E_k).
- N_i (N sub i): This is the population density of the lower lasing level (energy E_i).
- g_k (g sub k): This is the degeneracy of the upper lasing level E_k. Degeneracy means there are g_k distinct quantum states that all have the same energy E_k.
- g_i (g sub i): This is the degeneracy of the lower lasing level E_i.

The condition can be rewritten as

$$\frac{N_\mathrm{k}}{g_\mathrm{k}} > \frac{N_\mathrm{i}}{g_\mathrm{i}}$$

This means that the population density *per state* in the upper level must be greater than the population density *per state* in the lower level. If this condition is met, then an incoming photon of the correct frequency is more likely to cause a stimulated emission (atom goes from E_k to E_i, releasing a photon) than to be absorbed (atom goes from E_i to E_k, consuming a photon). This is the fundamental requirement for light amplification.

Page 9:

Continuing our discussion on "Slide 3: The Active Medium — Energy Levels and Gain," we build upon the concept of population inversion.

The first point here clarifies a term we just used: g_j (g sub j): degeneracy of level j. As mentioned, the degeneracy of an energy level "j" refers to the number of distinct quantum states that share that same energy E_j. For example, in atoms, electron spin or orbital angular momentum can lead to degenerate energy levels. These degeneracies are important because they affect the statistical likelihood of finding an atom in a particular energy level, and they influence the rates of transitions between levels.

The second point describes what happens when population inversion is achieved: "With inversion, stimulated-emission rate exceeds absorption rate:" This is expressed by the inequality:

$$W_\text{stim} = N_\text{k} B_{ki} \rho(\nu) > N_\text{i} B_{ik} \rho(\nu)$$

Let's carefully read this: " W_stim, equals N_k, times B_{ki}, times $\rho(\nu)$, which is greater than N_i, times B_{ik}, times $\rho(\nu)$".

Let's break down the terms:

- W_stim (W sub stim): This represents the net rate of stimulated transitions. More precisely, the left side is the rate of stimulated emission events per unit volume, and the right side is the rate of absorption events per unit volume.
- N_k (N sub k): Population density of the upper lasing level.
- N_i (N sub i): Population density of the lower lasing level.
- B_{ki} (B sub k i): This is the Einstein B-coefficient for stimulated emission from the upper level k to the lower level i. It quantifies the probability per

unit time, per unit energy density of the radiation field, that an atom in level k will be stimulated to emit a photon and transition to level i.

• B_{ik} (B sub i k): This is the Einstein B-coefficient for absorption, representing the probability for a transition from the lower level i to the upper level k upon interaction with a photon.

• $\rho(\nu)$ (rho of nu): This is the spectral energy density of the radiation field at the transition frequency ν (nu). It represents the amount of electromagnetic energy per unit volume per unit frequency interval. Its units are typically Joules per meter cubed per Hertz.

The Einstein B-coefficients B_{ki} and B_{ik} are related by their degeneracies: $g_k B_{ki} = g_i B_{ik}$. If we incorporate this into the population inversion condition $\frac{N_k}{g_k} > \frac{N_i}{g_i}$, it directly leads to $N_k B_{ki} > N_i B_{ik}$. Multiplying both sides by $\rho(\nu)$ gives the inequality shown. This confirms that when population inversion is achieved (considering degeneracies correctly), the rate of stimulated emission indeed exceeds the rate of absorption.

The third bullet point provides the units for these coefficients:

• B_{ik}, B_{ki}: Einstein B-coefficients have units of $[\mathrm{m^3\,J^{-1}s^{-2}}]$ (meters cubed per Joule per second squared).

• $\rho(\nu)$ (rho of nu): spectral energy density has units of $[\mathrm{J\,m^{-3}\,Hz^{-1}}]$ (Joules per meter cubed per Hertz).

Finally, the crucial outcome: "Result: passing electromagnetic wave is amplified instead of attenuated." This is the very definition of an active medium providing gain. Because stimulated emission events (which add photons identical to the incident photons) outnumber absorption events (which remove photons), the net effect is an increase in the intensity of the light wave as it propagates through the medium. This is the fundamental process that makes lasers possible.

Page 10:

This page, still part of "Slide 3: The Active Medium — Energy Levels and Gain," presents a very instructive diagram titled "Energy Level Populations and Stimulated Emission." This visual helps solidify the concepts we've been discussing.

Let's describe the diagram. The vertical axis represents "Energy (E)," increasing upwards. The horizontal axis represents "Population (N)" of the energy levels, increasing to the right.

Three discrete energy levels are shown as horizontal dotted lines: * E_0 (E sub zero) at the bottom, labeled "(Ground State)." * E_1 (E sub one) above it, labeled "(Lower Laser Level)." * E_2 (E sub two) at the top, labeled "(Upper Laser Level)."

Two different population distributions are depicted:

1. A solid blue line shows the "Boltzmann Distribution." It starts with a high population at the ground state E_0 and shows a decreasing population as energy increases to E_1 and then to E_2. This represents the normal situation in thermal equilibrium, where lower energy levels are more populated than higher ones.

2. A dashed red line illustrates "Population Inversion." This line shows that the population of the upper laser level E_2 is significantly greater than the population of the lower laser level E_1. The annotation confirms this: "Population Inversion ($N_\mathrm{k} > N_\mathrm{i}$)," where N_k would correspond to the population of E_2 and N_i to E_1. This inverted state is, of course, not the natural equilibrium state and must be achieved by pumping.

The diagram then illustrates the process of stimulated emission: An incoming photon, represented by an orange arrow labeled $h\nu$ (h nu), interacts with an atom that is in the upper laser level E_2 (where the population is high due to inversion). This interaction stimulates the atom to transition down to the lower laser level E_1.

Crucially, in this process, the atom emits a *second* photon that is identical to the incident photon – same frequency, same direction, same phase, and same polarization. This is shown by two orange arrows emerging downwards from E_2, labeled $2\,h\nu$ (two h nu), and the process is labeled "Stimulated Emission" with a green downward arrow indicating the atomic transition from E_2 to E_1.

The diagram effectively contrasts the normal Boltzmann distribution, where absorption would dominate, with the population-inverted scenario, where stimulated emission dominates for photons of energy $h\nu = E_2 - E_1$. This process of getting two photons out for one incident photon (when interacting with an excited atom) is the microscopic basis of light amplification in a laser. The key is to first establish that population inversion between E_2 and E_1.

Page 11:

Slide 4: Energy-Pumping Mechanisms — Achieving Inversion.

We now move to "Slide 4: Energy-Pumping Mechanisms — Achieving Inversion." We've established that population inversion is essential for laser action. This slide discusses the various methods, or mechanisms, used to pump energy into the active medium to create this non-equilibrium state.

The first method listed is "Optical pumping with broadband flashlamp — used in ruby lasers." Optical pumping means using light from an external source to excite the atoms or molecules in the active medium. A "broadband flashlamp," like a xenon or krypton flashlamp, emits intense pulses of light over a wide range of wavelengths. Some of this light will match the absorption bands of the active medium, exciting its constituents to higher energy levels, which then might decay into the upper laser level E_k, leading to population inversion. The very first laser, the ruby laser demonstrated by Theodore Maiman in 1960, used this method. It's still used for some solid-state and dye lasers, particularly when high pulse energies are needed.

The second mechanism is "Electrical discharge in low-pressure gas mixtures — He-Ne, Ar^+, CO_2." In gas lasers, a common pumping method is to pass an electrical current through the gas – an electrical discharge. Energetic electrons in the discharge collide with the gas atoms or molecules. These collisions can excite the atoms/molecules directly to the upper laser level, or to other levels that then transfer their energy to the upper laser level (often via collisions with another gas species, as in the Helium-Neon laser).

* "He-Ne" (Helium-Neon) lasers are classic examples, where helium atoms are excited by electron impact and then resonantly transfer their energy to neon atoms, creating inversion on specific neon transitions (like the famous red 632.8 nm line).

* "Ar^+" (Argon ion) lasers use a high-current discharge to ionize argon atoms and then excite the argon ions to states that can lase, producing blue and green light.

* "CO_2" (Carbon dioxide) lasers, which are very powerful infrared lasers, use an electrical discharge in a mixture of CO_2, nitrogen, and helium. Nitrogen molecules are excited by electron impact and then efficiently transfer their vibrational energy to CO_2 molecules, creating inversion.

The third type is "Chemical pumping — HF, DF, and excimer lasers." In chemical lasers, the energy for population inversion comes directly from the exothermic energy released during a chemical reaction.

"HF" (Hydrogen Fluoride) and "DF" (Deuterium Fluoride) lasers are examples where reactions like \(\text{F} + \text{H}_2 \rightarrow \text{HF}^{} + \text{H}\) produce molecules directly in excited states capable of lasing. These can be very high-power lasers.

* "Excimer lasers" (from "excited dimer") use molecules that are stable only in their excited state, like KrF (Krypton Fluoride) or ArF (Argon

Fluoride). An electrical discharge creates these excited molecules, which then lase as they dissociate to their unstable ground state. Excimer lasers typically emit in the ultraviolet and are used in applications like semiconductor lithography and eye surgery.

Fourth, we have "Laser-pumped solid-state or dye media — Ti:sapphire pumped by frequency-doubled Nd:YAG." This is a very important and widely used technique: using one laser to pump another. This allows for very efficient and selective excitation of the active medium.

* "Solid-state media" like Ti:sapphire (Titanium-doped sapphire, Ti:Al_2O_3) or Nd:YAG can be pumped by other lasers, often diode lasers or other solid-state lasers.

* "Dye media" (organic dyes in solution) are almost always optically pumped, frequently by another laser (like a pulsed Nd:YAG laser or a CW argon ion laser).

* The specific example given, "Ti:sapphire pumped by frequency-doubled Nd:YAG," is a common setup. A Nd:YAG laser produces infrared light (e.g., at 1064 nm). This light is then passed through a nonlinear crystal to double its frequency, producing green light (at 532 nm). This green light is efficiently absorbed by the Ti:sapphire crystal, pumping it to create a population inversion and enabling tunable laser output in the red and near-infrared.

Each of these pumping mechanisms has its own advantages, disadvantages, and specific applications, but all share the common goal of creating that crucial population inversion.

Page 12:

Continuing with "Slide 4: Energy-Pumping Mechanisms," this page highlights a crucial design consideration and notes a missing visual.

The first point states a "Key design goal: deposit energy selectively into upper laser level while depleting lower level swiftly (short τ_{eff})." Let's break this down because it encapsulates much of the art and science of laser design.

* "Deposit energy selectively into upper laser level": The pump energy should, as much as possible, go into populating the desired upper laser level E_{k} (or levels that quickly feed it). If the pump energy excites many other levels that don't contribute to the lasing transition, or that lead to losses, the laser will be inefficient. So, matching the pump source characteristics (e.g., wavelength for optical pumping, electron energy distribution for discharge pumping) to the absorption properties of the active medium is critical for selectivity.

* "while depleting lower level swiftly (short τ_{eff})" – τ_{eff} (tau sub effective) refers to the effective lifetime of the lower laser level E_{i}. For efficient and continuous population inversion $N_{\text{k}} > \left(\frac{g_{\text{k}}}{g_{\text{i}}}\right) N_{\text{i}}$, the population N_{i} of the lower laser level must be kept as low as possible. If atoms arriving in E_{i} (after stimulated or spontaneous emission from E_{k}) linger there for a long time, N_{i} will build up and can eventually terminate the inversion. Therefore, a key design goal is to ensure that the lower laser level E_{i} has a very fast relaxation pathway, typically to the ground state or some other lower energy level, so that it empties quickly. This "short τ_{eff}" for the lower level is a hallmark of an efficient four-level laser system, which we'll discuss more later. It prevents a "bottleneck" at the lower lasing level.

The second point indicates: "[IMAGE REQUIRED: Table-style graphic listing pump types versus representative lasers, with arrows from pump energy to specific transition levels.]"

Since this image is not present, I will describe what it would ideally convey. Such a graphic would be extremely helpful for students. It would likely be a table where, for instance, rows list different pump types (e.g., flashlamp, electrical discharge, diode laser pumping) and columns list representative laser systems (e.g., Ruby, He-Ne, Nd:YAG, Ti:sapphire).

Crucially, within the cells of this table, or accompanying it, there would be simplified energy level diagrams for each laser type. Arrows would illustrate how the specific pump energy is absorbed by the active medium (e.g., an arrow from a "pump band" down to the upper laser level E_{k}) and then indicate the lasing transition (an arrow from E_{k} down to E_{i}), and finally, the rapid decay from E_{i} to a lower level (often the ground state).

For example, for a He-Ne laser, it would show electron impact exciting Helium atoms, then resonant energy transfer from Helium to Neon atoms, populating a specific upper level in Neon. Then it would show the lasing transition in Neon, and subsequent decay pathways. For a diode-pumped Nd:YAG laser, it would show the absorption of the diode laser light by Neodymium ions into a pump band, followed by rapid non-radiative decay to the metastable upper laser level, then the lasing transition, and then decay from the lower laser level.

Such a visual would effectively connect the abstract concept of pumping mechanisms to concrete examples and their underlying energy level dynamics.

Page 13:

We now arrive at "Slide 5: Spontaneous vs. Stimulated Emission — Microscopic View." This slide revisits these two fundamental radiative processes, but from the perspective of their probabilities or rates per particle, which are characterized by the Einstein coefficients.

The first point concerns spontaneous emission: "Spontaneous emission probability per excited particle" is given by A_{ki} [s^{-1}].

Let's unpack this:

* "Spontaneous emission" is the process by which an atom or molecule in an excited energy level (say, level k) decays to a lower energy level (level i) on its own, without any external trigger, by emitting a photon. The emitted photon has an energy equal to the energy difference between the levels $(E_{\mathrm{k}} - E_{\mathrm{i}})$. Importantly, spontaneously emitted photons are radiated in random directions and have random phases. * "Probability per excited particle": A_{ki} (A sub k i) is the Einstein A-coefficient for the transition from level k to level i. It represents the probability per unit time that a single atom in level k will spontaneously decay to level i by emitting a photon. * [s^{-1}] (per second): The units of A_{ki} are inverse seconds, signifying a rate. The reciprocal of A_{ki}, which is $\frac{1}{A_{ki}}$, is the radiative lifetime (τ_{rad}, tau sub rad) of level k with respect to this specific transition to level i. If there are multiple spontaneous decay paths from level k, the total spontaneous decay rate is the sum of the A_{ki} coefficients for all possible lower levels i.

The second point addresses stimulated emission: "Stimulated emission probability density per excited particle" is given by $B_{ki}\rho(\nu)$ [s^{-1}].

Let's break this down:

* "Stimulated emission" occurs when an atom already in an excited state (level k) is "stimulated" by an incoming photon of the correct frequency $\nu = \frac{E_{\mathrm{k}} - E_{\mathrm{i}}}{h}$ to transition to a lower energy state (level i). In doing so, it emits a second photon that is an exact replica of the incident photon – same frequency, same direction, same phase, and same polarization. This is the process responsible for light amplification in lasers. * "Probability density per excited particle": The term $B_{ki}\rho(\nu)$ (B sub k i times rho of nu) gives the probability per unit time that a single atom in level k will undergo stimulated emission to level i, in the presence of a radiation field of spectral energy density $\rho(\nu)$ at the transition frequency. * B_{ki} (B sub k i) is the Einstein B-coefficient for stimulated emission. It quantifies the intrinsic strength of the transition's interaction with the radiation field leading to stimulated

emission. Its units depend on how $\rho(\nu)$ is defined (e.g., per unit frequency or per unit wavelength). As we saw on page 9, if $\rho(\nu)$ is energy per unit volume per unit frequency, B_{ki} has units like $\text{m}^{3}\,\text{J}^{-1}\text{s}^{-2}$. * $\rho(\nu)$ (rho of nu) is the spectral energy density of the incident radiation field at frequency ν. The more intense the stimulating field (i.e., the larger $\rho(\nu)$), the higher the rate of stimulated emission. * [s^{-1}] (per second): The product $B_{ki}\rho(\nu)$ has units of a rate (inverse seconds), representing the stimulated emission rate per particle.

The term "probability density" on the slide might be a bit imprecise if it refers to B_{ki} alone; B_{ki} itself is a coefficient, and the product $B_{ki}\rho(\nu)$ is the actual rate (or probability per unit time). The key is that the stimulated emission rate is proportional to both the B_{ki} coefficient (an atomic property) and the density of stimulating photons $\rho(\nu)$ (a field property).

This microscopic view, focusing on the A and B coefficients, is fundamental to understanding the competition between spontaneous and stimulated emission, which is at the heart of laser operation.

Page 14:

Continuing with "Slide 5: Spontaneous vs. Stimulated Emission — Microscopic View," this page explores the consequences of these processes in the context of a laser.

The first point states: "Inversion increases $B_{ki}\rho(\nu)$ → overwhelmingly directs emission into modes where $\rho(\nu)$ is large."

Let's analyze this statement carefully. "Inversion" refers to population inversion $\left(\frac{N_\text{k}}{g_\text{k}} > \frac{N_\text{i}}{g_\text{i}}\right)$. Inversion itself doesn't directly *increase* the value of B_{ki} (which is an atomic constant) or $\rho(\nu)$ (the radiation field density) for a single atom.

Rather, what inversion does is ensure that the *net rate of stimulated emission*, which is proportional to $(N_\text{k}B_{ki} - N_\text{i}B_{ik})\rho(\nu)$, becomes positive and significant. When this happens, stimulated emission becomes the dominant process by which excited atoms release their energy as photons.

The second part of the statement, "→ overwhelmingly directs emission into modes where $\rho(\nu)$ is large," is crucial. Stimulated emission is proportional to $\rho(\nu)$. In a laser cavity, the optical resonator is designed to support specific resonant modes, and within these modes, the photon density $\rho(\nu)$ can build up to very high values. Therefore, if population inversion exists, stimulated emission will preferentially add photons to those cavity modes that already have a high photon density. This is a positive feedback mechanism: more photons in a mode lead to more stimulated emission into that same mode, further increasing the photon density. This is how a laser "chooses" to emit light into specific, well-defined modes rather than randomly like spontaneous emission.

The second point reinforces this: "Optical resonator provides large $\rho(\nu)$ for its eigen-frequencies → feedback loop." The optical resonator is not just a pair of mirrors; it's a frequency-selective device. It has specific resonant frequencies, called "eigen-frequencies" (our longitudinal modes $\nu_{\mathrm{q}} = \frac{qc}{2\,d}$ from before), where light can build up to a high intensity, meaning a large $\rho(\nu)$. Photons at these eigen-frequencies are trapped effectively, making many passes through the gain medium. This high $\rho(\nu)$ at the eigen-frequencies then drives strong stimulated emission at these same frequencies, provided there's population inversion. This creates the "feedback loop": photons at resonant frequencies stimulate more of the same, leading to oscillation.

The third point describes the macroscopic result: "Above threshold, stimulated processes dominate, making output highly directional and coherent." "Above threshold" means the gain from stimulated emission is sufficient to overcome all the losses in the cavity (mirror transmission, scattering, absorption, etc.). Once this condition is met, laser oscillation begins, and stimulated emission becomes by far the dominant emission process. The consequences of stimulated emission dominating are precisely the unique characteristics of laser light:

• "Highly directional": Because stimulated photons are emitted in the exact same direction as the stimulating photons, and the resonator axis defines a preferred direction for photon buildup, the output laser beam is very well-collimated and has low divergence.

• "Highly coherent": Stimulated photons are also emitted with the same phase and frequency as the stimulating photons. This leads to both: – Temporal coherence: The light waves maintain a predictable phase relationship over time (related to monochromaticity). – Spatial coherence: The light waves maintain a predictable phase relationship across different points on the wavefront.

These properties distinguish laser light from the incoherent, randomly directed light produced by spontaneous emission in conventional light sources.

Page 15:

This page, still under "Slide 5: Spontaneous vs. Stimulated Emission — Microscopic View," presents a graph titled "Emission Rates vs. Photon Frequency (ν) for an Inverted Medium." This graph is excellent for visualizing how the interplay between the gain medium and the optical resonator shapes the emission spectrum.

Let's describe the graph:

The vertical axis is "Spectral Emission Rate" in arbitrary units. The horizontal axis is "Photon Frequency (ν)" (nu). The center of the x-axis is marked as ν_0 (nu sub zero), representing the center frequency of the atomic transition. Points like $\nu_0 - \Gamma_\mathrm{A}$ (nu sub zero minus Capital Gamma sub A) and $\nu_0 + \Gamma_\mathrm{A}$ (nu sub zero plus Capital Gamma sub A) are also marked, likely indicating some characteristic width associated with the atomic transition itself.

Three curves are plotted, illustrating different emission components:

1. The blue solid line is labeled "Spontaneous Emission ($\propto A_{ki}g(\nu)$)" – proportional to A_{ki} times $g(\nu)$. This curve is broad and bell-shaped, centered at ν_0. It represents the natural lineshape $g(\nu)$ of spontaneous emission from the inverted medium. Its width Γ_{A} is characteristic of the atomic transition (e.g., Doppler or pressure broadened).

2. The orange dashed line is labeled "Stimulated Emission (broadband/low $\rho(\nu)$)". This curve essentially follows the shape of the spontaneous emission profile. It shows what the stimulated emission rate would look like if the radiation density $\rho(\nu)$ were low and spectrally broad – it would simply mimic the atomic lineshape $g(\nu)$, scaled by $B_{ki}\rho(\nu)$.

3. The red solid line is the most important one. It's labeled "Stimulated Emission (cavity-enhanced $\rho(\nu)$)". This curve shows a dramatically sharp and intense peak, also centered at ν_0 (which is also indicated as a cavity resonance in this depiction). This illustrates the effect of the optical resonator. The resonator enhances the photon density $\rho(\nu)$ very significantly, but only at its specific resonant frequencies (cavity modes).

Several annotations provide further insight:

A legend on the left clarifies the curves.

"Stimulated rates $\propto B_{ki}g(\nu)\rho(\nu)$" – proportional to B_{ki} times $g(\nu)$ times $\rho(\nu)$. This reminds us of the dependence of stimulated emission on the atomic properties (B_{ki}, $g(\nu)$) and the field ($\rho(\nu)$).

"Inversion makes $B_{ki}\rho(\nu)$ dominate where $\rho(\nu)$ is large (e.g., cavity mode)." This is the key take-away. Population inversion ensures that stimulated emission can be the dominant process. The cavity ensures that $\rho(\nu)$ is very large specifically at the cavity mode frequencies. The combination of these two effects leads to the very strong, sharply peaked red curve.

An arrow points from the red peak to an annotation "$\rho(\nu)$ enhancement at cavity resonance (ν_0)".

What this graph beautifully demonstrates is that while the atomic transition itself has a natural (broader) lineshape $g(\nu)$ for both spontaneous and potentially stimulated emission, the presence of an optical resonator dramatically alters the *actual* stimulated emission spectrum.

The resonator acts like a filter and an amplifier for its own modes. Thus, in an inverted medium placed within a resonator, stimulated emission is overwhelmingly channeled into these narrow, high-intensity cavity modes, leading to the characteristic sharp spectral output of a laser. The broad spontaneous emission (blue curve) is still present, but it's usually a much weaker background "noise" compared to the intense, spectrally narrow laser light (red curve).

Page 16

Slide 6: Optical Resonator — Storage and Mode Selection

We now move to "Slide 6: Optical Resonator — Storage and Mode Selection." This slide delves deeper into the properties and functions of the optical resonator, which, as we've seen, is crucial for laser operation.

The first point states: "Two parallel mirrors form standing-wave cavity." This describes the classic Fabry-Pérot resonator, the simplest and most common type. When light waves are confined between two parallel mirrors, they reflect back and forth. If the distance between the mirrors is an integer multiple of half-wavelengths of the light, then the waves traveling in opposite directions will interfere constructively to form a "standing wave" pattern. This means specific points in the cavity will always have maximum light intensity, and others (nodes) will always have zero intensity. Only light that forms such standing waves can resonate effectively within the cavity.

The second point elaborates on this: "Allowed longitudinal modes satisfy" the condition:

$$\nu_{\mathrm{q}} = \frac{q\,c}{2\,d}$$

where $q \in \mathbb{Z}$ (q is an element of the set of integers). Let's break down this fundamental equation for longitudinal modes: * ν_{q} (nu sub q): This is the frequency of the q-th allowed longitudinal mode. * q: This is an integer (..., -2, -1, 0, 1, 2, ...), often called the mode number. It represents the number of half-wavelengths of the light that fit exactly into the cavity length. For optical frequencies and typical cavity lengths, q is a very large integer (e.g., 10^5 to 10^9). * c: This is the speed of light in the medium filling the cavity (or in vacuum if the cavity is empty). * d: This is the optical length of the cavity – the distance between the mirrors.

This equation tells us that only discrete frequencies, ν_{q}, can form stable standing waves and thus resonate within the cavity. These are the longitudinal modes. The frequency spacing between adjacent longitudinal modes (e.g., between ν_{q} and ν_{q+1}) is called the Free Spectral Range (FSR), and it's equal to

$$\frac{c}{2\,d}.$$

The third point introduces another type of mode: "Mirror radii and apertures set transverse mode pattern (TEM_{mn})." Besides the longitudinal modes (which determine the allowed frequencies), optical resonators also support "transverse modes." These describe the intensity distribution of the electromagnetic field in the plane perpendicular (transverse) to the direction of light propagation along the cavity axis. * TEM_{mn} (T E M sub m n) is the common designation for these Transverse ElectroMagnetic modes. 'm' and 'n' are integers (0, 1, 2, ...) that specify the number of nodes (points of zero intensity) in two orthogonal directions (e.g., x and y) across the beam profile. * The fundamental mode is TEM_{00} (T E M sub zero zero), which has a Gaussian intensity profile (brightest at the center, falling off smoothly). Higher-order modes (like TEM_{01} or TEM_{11}) have more complex patterns with one or more nodes. * The "mirror radii" (i.e., the curvature of the mirrors, if they are not flat) and any "apertures" (size-limiting openings) within the cavity play a crucial role in determining which transverse modes are stable (i.e., can propagate back and forth without excessive loss) and what their spot sizes are. Often, laser designers aim for TEM_{00} operation because it provides the best beam quality (lowest divergence, ability to be focused to the smallest spot).

The fourth point summarizes a key function: "Resonator acts as frequency-selective filter with linewidth determined by finesse F." The resonator doesn't just allow the discrete frequencies ν_{q}; it acts like a very sharp filter around each of these frequencies. The "linewidth" of each resonant mode (how narrow the peak is in frequency space) is determined by the "finesse" (F) of the cavity.

Finesse is a measure of the "sharpness" of the cavity resonances. It depends primarily on the reflectivity of the mirrors: higher reflectivity leads to higher finesse. A high-finesse cavity will have very narrow resonant linewidths, meaning it is highly frequency-selective. It also implies that photons make many round trips within the cavity before being lost, leading to a longer photon storage time.

Page 17:

Continuing our discussion of "Slide 6: Optical Resonator — Storage and Mode Selection," this page elaborates on two important characteristics: the Q-factor and the output coupling.

The first point highlights the "High Q-factor."

Q stands for Quality factor, a general concept used to describe resonators in many areas of physics and engineering (e.g., LCR circuits, microwave cavities, mechanical oscillators). For an optical resonator, the Q-factor is defined as:

$$Q = 2\pi \times \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

Let's read this: "Capital Q equals two pi, times the ratio of, energy stored in the cavity, divided by energy lost per optical cycle."

* "Energy stored": This is the electromagnetic energy contained within the resonant mode inside the cavity. * "Energy lost per cycle": This is the amount of energy that escapes or is dissipated from the cavity during one period of the light wave.

A "High Q-factor" means that the energy stored is large compared to the energy lost per cycle. In other words, high Q implies low losses and a long energy decay time (or photon lifetime) within the cavity. High Q cavities are essential for lasers because they allow the light intensity to build up significantly.

The Q-factor is directly related to the finesse (F) and the resonant frequency ν_0:

$$Q \approx \frac{\nu_0}{\Delta\nu_{\text{cav}}}$$

where $\Delta\nu_{\text{cav}}$ is the linewidth (FWHM) of the cavity resonance. So, a high Q means a narrow cavity linewidth.

The consequence of a high Q-factor is that it "enhances photon lifetime → greater amplification per pass."

Well, not quite "per pass," but rather "greater cumulative amplification." A longer photon lifetime means that photons make more round trips within the cavity before being lost. Each pass through the active medium provides an opportunity for stimulated emission. So, more round trips allow for a greater overall amplification of the light, making it easier to reach the lasing threshold (where gain overcomes total losses) and achieve efficient laser operation.

The second point concerns how we get useful light out of the laser: "Coupling mirror (partial reflector) allows controlled extraction of laser power."

For a laser to be useful, some of the light circulating within the cavity must be allowed to escape in a controlled manner. This is achieved by making one of the cavity mirrors (called the "output coupler") partially transmissive, meaning it reflects most of the light but transmits a small, well-defined fraction.

The choice of the output coupler's reflectivity R is a critical design parameter. * If R is too high (too close to 100%), very little light escapes, resulting in low output power, even if the internal cavity intensity is high. * If R is too low (too much transmission), the losses from the cavity will be too high. This might prevent the laser from reaching threshold at all, or it might lead to inefficient operation because photons don't spend enough time in the cavity to be significantly amplified.

So, there's an optimum reflectivity for the output coupler that maximizes the extracted laser power for a given active medium and pump level. This "controlled extraction" is what gives us the usable laser beam.

Page 18:

This page provides an excellent visual summary for "Slide 6: Optical Resonator — Storage and Mode Selection," combining a schematic of the cavity with a frequency-domain representation of mode selection.

At the top of the slide, under the heading "Two parallel mirrors form a standing-wave cavity:", we see a diagram of a Fabry-Pérot resonator.

On the left is "M1 ($R \approx 100\%$)" – Mirror 1 with reflectivity approximately 100%. This is the high reflector.

On the right is "M2 (Output Coupler, $R < 100\%$)" – Mirror 2, the output coupler, with reflectivity less than 100%, allowing some light to be transmitted.

The distance between the mirrors is "d (cavity length)."

Inside the cavity, several blue sinusoidal patterns are drawn, representing different standing wave patterns (longitudinal modes) of light. Orange arrows indicate the forward and backward propagating waves that form these standing waves. For example, one mode shows three half-wavelengths fitting into d, another might show four.

A red arrow labeled "Output Laser Beam" is shown emerging from M2, signifying the useful laser output.

Below this physical schematic, we see a graph titled "Frequency Domain: Mode Selection."

The vertical axis is labeled "Gain / Intensity."

The horizontal axis is labeled "Frequency (ν)".

A broad, orange, bell-shaped curve represents the "Gain Profile of Active Medium." This shows the range of frequencies over which the active medium can provide amplification, peaking at some central frequency.

Superimposed on this are vertical blue lines. These represent the discrete "Allowed longitudinal modes" of the cavity, which we know are spaced by $\Delta\nu = \frac{c}{2\,d}$ (Delta nu equals $\frac{c}{2\,d}$). The slide labels some of these modes as ν_{q-1}, ν_q, and ν_{q+1}. The spacing between them is explicitly labeled "$\Delta\nu = \frac{c}{2\,d}$".

A dashed horizontal line is labeled "Lasing Threshold." This line represents the level of gain required to overcome all the losses in the cavity.

The crucial insight from this graph is that lasing will only occur for those longitudinal cavity modes (blue lines) whose gain (as given by the height of the orange gain profile at that mode's frequency) exceeds the lasing threshold. In the diagram, several modes under the peak of the gain curve are shown to be above the threshold, implying that this laser would operate on multiple longitudinal modes simultaneously. If the gain profile were narrower, or the threshold higher, or the modes more widely spaced, fewer modes (or even just one) might lase.

Below the graph, text reiterates key concepts:

"Allowed longitudinal modes: $\nu_{\mathrm{q}} = \frac{qc}{2\,d}, \quad q \in \mathbb{Z}$"

"Resonator acts as a frequency-selective filter."

"Mode linewidth is determined by the cavity Finesse (F)."

This entire page beautifully ties together the physical structure of the resonator with its frequency-selective properties and its interaction with the gain medium to determine the actual lasing output.

Page 19:

Slide 7: Propagation Through Gain Medium — Intensity Evolution

Now we transition to "Slide 7: Propagation Through Gain Medium — Intensity Evolution." This slide begins to quantify how the intensity of a light beam changes as it travels through an active (or absorbing) medium.

The first point sets the scene: "Consider monochromatic beam frequency ν (nu) along z." We are looking at a light beam of a single, well-defined frequency ν, propagating in a specific direction, which we label as the z-axis.

The second point introduces the "Differential intensity change" with a fundamental equation:

$$\frac{dI(\nu,z)}{dz} = -\alpha(\nu)I(\nu,z)$$

Let's read this carefully: "dee capital I of nu comma z, by dee z, equals minus alpha of nu, times capital I of nu comma z."

Now, let's deconstruct this differential equation, which is a form of the Beer-Lambert law:

* $I(\nu, z)$ (Capital I of nu comma z): This is the intensity of the light beam at frequency ν and at position z along its path. Intensity is power per unit area (e.g., Watts per square centimeter).

* $\frac{dI(\nu,z)}{dz}$: This is the derivative of the intensity with respect to position z. It represents the rate at which the intensity changes as the beam propagates an infinitesimal distance dz.

* $\alpha(\nu)$ (alpha of nu): This is the crucial parameter called the "absorption coefficient" (or attenuation coefficient) of the medium at frequency ν. Its units are typically inverse length (e.g., cm^{-1}, per centimeter). * If $\alpha(\nu)$ is positive, then $\frac{dI}{dz}$ is negative (since I is positive), meaning the intensity decreases as the beam propagates. This corresponds to absorption or attenuation. *If* $\alpha(\nu)$ is negative, which can happen in an active medium with population inversion, then $-\alpha(\nu)$ is positive. In this case, $\frac{dI}{dz}$ is positive, meaning the intensity increases* as the beam propagates. This corresponds to amplification or gain.

So, the sign of $\alpha(\nu)$ determines whether we have loss or gain.

The third point gives the "Solution over length z" for this differential equation:

$$I(\nu, z) = I(\nu, 0)\exp[-\alpha(\nu)z]$$

Let's read this: "Capital I of nu comma z, equals capital I of nu comma zero, times the exponential of, minus alpha of nu, times z."

Breaking it down:

* $I(\nu, z)$: The intensity at frequency ν after propagating a distance z through the medium.

* $I(\nu, 0)$ (Capital I of nu comma zero): The initial intensity of the beam at the starting point, $z = 0$.

* exp: The exponential function.

* $-\alpha(\nu)z$: The argument of the exponential.

This solution shows that:

* If $\alpha(\nu)$ is positive (absorption), the intensity decays exponentially with distance z: $I(z) = I(0)e^{-|\alpha|z}$.

* If $\alpha(\nu)$ is negative (gain), let $\alpha(\nu) = -g(\nu)$ where $g(\nu)$ is a positive gain coefficient. Then the equation becomes $I(z) = I(0)\exp[g(\nu)z]$. The intensity grows exponentially with distance z. This exponential growth is the essence of light amplification in a laser's active medium.

This framework is fundamental for describing how light interacts with any medium, whether it's a simple absorber or a complex gain medium in a laser.

Page 20:

Continuing with "Slide 7: Propagation Through Gain Medium — Intensity Evolution," this page gives us the explicit form of the absorption (or gain) coefficient $\alpha(\nu)$ in terms of atomic parameters.

The first point defines the "Absorption (or gain) coefficient" with the equation:

$$\alpha(\nu) = \left[N_{\mathrm{i}} - \left(\frac{g_{\mathrm{i}}}{g_{\mathrm{k}}}\right) N_{\mathrm{k}}\right] \sigma(\nu)$$

Let's read this: "alpha of nu, equals, open square bracket, N_i, minus, open parenthesis, $\frac{g_\mathrm{i}}{g_\mathrm{k}}$, close parenthesis, times N_k, close square bracket, all times $\sigma(\nu)$."

Now, let's deconstruct this very important expression: * $\alpha(\nu)$ (alpha of nu): The absorption coefficient at frequency ν.

* N_i (N sub i): Population density of the lower energy level involved in the transition.

* N_k (N sub k): Population density of the upper energy level involved in the transition.

* g_i (g sub i): Degeneracy of the lower level.

* g_k (g sub k): Degeneracy of the upper level.

* $\sigma(\nu)$ (sigma of nu): This is the "stimulated-transition cross-section" at frequency ν, which we'll discuss in the next bullet.

The term in the square brackets, $\left[N_\mathrm{i} - \left(\frac{g_\mathrm{i}}{g_\mathrm{k}}\right) N_\mathrm{k}\right]$, determines the sign of $\alpha(\nu)$:

* If $N_\mathrm{i} > \left(\frac{g_\mathrm{i}}{g_\mathrm{k}}\right) N_\mathrm{k}$: This means $\frac{N_\mathrm{i}}{g_\mathrm{i}} > \frac{N_\mathrm{k}}{g_\mathrm{k}}$, i.e., the population per state in the lower level is greater than in the upper level (the normal situation or absorption dominates). The term in the bracket is positive. Thus, $\alpha(\nu)$ is positive, leading to absorption (attenuation of light).

* If $N_\mathrm{i} < \left(\frac{g_\mathrm{i}}{g_\mathrm{k}}\right) N_\mathrm{k}$: This means $\frac{N_\mathrm{i}}{g_\mathrm{i}} < \frac{N_\mathrm{k}}{g_\mathrm{k}}$, i.e., population inversion exists (population per state in the upper level is greater). The term in the bracket is negative. Thus, $\alpha(\nu)$ is negative, leading to gain (amplification of light).

This expression elegantly combines the populations of the two levels, their degeneracies, and the interaction strength (via $\sigma(\nu)$) to determine whether the medium absorbs or amplifies light at frequency ν.

The second bullet point defines: "σ(ν) (sigma of nu): stimulated-transition cross-section [cm²]" (centimeters squared). The cross-section $\sigma(\nu)$ is a measure of the effective "area" that an atom or molecule presents to an incoming photon for a stimulated transition (either stimulated emission or absorption) to occur at frequency ν. A larger cross-section means a stronger interaction and thus a larger absorption or gain coefficient for a given population difference. It's an intrinsic property of the specific atomic or molecular transition and depends on frequency, typically peaking at the resonant frequency of the transition and having a lineshape (e.g., Lorentzian or Gaussian) around it.

The third bullet point explicitly states the condition for gain: "If inversion satisfies $N_\text{k} > \left(\frac{g_\text{k}}{g_\text{i}}\right) N_\text{i} \Rightarrow \alpha(\nu) < 0$." Let's read this: "If N_k is greater than, open parenthesis $\frac{g_\text{k}}{g_\text{i}}$ close parenthesis times N_i, then this implies that $\alpha(\nu)$ is less than zero." This is just a rearrangement of the population inversion condition we saw earlier. If $\frac{N_\text{k}}{g_\text{k}} > \frac{N_\text{i}}{g_\text{i}}$, then the term $\left[N_\text{i} - \left(\frac{g_\text{i}}{g_\text{k}}\right) N_\text{k}\right]$ in the expression for $\alpha(\nu)$ becomes negative, because $\left(\frac{g_\text{i}}{g_\text{k}}\right) N_\text{k}$ will be greater than N_i. Hence, $\alpha(\nu)$ becomes negative.

And the consequence, indicated by an arrow: "→ beam is amplified." When $\alpha(\nu)$ is negative, the exponent in $I(z) = I(0)\exp[-\alpha(\nu)z]$ becomes positive, leading to exponential growth of intensity.

The "---" indicates the end of this slide's content. This expression for $\alpha(\nu)$ is central to understanding laser gain.

Page 21:

Slide 8: Round-Trip Analysis — Defining Gain & Losses.

We now move to "Slide 8: Round-Trip Analysis — Defining Gain & Losses." So far, we've considered a single pass of light through the active medium.

In a laser, light makes many round trips within the optical resonator. This slide starts to analyze what happens in one full round trip.

The first point sets the parameters: "Active medium length L; resonator length d (possibly $d > L$)." • L (Capital L): The length of the active medium, the region where gain occurs. • d (small d): The total length of the optical resonator, i.e., the distance between the mirrors.

It's important to note that the active medium might not fill the entire resonator ($L \leq d$). For instance, in a gas laser, the discharge tube (active medium) is typically shorter than the mirror separation.

The second point defines the "Pure gain over round-trip (no extraneous loss):"

$$G(\nu) = \exp[-2\alpha(\nu)L]$$

Let's read this: "Capital G of nu, equals the exponential of, minus two times $\alpha(\nu)$, times capital L." Let's analyze this: • $G(\nu)$ (Capital G of nu): This is the factor by which the light intensity is multiplied after one complete round trip *solely due to interaction with the active medium of length L*. The beam passes through the active medium twice in one round trip (e.g., forward pass of length L, reflection, backward pass of length L). • $\alpha(\nu)$: This is the absorption coefficient we defined on the previous page. If $\alpha(\nu)$ is negative (due to population inversion), say $\alpha(\nu) = -g(\nu)$ where $g(\nu)$ is the positive gain coefficient, then the exponent becomes $-2\big(-g(\nu)\big)L = 2\,g(\nu)L$. In this case,

$$G(\nu) = \exp[2\,g(\nu)L]$$

which will be greater than 1, representing net amplification over the round trip. • The "$-2\alpha(\nu)L$" assumes $\alpha(\nu)$ is defined such that it's positive for absorption. So, if there's gain, $\alpha(\nu)$ itself is negative.

This $G(\nu)$ represents the ideal gain if there were no other losses in the cavity.

The third point introduces reality: "Real cavities have distributed losses summarised by dimensionless γ." (gamma). In any real laser resonator, there are always losses other than just the potential absorption within the active medium itself (if it weren't pumped). These "extraneous" losses are unavoidable. γ (gamma) is a dimensionless parameter that quantifies the total fractional energy loss per round trip due to all these other factors.

The fourth point lists the "Sources" of these losses γ: • "Mirror transmission $(1 - R)$": Mirrors are not perfect reflectors. Even the high reflector might have a reflectivity R slightly less than 1. The output coupler is *designed* to transmit a certain fraction ($T = 1 - R$, neglecting absorption in the mirror) of the light to produce the laser beam; this is a deliberate and necessary loss from the cavity's perspective, contributing to γ. • "Window absorption": If the active medium is enclosed by windows (e.g., Brewster windows on a gas laser tube), these optical components can absorb a small fraction of the light passing through them. • "Scattering": Imperfections on the surfaces of mirrors or windows, or scattering centers within the active medium itself, can scatter light out of the main beam path, contributing to loss. • "Diffraction clipping": The laser beam has a finite size. If the mirrors or other apertures within the cavity are too small, some of an expanding beam's energy can be lost ("clipped") at the edges due to diffraction. Higher-order transverse modes are particularly susceptible to this.

All these loss mechanisms are lumped together into the round-trip loss factor γ. For lasing to occur, the round-trip gain $G(\nu)$ must be large enough to overcome these total round-trip losses represented by γ.

Page 22:

Continuing with "Slide 8: Round-Trip Analysis — Defining Gain & Losses," this page builds on the concepts of gain and loss to describe the net effect over a round trip.

The first bullet point considers the "Exponential decay of intensity in empty resonator:" $I = I_0 e^{-\gamma y}$

Let's read this: "Capital I equals I sub zero, times $e^{-\gamma y}$." And it's clarified: "where y: number of round-trips." This equation describes how the intensity of light would decay in a passive cavity – one that either has no active medium, or where the active medium is not pumped (so $\alpha(\nu)$ is positive or zero).

* I_0 (I sub zero) is the initial intensity. * I is the intensity after 'y' round trips. γ (gamma) here is the dimensionless loss per round trip*. The form $e^{-\gamma y}$ implies that $(1-\gamma_{\text{fractional_\text{loss}}}) \approx e^{-\gamma_{\text{exponent_\text{loss}}}}$. More precisely, if γ represents the fractional loss per round trip, then after y round trips, $I = I_0(1-\gamma)^y$. For small γ, $(1-\gamma) \approx e^{-\gamma}$, so $(1-\gamma)^y \approx (e^{-\gamma})^y = e^{-\gamma y}$. So, γ here is effectively the exponential decay constant per round trip.

This shows that in the absence of gain, the light intensity within the cavity will die out exponentially due to these inherent losses.

The second bullet point then considers the "Combined effect (single round-trip)" when both gain from the active medium and these other cavity losses are present: $I(\nu, 2\,d) = I(\nu, 0)\exp[-2\alpha(\nu)L - \gamma]$

Let's read this: "Capital I of ν comma two d, equals capital I of ν comma zero, times the exponential of, open square bracket, minus two $\alpha(\nu)$ times capital L, minus γ, close square bracket."

Let's analyze this crucial equation: * $I(\nu, 0)$: The intensity of light at frequency ν at the start of a round trip. * $I(\nu, 2\,d)$: The intensity after one complete round trip (covering a total cavity path length of $2\,d$, though the gain medium itself is length L, traversed twice). * The exponent $[-2\alpha(\nu)L - \gamma]$ determines the net change in intensity. * $-2\alpha(\nu)L$: This is the gain term from the active medium over one round trip (as defined on the previous slide by $G(\nu) = \exp[-2\alpha(\nu)L]$). If $\alpha(\nu)$ is negative (gain), this term is positive. * $-\gamma$: This represents the effect of all other losses per round trip. Since γ is defined as a loss factor, it appears with a minus sign in the exponent for intensity reduction.

For the laser to oscillate, the intensity must at least remain constant, and ideally grow, from one round trip to the next. This means the overall argument of the exponential must be greater than or equal to zero:

$$-2\alpha(\nu)L - \gamma \geq 0$$

This implies that the gain term must overcome the loss term:

$$-2\alpha(\nu)L \geq \gamma$$

Since $\alpha(\nu)$ is negative for gain (say, $\alpha(\nu) = -g(\nu)$ where $g(\nu)$ is positive gain coefficient), this condition becomes:

$$2\, g(\nu)L \geq \gamma$$

This is the fundamental condition for lasing threshold: the round-trip gain $(2\, g(\nu)L)$ must equal or exceed the round-trip losses (γ). When gain exactly equals loss, the laser is at threshold. When gain exceeds loss, the intensity builds up.

The "---" indicates the end of this slide. This equation beautifully summarizes the balance of gain and loss in a laser cavity.

Page 23:

Now we arrive at "Slide 9: Laser Threshold — Step-by-Step Derivation." This slide formalizes the condition for net amplification and derives the threshold population inversion.

The heading "Condition for Net Amplification" sets the stage.

Step 1 is: "Net gain required:"

$$-2\, L\alpha(\nu) > \gamma$$

Let's read this: "Minus two times capital L times alpha of nu, is greater than gamma." This is precisely the condition we derived at the end of the last page for the light intensity to grow over a round trip.

• $-2\,L\alpha(\nu)$: This represents the round-trip gain provided by the active medium of length L. Remember, $\alpha(\nu)$ is the absorption coefficient; if it's negative (due to population inversion), then $-2\,L\alpha(\nu)$ becomes a positive gain factor in the exponent. • γ: This is the total dimensionless loss per round trip from all other sources (mirrors, scattering, etc.).

So, for net amplification, the round-trip gain must exceed the round-trip losses.

Step 2 is: "Substitute $\alpha(\nu)$ expression:"

We recall from page 20 that the absorption coefficient $\alpha(\nu)$ is given by:

$$\alpha(\nu) = \left[N_{\mathrm{i}} - \frac{g_{\mathrm{i}}}{g_{\mathrm{k}}} N_{\mathrm{k}}\right] \sigma(\nu)$$

Substituting this into our net gain condition:

$$-2\,L\left[N_{\mathrm{i}} - \frac{g_{\mathrm{i}}}{g_{\mathrm{k}}} N_{\mathrm{k}}\right] \sigma(\nu) > \gamma$$

We can bring the minus sign inside the square brackets by reversing the terms:

$$2\,L\left[\frac{g_{\mathrm{i}}}{g_{\mathrm{k}}} N_{\mathrm{k}} - N_{\mathrm{i}}\right] \sigma(\nu) > \gamma$$

The slide presents a slightly different algebraic form, but one that is equivalent if we define the population inversion carefully. The slide writes:

$$2\,L\left[N_{\mathrm{k}} - \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}}\right] \sigma(\nu) > \gamma$$

Let's analyze the term in the square brackets on the slide: $\left[N_{\mathrm{k}} - \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}}\right]$. For this term to be positive (which is needed for the left side to represent gain), we need

$$N_{\mathrm{k}} > \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}}.$$

This is exactly the condition for population inversion

$$\frac{N_{\mathrm{k}}}{g_{\mathrm{k}}} > \frac{N_{\mathrm{i}}}{g_{\mathrm{i}}}.$$

So, if we define an "effective population inversion density"

$$\Delta N_{\mathrm{eff}} = N_{\mathrm{k}} - \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}},$$

then the condition becomes

$$2\, L \Delta N_{\mathrm{eff}} \sigma(\nu) > \gamma.$$

This form is consistent and often used. The key is that the term $\left[N_{\mathrm{k}} - \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}}\right]$ must be positive for gain to occur.

Step 3 is: "Define inversion density".

This foreshadows that the term we just discussed, $\left[N_{\mathrm{k}} - \frac{g_{\mathrm{k}}}{g_{\mathrm{i}}} N_{\mathrm{i}}\right]$, will be formally defined as the population inversion density relevant for this gain equation. We'll see this explicit definition on the next page. This step-by-step derivation is leading us directly to the minimum population inversion required for lasing.

Page 24:

Slide 9: Laser Threshold — Step-by-Step Derivation

Continuing with "Slide 9: Laser Threshold — Step-by-Step Derivation," this page completes the derivation.

First, we see the definition of the inversion density, following from Step 3 on the previous page:

$$\Delta N = N_\mathrm{k} - \frac{g_\mathrm{k}}{g_\mathrm{i}} N_\mathrm{i}$$

Let's read this: "Capital Delta N equals N sub k, minus, open parenthesis g sub k divided by g sub i close parenthesis, times N sub i."

This ΔN (Capital Delta N) is the specific definition of population inversion density that makes the gain expression from the previous page $2\,L\,\Delta N\,\sigma(\nu)$. When this ΔN is positive, we have gain.

Step 4 is to find the "Threshold inversion." This is the minimum value of ΔN required for lasing to begin. It's found by setting the round-trip gain equal to the round-trip losses (the threshold condition):

$$2\,L\,\Delta N_{thr}\,\sigma(\nu) = \gamma$$

Solving for ΔN_{thr} (Delta N sub t h r), we get:

$$\Delta N_{thr} = \frac{\gamma}{2\,\sigma(\nu)\,L}$$

Let's read this: "Capital Delta N sub t h r, equals gamma, divided by, the product of two, sigma of nu, and capital L."

Let's deconstruct this important result for the threshold population inversion density: * γ (gamma): The dimensionless round-trip loss. Higher losses mean a higher threshold inversion is needed. * $\sigma(\nu)$ (sigma of nu): The stimulated-transition cross-section at frequency ν. A larger cross-section (stronger interaction) means a lower threshold inversion is needed. * L (Capital L): The length of the active medium. A longer gain medium means a lower threshold inversion density is needed (as the total gain is proportional to L).

This formula is fundamental to laser design. It tells us quantitatively what level of population inversion we must achieve in our active medium, given its properties (σ, L) and the quality of our resonator (γ).

Step 5 states: "Laser oscillation possible only if $\Delta N > \Delta N_{thr}$". This is boxed to emphasize its importance: "Capital Delta N must be greater than Capital Delta N sub t h r." The actual population inversion density (ΔN) achieved in the medium by pumping must exceed this calculated threshold value (ΔN_{thr}) for the laser to start oscillating and produce output. If ΔN is less than ΔN_{thr}, the gain will not be sufficient to overcome the losses, and lasing will not occur.

Finally, the slide clarifies the units of the symbols involved:

"All symbols: ΔN [cm^{-3}], σ [cm^{2}], L [cm], γ [dimensionless]."

* ΔN (Delta N) is a population inversion density, in units of per centimeter cubed. * σ (sigma) is a cross-section, in units of centimeters squared. * L is a length, in units of centimeters. * γ (gamma) is a dimensionless fractional loss per round trip.

These units are consistent, making ΔN_{thr} indeed a density (cm^{-3}).

This threshold condition is one of the most important results in basic laser theory.

Page 25:

Slide 10: Numerical Illustration — He-Ne Laser Example

Now we proceed to "Slide 10: Numerical Illustration — He-Ne Laser Example." This slide applies the threshold inversion formula we just derived to a practical example, the Helium-Neon laser, which is a very common and well-understood laser system.

First, the "Parameters" for this specific He-Ne laser example are given:

- "$L = 10\,\text{cm}$" (Capital L equals 10 centimeters). This is the length of the active gain medium (the gas discharge tube).

- "$\gamma = 0.10$" (10% loss per round-trip). This means that 10% of the light intensity is lost from the cavity on each round trip due to factors like mirror transmission, scattering, etc.

- "$\sigma = 1.0 \times 10^{-12}\,\text{cm}^2$" (sigma equals one point zero times ten to the minus twelve centimeters squared). This is the stimulated emission cross-section for the lasing transition in neon (likely the 632.8 nm red line). This is a relatively large cross-section, which is favorable for achieving laser action. Note that $\sigma(\nu)$ is just written as σ, implying it's evaluated at the peak of the gain curve.

Next, we "Plug into threshold formula." The formula is:

$$\Delta N_{\text{thr}} = \frac{\gamma}{2\sigma L}$$

(Capital Delta N sub t h r equals gamma, divided by the product of two sigma L).

Let's perform the calculation with the given values:

$$\Delta N_{\text{thr}} = \frac{0.10}{2 \times \left(1.0 \times 10^{-12}\, \text{cm}^{2\right)} \times \left(10\, \text{cm}\right)}$$

$$\text{Denominator} = 2 \times 1.0 \times 10 \times 10^{-12}\,\text{cm}^3 = 20 \times 10^{-12}\,\text{cm}^3$$
$$= 2 \times 10^{-11}\,\text{cm}^{3.}$$

$$\Delta N_{\text{thr}} = \frac{0.10}{2 \times 10^{-11}\,\text{cm}^3}$$

$$\Delta N_{\text{thr}} = \frac{1 \times 10^{-1}}{2 \times 10^{-11}}\,\text{cm}^{-3}$$

$$\Delta N_{\text{thr}} = \frac{1}{2} \times 10^{10}\,\text{cm}^{-3}$$

$$\Delta N_{\text{thr}} = 0.5 \times 10^{10}\,\text{cm}^{-3} = 5 \times 10^{9}\,\text{cm}^{-3}.$$

The slide shows the calculation as: "$\Delta N_{\text{thr}} = \frac{0.10}{2\times1.0\times10^{-12}\times10} = 5 \times 10^{9}\,\text{cm}^{-3}$" (equals five times ten to the nine per centimeter cubed).

Our calculation matches the slide. This means that for this specific He-Ne laser to begin lasing, a population inversion density of 5 billion atoms per cubic centimeter must be achieved between the upper and lower lasing levels of neon (factoring in their degeneracies as per our definition of ΔN).

This numerical example helps to make the abstract formula for threshold inversion more concrete.

Page 26:

Continuing with "Slide 10: Numerical Illustration — He-Ne Laser Example," this page contextualizes the threshold inversion density we just calculated.

The first point provides information about the total number of neon atoms available: "Neon partial pressure $p = 0.2\,\text{mbar} \Rightarrow \text{total neon density } N_{\text{Ne}} \approx 5 \times 10^{15}\,\text{cm}^{-3}." (p equals zero point two millibar, implies \(N_{\text{Ne}}$, the total Neon density, is approximately five times ten to the fifteen per centimeter cubed). A partial pressure of 0.2 millibar (which is 20 Pascals) for neon in a typical He-Ne gas mixture at operating temperature (a few hundred Kelvin) would indeed correspond to a total neon number density of this order of magnitude $\left(\frac{N}{V} = \frac{P}{k_{\text{B}}T}\right)$. So, we have a vast reservoir of about 5 quadrillion neon atoms per cubic centimeter.

The second point then calculates the "Required inversion fraction:" This is the ratio of the threshold population inversion density ΔN_{thr} to the total neon density N_{Ne}. "$\frac{\Delta N_{\text{thr}}}{N_{\text{Ne}}} \approx \frac{5\times10^{9}\,\text{cm}^{-3}}{5\times10^{15}\,\text{cm}^{-3}}$" Let's do the math:

$$\frac{5 \times 10^{9}}{5 \times 10^{15}} = \frac{5}{5} \times \frac{10^{9}}{10^{15}} = 1 \times 10^{9-15} = 1 \times 10^{-6}.$$

The slide result is: "$\approx 1 \times 10^{-6}$" (approximately one times ten to the minus six). This is a very small fraction! It means that only about one in a million neon atoms needs to be in the effectively inverted state (contributing to ΔN_{thr}) for the laser to reach threshold.

The third point provides the crucial interpretation of this small fraction, indicated by an arrow: "→ remarkably small fraction suffices because gain cross-section is large and losses moderate." This is a key insight into why He-Ne lasers are practical and can operate continuously with relatively modest pumping.

- "gain cross-section is large": As we noted, $\sigma \approx 10^{-12}$ cm^2 is a fairly large value. This means that each inverted atom is quite effective at participating in stimulated emission. If σ were much smaller, a much larger ΔN_{thr} would be needed for the same L and γ.

- "losses moderate": A 10% round-trip loss $(\gamma = 0.10)$ is a reasonable value for a well-constructed cavity. If losses were significantly higher, ΔN_{thr} would also need to be higher.

Because of these favorable factors, only a tiny fraction of the available neon atoms needs to be actively participating in the population inversion at any given time. This makes it feasible to sustain this level of inversion continuously with an electrical discharge pump.

This example illustrates how the interplay of atomic properties (σ), cavity design (L, γ), and pump effectiveness determines the practicality of a laser system.

Page 27:

This page concludes "Slide 10: Numerical Illustration — He-Ne Laser Example" with a final important takeaway.

The bullet point states: "Demonstrates practicality of continuous-wave operation in He-Ne gas discharge."

This directly follows from our finding on the previous page that only a very small fraction (around 10^{-6}) of neon atoms needs to be involved in the population inversion to reach the lasing threshold.

Because such a small fraction is required:

* The pumping mechanism (the electrical discharge in the He-Ne gas mixture) does not need to be extraordinarily powerful or efficient to achieve and maintain this level of inversion.

* It's possible to sustain this inversion continuously over time, rather than only in short pulses.

This is why He-Ne lasers are well-known for their ability to operate in "continuous-wave" (CW) mode, meaning they produce a steady, uninterrupted laser beam. If a very large fraction of atoms needed to be inverted, it might only be possible to achieve this for brief periods using intense pulsed pumping, as is the case for some other laser systems (e.g., those with very low gain cross-sections or very high losses, or 3-level systems).

The He-Ne laser, being one of the earliest gas lasers and one of the first to achieve CW operation at visible wavelengths, beautifully exemplifies how laser design principles can lead to practical and reliable devices.

The "---" indicates the end of the content for this slide and this specific numerical example.

Page 28:

We now advance to "Slide 11: Frequency Dependence of Gain — Line Shape." So far, we've often treated the gain coefficient or cross- section as a single value. However, in reality, these quantities are frequency- dependent, and this dependence is crucial for understanding the spectral characteristics of lasers.

The first point states: "Stimulated-transition cross-section relates to Einstein B-coefficient and homogeneous/inhomogeneous line profile $g(\nu - \nu_0)$." Let's break this down:

- $\sigma(\nu)$, the stimulated-transition cross-section, is not a constant but varies with the frequency ν (nu) of the light.

- It's fundamentally related to the Einstein B-coefficient (B_{ki} for stimulated emission or B_{ik} for absorption), which represents the intrinsic strength of the radiative transition between two energy levels.

- The frequency dependence comes from the "line profile" or "lineshape function," denoted here as $g(\nu - \nu_0)$ (g of nu minus nu zero). This function describes the shape of the spectral line associated with the transition. It's typically peaked at a central resonant frequency ν_0 (nu zero) and falls off as the frequency ν moves away from ν_0. The integral of $g(\nu - \nu_0)$ over all frequencies is usually normalized to unity.

- The lineshape $g(\nu - \nu_0)$ can arise from two main types of broadening mechanisms: – Homogeneous broadening: All atoms or molecules in the active medium behave identically, having the same resonant frequency and linewidth. – Inhomogeneous broadening: Different atoms or molecules (or groups of them) have slightly different resonant frequencies, leading to an overall broader line composed of many narrower individual contributions.

The second point provides an expression for the "Unsaturated gain coefficient," $\alpha(\nu)$, which now explicitly shows its frequency dependence:

$$\alpha(\nu) = \Delta N\, \sigma_{ik}(\nu) = \Delta N \left(\frac{h\nu}{c}\right) B_{ik}\, g(\nu - \nu_0)$$

Let's read this: "alpha of nu, equals Capital Delta N times sigma sub i k of nu, which equals Capital Delta N, times, open parenthesis h nu divided by c close parenthesis, times B sub i k, times g of nu minus nu zero."

Here, $\alpha(\nu)$ is being used to denote the gain coefficient (positive for gain).

- ΔN: This is the population inversion density, as defined previously (e.g., $N_k - \frac{g_k}{g_i} N_i$ for the levels k and i).

- $\sigma_{ik}(\nu)$ (sigma sub i k of nu): This is the frequency-dependent stimulated emission cross-section (even though subscript is ik, with ΔN for inversion, it's effectively for stimulated emission from k to i).

- The second part of the equation, $\Delta N \left(\frac{h\nu}{c}\right) B_{ik}\, g(\nu - \nu_0)$, expresses this cross-section in terms of more fundamental parameters: – h: Planck's constant. – ν: Frequency of light. – c: Speed of light. – B_{ik}: The Einstein B-coefficient for the transition (again, with ΔN, context implies B_{ki} for emission). – $g(\nu - \nu_0)$: The lineshape function.

This equation shows that the gain coefficient $\alpha(\nu)$ directly inherits its frequency dependence from the lineshape function $g(\nu - \nu_0)$. The gain will be strongest where $g(\nu - \nu_0)$ is largest.

The third point confirms this: "Peak amplification at line centre ν_0." Naturally, since the lineshape function $g(\nu - \nu_0)$ is typically peaked at the central resonant frequency ν_0 of the atomic or molecular transition, the gain coefficient $\alpha(\nu)$ will also be maximum at $\nu = \nu_0$. Lasers will preferentially oscillate at or near this peak gain frequency, if allowed by the resonator modes.

The fourth point begins to specify the types of lineshapes: "Homogeneous broadening → Lorentz profile, width Γ_L." (Capital Gamma sub L).

- "Homogeneous broadening" occurs when all active atoms or molecules in the sample experience the same local environment and thus have identical transition frequencies and linewidths. Examples of homogeneous broadening mechanisms include: – Natural broadening (or lifetime broadening): Due to the finite lifetime of the excited state (related to the spontaneous emission rate A_{ki}), there's an inherent uncertainty in its energy, leading to a broadening of the spectral line. This is a fundamental limit. – Collisional broadening (or pressure broadening): In gases and liquids, collisions between active atoms/molecules and other particles can interrupt the phase of the emitted light wave or shorten the effective lifetime of the states, leading to broadening. This type of broadening increases with pressure. – In solids, interaction with lattice vibrations (phonons) can also lead to homogeneous broadening.

• When homogeneous broadening is dominant, the lineshape function $g(\nu - \nu_0)$ typically takes the form of a "Lorentz profile" (also known as a Lorentzian or Breit-Wigner lineshape).

• Γ_{L} (Capital Gamma sub L) represents the characteristic width of this Lorentzian profile, usually the Full Width at Half Maximum (FWHM).

Page 29:

Slide 11: Frequency Dependence of Gain — Line Shape

Continuing with "Slide 11: Frequency Dependence of Gain — Line Shape," this page discusses the other major type of broadening.

The bullet point states: "Inhomogeneous broadening (Doppler) → Gaussian profile, width $\Delta\nu_{\text{D}}$." (Delta nu sub D).

Let's break this down:

"Inhomogeneous broadening" occurs when different atoms or molecules (or groups of them) within the active medium experience slightly different local environments or conditions, causing their individual resonant frequencies ν_0 to be slightly shifted relative to each other. The overall observed spectral line is then a superposition (an envelope) of many narrower, homogeneously broadened lines from these different groups of atoms.

The most prominent example of inhomogeneous broadening, especially in gas lasers, is "Doppler broadening." Gas atoms are in constant random thermal motion. Atoms moving towards an observer (or a light beam) see the light frequency up-shifted (blueshifted), while atoms moving away see it down-shifted (redshifted) due to the Doppler effect. Since there's a distribution of velocities (typically a Maxwell-Boltzmann distribution), there will be a corresponding distribution of perceived resonant frequencies.

When Doppler broadening is the dominant inhomogeneous mechanism, the resulting lineshape function $g(\nu - \nu_0)$ takes the form of a "Gaussian profile."

$\Delta\nu_{\mathrm{D}}$ (Capital Delta nu sub D) represents the characteristic width (FWHM – Full Width at Half Maximum) of this Gaussian profile due to Doppler broadening. This width is proportional to the central frequency ν_0 and the square root of $\frac{\text{Temperature}}{\text{Mass of the atom/molecule}}$. So, higher temperatures and lighter particles lead to greater Doppler broadening.

Understanding whether a laser transition is primarily homogeneously or inhomogeneously broadened is crucial because it affects how the laser behaves, particularly in terms of its power output characteristics and its ability to operate on single or multiple frequencies (modes).

For example, in an inhomogeneously broadened gain medium, different cavity modes can be amplified by different velocity groups of atoms, a phenomenon known as "spatial hole burning" when considering standing waves, or spectral hole burning if a narrow frequency burns a "hole" in the gain profile.

Page 30:

This page, still part of "Slide 11: Frequency Dependence of Gain — Line Shape," provides a visual "Comparison of Normalized Lorentzian and Gaussian Line Shapes."

Let's describe the graph:

The vertical axis is labeled "Normalized Line Shape $g(\nu - \nu_0)$" (g of nu minus nu zero). It ranges from 0 to 1.0, indicating that both lineshapes are normalized to have a peak value of 1 at the line center.

The horizontal axis is labeled "$\nu - \nu_0$" (nu minus nu zero), representing the frequency offset from the line center ν_0. The center is at 0. The axis is also marked with units like $-2\Gamma_1$, $-\Gamma_1$, 0, Γ_1, $2\Gamma_1$, where Γ_1 (Capital Gamma sub 1) is likely representing the Half Width at Half Maximum (HWHM) or is related to the FWHM. (The previous slides used Γ_L and $\Delta\nu_\mathrm{D}$ for FWHM).

Two curves are plotted:

1. The blue curve is labeled "Lorentzian (Homogeneous)." It is sharply peaked at $\nu - \nu_0 = 0$, with a value of 1.0. As you move away from the center, it falls off, but its distinguishing feature is its relatively "heavy" or "broad" wings. That is, the Lorentzian profile decays more slowly than a Gaussian in the far wings (large $|\nu - \nu_0|$). Specifically, a Lorentzian falls off as

$$\frac{1}{(\nu - \nu_0)^2 + \left(\frac{\Gamma}{2}\right)^2},$$

so for large offsets, it’s like

$$\frac{1}{(\nu - \nu_0)^2}.$$

2. The red curve is labeled "Gaussian (Inhomogeneous)." It is also peaked at $\nu - \nu_0 = 0$ with a value of 1.0. Compared to the Lorentzian (when they have the same FWHM), the Gaussian appears more "pointed" near the peak and falls off much more rapidly in the wings. A Gaussian profile has the form

$$\exp\left[-\left(\frac{\nu - \nu_0}{w}\right)^2\right],$$

where w is related to the width. This exponential decay in the wings is much faster than the algebraic decay of the Lorentzian.

An important annotation on the graph states: "Plot assumes equal FWHM: $\Gamma_{\mathrm{L}} = \Delta\nu_{\mathrm{D}} =$ FWHM." This means that for this specific comparison, the parameters of the Lorentzian (Γ_{L}, its Full Width at Half Maximum) and the Gaussian ($\Delta\nu_{\mathrm{D}}$, its FWHM) have been chosen such that their FWHMs are identical. This allows for a fair visual comparison of their shapes. If they didn't have the same FWHM, one would simply look broader than the other overall.

This visual comparison is very instructive. While both are bell-shaped curves, the difference in their wing behavior is significant and has implications for phenomena like gain saturation and the extent of the frequency range over which a laser might still have some gain, even if small.

Page 31:

Slide 12: Resonator Loss Factor γ — Frequency Behaviour

We now proceed to "Slide 12: Resonator Loss Factor γ — Frequency Behaviour." Just as the gain $\alpha(\nu)$ is frequency-dependent, the total round-trip loss factor γ can also vary with frequency, $\gamma(\nu)$. This slide explores the reasons for this.

The first point notes: "Mirror reflectivity depends on wavelength via coating design." Laser mirrors are typically not simple metallic reflectors but use "dielectric coatings." These consist of multiple thin layers of transparent dielectric materials with different refractive indices, carefully designed with specific thicknesses (often quarter-wavelengths or half-wavelengths). These layers create interference effects that can lead to extremely high reflectivity (e.g., $> 99.9\%$) but usually only over a specific range of wavelengths (or frequencies). Outside this design range, the reflectivity can drop significantly. Since the loss factor γ includes contributions from mirror transmission $(1 - R)$, any frequency dependence of R will directly translate into a frequency dependence of γ.

The second point is: "Diffraction losses vary with mode order → higher-order modes often suppressed." As we discussed, optical resonators support transverse modes TEM_{mn}. Higher-order transverse modes (those with larger m and n values) generally have a larger spatial extent – they are wider beams. If the mirrors or other components in the cavity have a finite aperture (size), these wider higher-order modes are more likely to be "clipped" at the edges, leading to "diffraction losses." The fundamental TEM_{00} mode, being the most compact, typically experiences the lowest diffraction losses. While this isn't a direct dependence on frequency ν itself, if the laser is trying to operate on a higher-order mode (which might have a slightly different resonant frequency due to modal dispersion), its associated diffraction loss might be higher. More commonly, this effect is used to suppress higher-order modes by appropriately sizing apertures, encouraging the laser to operate in the desired TEM_{00} mode. So, γ effectively becomes higher for these undesired modes.

The third point is crucial for understanding cavity behavior: "Total $\gamma(\nu)$ therefore has minima aligned with resonator eigen-frequencies." This refers to the intrinsic behavior of the Fabry-Pérot resonator itself. The cavity is designed to resonate at specific longitudinal mode frequencies ν_q (its eigen-frequencies). At these resonant frequencies, light waves constructively interfere, leading to efficient energy storage and thus *minimal loss* for those specific frequencies. For frequencies that are slightly off-resonance, destructive interference occurs, leading to higher losses from the cavity. So, the resonator itself imposes a frequency-dependent loss that looks like a series of sharp dips (low loss) at each ν_q, and higher loss in between. This is often described by the Airy function for a Fabry-Pérot interferometer's transmission, which implies the loss within the cavity is minimized at these resonances.

The fourth point brings gain and loss together: "Intersection of $-2\,L\alpha(\nu)$ and $\gamma(\nu)$ curves dictates actual laser spectrum." Recall the threshold condition for lasing: the round-trip gain (let's call it $G_{RT}(\nu) = -2\,L\alpha(\nu)$, where $\alpha(\nu)$ is negative for gain) must exceed the round-trip losses $\gamma(\nu)$. So, $G_{RT}(\nu) > \gamma(\nu)$. The actual frequencies at which the laser will oscillate are those where this condition is met.

* $-2\,L\alpha(\nu)$ (or $2\,Lg(\nu)$ if g is positive gain coefficient) represents the gain profile, typically a bell-shaped curve determined by the atomic lineshape. * $\gamma(\nu)$ represents the total loss profile, which will have minima at the cavity eigen-frequencies and may also vary more broadly due to mirror coatings.

The "intersection" of these two curves (or more accurately, the regions where the gain curve lies above the loss curve) will determine the range of frequencies that can lase. The laser will tend to oscillate most strongly where the difference [Gain(ν) – Loss(ν)] is maximized.

Page 32:

Slide 12: Resonator Loss Factor γ — Frequency Behaviour

Continuing with "Slide 12: Resonator Loss Factor γ — Frequency Behaviour," this page discusses an important consequence for laser operation. The single bullet point here states: "Single-mode operation requires cavity and gain bandwidth to support only one intersection."

Let's elaborate on this critical concept. As we saw from the previous discussion, lasing occurs at frequencies where the gain curve $G_{\mathrm{RT}}(\nu)$ is above the loss curve $\gamma(\nu)$. The loss curve $\gamma(\nu)$ has minima at each of the cavity's longitudinal mode frequencies ν_{q}. If the gain profile $G_{\mathrm{RT}}(\nu)$ (determined by the active medium's lineshape) is broad enough to be above the loss minima of several adjacent longitudinal modes, then the laser will oscillate simultaneously on all those modes. This is "multi-mode operation." The output spectrum would consist of several discrete frequencies corresponding to these lasing modes. For many applications, particularly in high-resolution spectroscopy or for achieving very high coherence, "single-mode operation" is desired, meaning the laser outputs only a single longitudinal mode (and typically also a single transverse mode, TEM_{00}). To achieve this, the laser system must be designed such that the condition $G_{\mathrm{RT}}(\nu) > \gamma(\nu)$ is satisfied for *only one* longitudinal mode. The "one intersection" in the slide's phrasing refers to the gain curve only rising above the loss curve for a single one of the cavity's resonant frequencies.

This can be achieved in several ways:

1. **Short cavity:** Make the cavity length d very small. This increases the spacing between longitudinal modes (Free Spectral Range, FSR = $\frac{c}{2\,d}$). If the FSR becomes comparable to or larger than the gain bandwidth of the active medium, then only one mode might fall within the region of sufficient gain.

2. **Narrow gain bandwidth:** If the active medium inherently has a very narrow gain profile, it might only provide gain over a range smaller than the FSR, naturally selecting a single mode.

3. **Introduce intracavity frequency-selective loss elements:** This is a common technique. Elements like an etalon (another Fabry-Pérot device) or a birefringent filter can be placed inside the laser cavity. These elements introduce additional losses that are highly frequency-dependent, effectively making $\gamma(\nu)$ very high for all but one (or a few closely spaced) desired longitudinal modes. This "forces" the laser to operate on the single mode that experiences the lowest net loss.

Achieving stable single-mode operation is a significant topic in laser design and is essential for many advanced spectroscopic applications.

Page 33:

This page presents a graph that beautifully illustrates the concept from "Slide 12: Resonator Loss Factor $\gamma(\nu)$ — Frequency Behaviour," specifically how the gain and loss profiles interact to determine the lasing spectrum.

The graph is titled "Resonator Loss Factor $\gamma(\nu)$ — Frequency Behaviour." * The vertical axis is labeled "Gain / Loss," ranging from 0.0 to 1.0 in arbitrary units. * The horizontal axis is labeled "Frequency (ν)" (nu). Several points are marked along this axis relative to a central frequency ν: $\nu - 3$FSR, $\nu - 2$FSR, $\nu - 1$FSR, ν, $\nu + 1$FSR, $\nu + 2$FSR, $\nu + 3$FSR, where FSR stands for Free Spectral Range, the spacing between cavity modes.

Three curves are plotted:

1. The blue solid curve, which is broad and shaped like an inverted parabola, is labeled "$-2\,L\alpha(\nu)$ (Gain)." This represents the gain profile of the active medium. It's highest at the central frequency ν and falls off to the sides. (As before, $-2\,L\alpha(\nu)$ is positive because $\alpha(\nu)$ is negative for gain).

2. The red solid curve is labeled "$\gamma(\nu)$ (Total Loss)." This curve shows a series of sharp dips, or minima. These minima occur precisely at the frequencies $\nu - 3$FSR, $\nu - 2$FSR, ..., $\nu + 3$FSR, etc., which represent the resonant eigen-frequencies of the optical cavity. Between these resonant frequencies, the loss $\gamma(\nu)$ is significantly higher. This illustrates the frequency-selective nature of the resonator – it has low loss only at its modes.

3. Several vertical green lines are drawn, each topped with a small yellow dot at its base. These are labeled "Lasing Region (Gain > Loss)." These green lines exist *only* at the cavity resonant frequencies (where the red loss curve dips) *and* where the blue gain curve is higher than the red loss curve at that specific mode frequency. The yellow dots seem to indicate the magnitude of the gain available at those specific lasing modes.

What this graph vividly demonstrates is the condition for lasing:

Lasing occurs only at those discrete cavity mode frequencies where the gain provided by the active medium (blue curve) exceeds the total losses of the cavity at that mode frequency (the minima of the red curve).

In this particular illustration: * At the central frequency ν, the gain is well above the loss minimum, so this mode will lase strongly. * The modes at $\nu \pm 1$FSR also have gain significantly above their loss minima, so they will lase. * The modes at $\nu \pm 2$FSR also appear to have gain just above their loss minima, so they might also lase, perhaps more weakly. *The modes at $\nu \pm 3$*FSR have gain (blue curve) that is below *the loss minimum (red curve) at those frequencies. Therefore, these modes will* not* lase.

This laser would therefore be operating on multiple longitudinal modes (at least 5 in this depiction). To achieve single-mode operation, one would need to modify the system (e.g., by narrowing the gain curve or introducing additional losses) such that only one of these green "Lasing Region" lines remains. This plot is a cornerstone for understanding laser spectral output.

Page 34:

Slide 13: Enter Rate Equations — Tracking Populations & Photons

We now transition to a new topic with "Slide 13: Enter Rate Equations — Tracking Populations & Photons." While our previous analysis of gain and threshold was based on steady-state conditions and energy balance, rate equations provide a more dynamic picture of how the populations of energy levels and the number of photons in the cavity evolve over time.

The first point sets the context: "Consider four-level system—simplest practical laser scheme." A four-level laser system (as opposed to a three-level system) is often considered the simplest *practical* scheme for achieving efficient continuous-wave (CW) lasing. In a typical four-level scheme:

* Atoms are pumped from a ground state $|0\rangle$ to a higher energy pump band $|3\rangle$.

* They then rapidly decay (non-radiatively) from $|3\rangle$ to a metastable upper laser level $|2\rangle$.

* The lasing transition occurs from $|2\rangle$ to a lower laser level $|1\rangle$.

* Crucially, level $|1\rangle$ then rapidly decays (non-radiatively) to the ground state $|0\rangle$.

The advantage is that the lower laser level $|1\rangle$ remains largely unpopulated because of its fast decay. This makes it much easier to achieve population inversion ($N_2 > N_1$) between levels $|2\rangle$ and $|1\rangle$, compared to a three-level system where the lower laser level *is* the ground state (requiring more than half the atoms to be pumped out of the ground state for inversion). Examples of four-level lasers include Nd:YAG.

The second point is "Define:" followed by a list of parameters that will be used in the rate equations.

* "P: pump rate into level $|2\rangle$ [atoms cm^{-3} s^{-1}]." (Capital P, into ket $|2\rangle$. Units: atoms per centimeter cubed per second). This P represents the rate at which atoms are supplied to the upper laser level $|2\rangle$ per unit volume per unit time, due to the external pumping mechanism. Even if pumping is to a higher level $|3\rangle$ (as in the diagram on page 36), if the decay from $|3\rangle$ to $|2\rangle$ is very fast, P can be considered the effective rate of population arrival into $|2\rangle$.

* "R_i: non-radiative relaxation rate out of level $|i\rangle$ [s^{-1}]." (R sub i, out of ket $|i\rangle$. Units: per second). This R_{i} represents the rate constant for atoms in energy level $|i\rangle$ to decay to lower levels via processes that do *not* involve the emission of a photon (e.g., collisions, phonon emission in solids). For instance, R_2 would be the non-radiative decay rate

from the upper laser level $|2\rangle$, and R_1 would be the (desirably fast) non-radiative decay rate from the lower laser level $|1\rangle$.

* "$A_{2\ 1}$: spontaneous rate between $|2\rangle$ and $|1\rangle$ [s^{-1}]." (A sub two one, between ket $|2\rangle$ and ket $|1\rangle$. Units: per second). This is the Einstein A-coefficient for spontaneous emission from the upper laser level $|2\rangle$ directly to the lower laser level $|1\rangle$. This process contributes to populating level $|1\rangle$ and depopulating level $|2\rangle$, but the emitted photons are typically lost from the lasing mode (emitted in random directions).

These definitions set up the variables we will use to describe the dynamics of the laser system.

Page 35:

Slide 13: Enter Rate Equations — Tracking Populations & Photons

Continuing with "Slide 13: Enter Rate Equations — Tracking Populations & Photons," this page defines one more crucial variable and states an important approximation.

The first bullet point defines: "n: photon density inside cavity [photons cm^{-3}]."

(small n. Units: photons per centimeter cubed).

This "n" represents the number of photons per unit volume that are present in the lasing mode(s) within the optical resonator. The time evolution of this photon density, $\frac{dn}{dt}$, will be one of the key equations in our rate equation set, as it describes how the laser light builds up or decays.

The second bullet point makes a critical clarification about the applicability of the rate equation approach: "Neglect quantum coherence → population-rate approximation valid for many cw lasers."

This is a very important underlying assumption.

* "Neglect quantum coherence": In a full quantum mechanical description of light-matter interaction (e.g., using the density matrix formalism), one considers not only the populations of the energy levels (diagonal elements of the density matrix) but also the "coherences" between levels (off-diagonal elements). These coherences describe the phase relationships induced by the light field between the quantum states. *"Population-rate approximation": Rate equations, by their nature, only deal with the* populations *of energy levels (*N_1, N_2*,* etc.) and the number* (or density 'n') of photons. They do not explicitly track these quantum coherences. This is a simplification. * "valid for many cw lasers": This approximation is generally valid under conditions where any induced coherences decay very rapidly (i.e., dephasing times are very short) compared to the timescales of population changes due to pumping and stimulated emission. This is often true for continuous-wave (CW) lasers operating well above threshold, where the photon field is strong and can be treated somewhat classically, and for systems where collisions or other interactions cause rapid dephasing.

For situations involving very short pulses, or phenomena where quantum coherence is paramount (like Rabi oscillations or self-induced transparency), a more sophisticated treatment beyond simple rate equations (such as the Maxwell-Bloch equations) would be necessary. However, for understanding the basic dynamics, threshold conditions, and steady-state output power of many common lasers, the rate equation approach is both powerful and intuitive.

Page 36:

This page, still part of "Slide 13," presents the "Four-Level Laser System Diagram" along with "Rate Equation Parameters," visually laying out the energy levels and transitions we'll be modeling.

Let's examine the diagram:

Four distinct energy levels are shown as horizontal lines, labeled using ket notation: * $|0\rangle$ (Ground State) at the very bottom. * $|1\rangle$ (Lower Lasing Level) above the ground state. * $|2\rangle$ (Upper Lasing Level) above level $|1\rangle$. This is a metastable state from which lasing will occur. * $|3\rangle$ (Pump Band) at the highest energy shown.

Various arrows depict the movement of atomic populations between these levels: * Pumping: A thick green horizontal arrow labeled $P\left[\text{atoms cm}^{-3}\text{ s}^{-1}\right]$ (P in units of atoms per centimeter cubed per second) originates from the ground state $|0\rangle$ and points towards the pump band $|3\rangle$, with the process labeled "Optical Absorption." This indicates that the pump energy excites atoms from the ground state up to level $|3\rangle$. * Fast Relaxation to Upper Laser Level: A dashed gray arrow points downwards from $|3\rangle$ to $|2\rangle$, labeled "Fast Relaxation." This signifies that atoms pumped to $|3\rangle$ quickly and efficiently (often non-radiatively) transfer to the upper laser level $|2\rangle$. This is a common feature in practical four-level systems; level $|2\rangle$ is where the population accumulates for lasing. * Lasing Transition: A prominent red curved arrow points downwards from $|2\rangle$ to $|1\rangle$. This is labeled "Lasing A_{21} (+ Stim. Em.)." This represents the transition where laser light is produced. It includes both spontaneous emission (rate A_{21}) and, crucially, stimulated emission (which will depend on the photon density n). * Decay from Upper Laser Level (Non-radiative): Another arrow, a blue curved one labeled R_2 (Non-rad. Decay), is shown originating from level $|2\rangle$ and pointing downwards, possibly towards $|1\rangle$ or even bypassing to $|0\rangle$. This represents non-radiative decay paths from the upper laser level that do not contribute to lasing photons. (The earlier definition of R_{i} was generic; here R_2 is specifically for level 2). * Fast Decay of Lower Laser Level: A crucial blue curved arrow, labeled R_1 (Fast Decay), points downwards from the lower lasing level $|1\rangle$ to the ground state $|0\rangle$. This rapid depopulation of level $|1\rangle$ is essential for maintaining population inversion between $|2\rangle$ and $|1\rangle$.

This diagram clearly illustrates the population flow in a typical four-level laser:

1. Pump from $|0\rangle$ to $|3\rangle$. 2. Fast relaxation from $|3\rangle$ to $|2\rangle$ (upper laser level). 3. Lasing transition (stimulated and spontaneous emission) from $|2\rangle$ to $|1\rangle$. 4. Fast relaxation from $|1\rangle$ (lower laser level) to $|0\rangle$.

The rates P, A_{21}, R_1, and R_2 (and the stimulated emission rate, which will involve a B-coefficient and photon density n) will form the basis of our rate equations.

Page 37:

Slide 14: Complete Rate-Equation Set

Now we arrive at "Slide 14: Complete Rate-Equation Set." This slide presents the coupled differential equations that describe the time evolution of the populations of the lower and upper laser levels, and the photon density in the cavity, based on the four-level system we just saw.

We'll denote N_1 as the population density of level $|1\rangle$ (the lower laser level) and N_2 as the population density of level $|2\rangle$ (the upper laser level). n is the photon density in the lasing mode. Let's use B' as a shorthand for $B_{21}\, h\nu$, where B_{21} is an Einstein-like coefficient and $h\nu$ is the photon energy, such that $B'n$ represents a rate of stimulated transition.

First, for the "Population of lower laser level $|1\rangle$:"

The rate equation is:

$$\frac{dN_1}{dt} = (N_2 - N_1)B_{21}\, n\, h\nu + N_2\, A_{21} - N_1\, R_1$$

Let's read this: "d N one by d t, equals, open parenthesis N two minus N one close parenthesis, times B two one n h nu, plus N two A two one, minus N one R one."

Let's break down each term on the right-hand side, which contributes to the rate of change of N_1:

* $(N_2 - N_1)B_{21}\, n\, h\nu$: This term represents the net rate of population change in N_1 due to stimulated processes (emission from N_2 to N_1 and absorption from N_1 to N_2). If $(N_2 - N_1)$ is positive (inversion), this term is positive, meaning N_1 increases due to net stimulated emission from N_2. If $(N_2 - N_1)$ is negative, N_1 decreases due to net absorption to N_2. This formulation implies B_{21} accounts for both processes with the $(N_2 - N_1)$ factor.

* $N_2\, A_{21}$: This term is straightforward. Atoms in the upper level N_2 spontaneously decay to level N_1 at a rate A_{21}, thus increasing the population of N_1.

* $-N_1\, R_1$: This term represents the decay of population from level N_1 (e.g., to the ground state) via non-radiative processes, at a rate R_1. This decreases the population of N_1. This rapid decay is vital for a four-level laser.

Second, for the "Population of upper laser level $|2\rangle$:"

The rate equation is:

$$\frac{dN_2}{dt} = P - (N_2 - N_1)B_{21}\, n\, h\nu - N_2\, A_{21} - N_2\, R_2$$

Let's read this: "d N two by d t, equals P, minus, open parenthesis N two minus N one close parenthesis, times B two one n h nu, minus N two A two one, minus N two R two."

Breaking down the terms:

* P: This is the pump rate, supplying population to the upper level N_2.

* $-(N_2 - N_1)B_{21}\, n\, h\nu$: This is the same stimulated transition term as above, but with a minus sign. If there's net stimulated emission from N_2 to

N_1 (i.e., $N_2 - N_1 > 0$), this term is negative, representing the depopulation of N_2.

* $-N_2 A_{21}$: Spontaneous emission from N_2 to N_1 also depopulates N_2.

* $-N_2 R_2$: Non-radiative decay from N_2 (to levels other than N_1 through the lasing transition, or to N_1 non-radiatively) at a rate R_2 also depopulates N_2.

Third, for the "Photon density inside cavity:"

The equation for $\frac{dn}{dt}$ will be on the next slide. These two equations for $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ describe how the populations of the lasing levels change due to pumping, spontaneous emission, stimulated emission/absorption, and non-radiative decay. The term $(N_2 - N_1)B_{21}\, n\, h\nu$ couples these two equations together and also couples them to the photon density n.

Page 38:

Continuing with "Slide 14: Complete Rate-Equation Set," this page presents the third crucial rate equation: the one for the photon density n inside the cavity.

The equation is:

$$\frac{dn}{dt} = -\beta n + (N_2 - N_1)B_{21} n h\nu$$

Let's read this: "d n by d t, equals minus beta n, plus, open parenthesis N two minus N one close parenthesis, times B two one n h nu."

Let's break down the terms on the right-hand side, which contribute to the rate of change of the photon density n:

* $-\beta n$ (minus beta n): This term represents the loss of photons from the cavity. * n is the photon density (photons per unit volume). * β (beta) is the photon loss rate constant, with units of s^{-1} (per second). It accounts for all

mechanisms by which photons are removed from the lasing mode, such as transmission through the output coupler (useful output), absorption within cavity components, and scattering out of the mode. β is the reciprocal of the photon lifetime (τ_p or τ_c) in the cavity: $\beta = \frac{1}{\tau_\mathrm{p}}$. * $+(N_2 - N_1)B_{21}nh\nu$: This is the source term for photons, representing the net rate of photon generation per unit volume due to stimulated emission. * $(N_2 - N_1)$: The effective population inversion density. If this is positive, net stimulated emission occurs. * $B_{21}nh\nu$: This must represent the rate of stimulated emission events per unit of this inversion that contributes one photon to n for each event. The presence of n itself in this term ($B_{21}nh\nu$) makes the generation rate proportional to the existing photon density, which is the hallmark of stimulated emission – photons stimulate the creation of more identical photons. The factor $B_{21}h\nu$ here acts as a rate coefficient (with units of volume per time, e.g., cm³/s) when multiplied by the population inversion $(N_2 - N_1)$.

Following the equation, two important points are made: 1. β [s^{-1}]: photon loss rate due to mirror transmission + intracavity absorption. This confirms our interpretation of β as the total photon loss rate constant from the cavity, encompassing all loss mechanisms.

2. Coupling between matter and field explicitly visible via $(N_2 - N_1)B_{21}nh\nu$. *When it appears with a plus sign in* $\frac{dn}{dt}$, it means photons are being created*. *When it appears with a minus sign in* $\frac{dN_2}{dt}$ (as on the previous page), it means the upper laser level population N_2 is being depleted*. *When it appears with a plus sign in* $\frac{dN_1}{dt}$ (previous page), it means the lower laser level population N_1 is being populated*.

This single term beautifully encapsulates the energy conversion process: atomic excitation energy (from the population inversion) is converted into electromagnetic energy in the form of photons in the lasing mode. The strength of this conversion process depends on the magnitude of the inversion $(N_2 - N_1)$ and, critically, on the existing photon density n (the stimulation).

The "---" indicates the end of this slide. This set of three coupled rate equations (for N_1, N_2, and n) forms a powerful, albeit simplified, model for understanding laser dynamics.

Page 39:

Now we move to "Slide 15: Relating β to Cavity Loss γ." We have two different ways of describing losses in the laser cavity: * β (beta): The photon loss rate constant (s^{-1}) from the rate equations, describing continuous exponential decay of photons. * γ (gamma): The dimensionless fractional energy loss per round-trip, used in our earlier threshold analysis.

This slide aims to connect these two parameters.

The first point defines "One round-trip time:"

$$T = \frac{2\,d}{c}$$

Let's read this: "Capital T equals two d, divided by c." * T (Capital T): The time it takes for light to complete one full round trip inside the optical resonator. * d (small d): The length of the resonator (mirror-to-mirror distance). * c (small c): The speed of light in the medium filling the cavity.

This is a standard definition for the round-trip time.

The second point reiterates: "Fractional energy loss per round-trip γ." (gamma). This is the γ we used before, for example, in

$$\Delta N_{\text{thr}} = \frac{\gamma}{2\sigma L}$$

It represents the fraction of photons (or energy) lost from the cavity during one round-trip time T.

The third point says: "Equate exponential decays:" We have two perspectives on how photon intensity (or number) decays in a passive cavity:

1. From the rate equation

$$\frac{dn}{dt} = -\beta n,$$

the solution is an exponential decay of the photon density $n(t)$ over continuous time t:

$$n(t) = n(0)e^{-\beta t}$$

2. From the round-trip loss perspective, if γ is the fractional loss per round trip, then after y round trips, the intensity I is $I_0(1-\gamma)^y$. If γ is small, $(1-\gamma) \approx e^{-\gamma}$. So, after y round trips, $I \approx I_0(e^{-\gamma})^y = I_0 e^{-\gamma y}$.

The slide uses this exponential form directly:

$$I = I_0 e^{-\gamma y} \quad \text{with} \quad y = \frac{t}{T}$$

Here, y is the number of round trips, which can be expressed as continuous time t divided by the time per round trip T.

So, substituting $y = \frac{t}{T}$, the round-trip decay equation becomes:

$$I(t) = I_0 \exp\left(-\gamma \frac{t}{T}\right)$$

Now, if $n(t)$ from the rate equation and $I(t)$ from the round-trip analysis represent the same decaying quantity (photon number or energy), their exponential decay rates must be equivalent.

Comparing the exponents:

$$\beta t = \gamma \frac{t}{T}$$

Dividing by t (assuming $t \neq 0$), we get:

$$\beta = \frac{\gamma}{T}$$

This provides the crucial link between the continuous decay rate β and the per-round-trip loss γ.

Page 40:

Continuing from "Slide 15: Relating β to Cavity Loss γ," this page formalizes the relationship we just derived.

The first point begins with "Hence," indicating a conclusion from the previous step.

The derived relationship is:

$$\beta = \frac{\gamma}{T} = \gamma\left(\frac{c}{2\,d}\right)$$

Let's read this: " β equals γ divided by capital T, which equals γ, times, open parenthesis c divided by two d close parenthesis."

This follows directly from $\beta = \frac{\gamma}{T}$ and substituting the expression for the round-trip time $T = \frac{2\,d}{c}$.

So,

$$\beta = \frac{\gamma}{(2\,d/c)} = \frac{\gamma c}{2\,d}$$

This equation provides a direct mathematical link between:

* β (beta): The photon loss rate constant from the rate equations (units s^{-1}). * γ (gamma): The dimensionless fractional loss per round-trip. * c: The speed of light. * d: The cavity length.

The second bullet point emphasizes the significance of this relationship:

"Convenient link between macroscopic loss coefficient γ and microscopic photon decay rate β."

* "macroscopic loss coefficient γ": Gamma is often determined from macroscopic properties of the resonator, such as mirror reflectivities ($R_{\text{output\_\text{coupler}}}$, $R_{\text{high\_\text{reflector}}}$) and estimated scattering/absorption losses. For instance, γ might be approximated as $(1 - R_1 R_2)$ + other losses. It describes the overall performance of the cavity in terms of how much energy it loses per round trip. * "microscopic photon decay rate β": Beta is a parameter that appears in the differential rate equations, which model the laser dynamics at a more "microscopic" level (tracking population and photon densities over continuous time).

This equation,

$$\beta = \frac{\gamma c}{2\, d}$$

allows us to bridge these two perspectives. We can estimate γ from the physical design of the cavity and then calculate β to use in the rate equations, or vice versa. This ensures consistency between different models of laser operation.

The "---" indicates the end of this slide's content. This connection is vital for quantitative laser modeling.

Page 41:

Now we move to "Slide 16: Steady-State Solutions — Pump Power Balance." After understanding the dynamic rate equations, it's often useful to analyze the laser's behavior in the "steady state," where populations and photon density are constant over time. This is relevant for continuous-wave (CW) lasers.

The first point defines the "Stationary regime:"

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dn}{dt} = 0.$$

$$\left(\frac{dN_1}{dt}, \frac{dN_2}{dt}, \frac{dn}{dt} \text{ all equal zero}\right)$$

This is the mathematical condition for steady state: all time derivatives of the system variables (N_1, N_2, and n) are set to zero. The system has reached an equilibrium where the rates of processes populating each level or the photon mode are exactly balanced by the rates of processes depopulating them.

The second point states: "Adding population equations yields pump-relaxation balance:"

Let's recall the rate equations for N_1 and N_2 from page 37 (using B' as shorthand for $B_{21}h\nu$):

$$\frac{dN_1}{dt} = (N_2 - N_1)B'n + N_2A_{21} - N_1R_1 = 0 \quad \text{(in steady state)}$$

$$\frac{dN_2}{dt} = P - (N_2 - N_1)B'n - N_2A_{21} - N_2R_2 = 0 \quad \text{(in steady state)}$$

If we add these two equations:

$$[(N_2 - N_1)B'n + N_2A_{21} - N_1R_1] \\ +[P - (N_2 - N_1)B'n - N_2A_{21} - N_2R_2] = 0 + 0$$

The terms $(N_2 - N_1)B'n$ cancel out.

The terms N_2A_{21} also cancel out.

We are left with:

$$-N_1R_1 + P - N_2R_2 = 0.$$

Rearranging this gives the equation on the slide:

$$P = N_1R_1 + N_2R_2.$$

This equation provides a simple balance: in steady state, the total pump rate P (atoms per unit volume per second being supplied to the upper laser level system) is exactly balanced by the sum of relaxation rates out of levels N_1 and N_2 via processes R_1 and R_2 respectively. These R_1 and R_2 are typically non-lasing decay paths. This equation essentially says that all atoms pumped into the system must eventually find a way out through these relaxation channels, *in addition* to any lasing processes.

The third point states: "Adding upper-level and photon equations gives:"

Let's take the steady-state equations for $\frac{dN_2}{dt}$ and $\frac{dn}{dt}$:

(1)

$$\frac{dN_2}{dt} = P - (N_2 - N_1)B'n - N_2A_{21} - N_2R_2 = 0,$$

(2)

$$\frac{dn}{dt} = -\beta n + (N_2 - N_1)B'n = 0.$$

From equation (2), assuming $n \neq 0$ (i.e., the laser is operating above threshold), we can divide by n to get:

$$-\beta + (N_2 - N_1)B' = 0 \quad \Rightarrow \quad (N_2 - N_1)B' = \beta.$$

So, the term $(N_2 - N_1)B'n$ in equation (1) can be replaced by βn.

Substituting this into equation (1):

$$P - \beta n - N_2A_{21} - N_2R_2 = 0.$$

Rearranging this gives the equation on the slide:

$$P = \beta n + N_2(A_{21} + R_2).$$

This is another crucial power balance equation. It states that the total pump rate P is channeled into two main paths:

- βn: This term represents the rate at which photons are lost from the cavity (per unit volume). In steady state above threshold, this loss must be balanced by generation, and a significant fraction of this βn is the useful laser output. So, βn is proportional to the output laser power.

- $N_2(A_{21} + R_2)$: This term represents the rate at which atoms are lost from the upper laser level N_2 due to spontaneous emission ($N_2 A_{21}$) and non-radiative decay ($N_2 R_2$). These are generally "waste" channels from the perspective of laser output, as they consume pumped atoms without contributing to coherent photons in the lasing mode.

The fourth point says: "Lower-level relaxation must satisfy"

This refers to the steady-state condition for $\frac{dN_1}{dt} = 0$:

$$(N_2 - N_1)B'n + N_2 A_{21} - N_1 R_1 = 0.$$

Again, substituting $(N_2 - N_1)B'n = \beta n$, we get:

$$\beta n + N_2 A_{21} - N_1 R_1 = 0.$$

This will be shown on the next page. These power balance equations provide great insight into laser efficiency and operation.

Page 42

Continuing with "Slide 16: Steady-State Solutions — Pump Power Balance," this page presents the result for the lower-level relaxation balance and provides a physical interpretation.

The equation derived from $\frac{dN_1}{dt} = 0$ in steady state (as we worked out on the previous page) is:

$$N_1 R_1 = \beta n + N_2 A_{21}$$

Let's re-verify the terms for clarity:

* $N_1 R_1$: This is the rate at which population leaves the lower laser level $|1\rangle$ (population density N_1) via relaxation processes (rate constant R_1). * βn: This represents the net rate of stimulated emission from $|2\rangle$ to $|1\rangle$ that results in photons being added to the lasing mode n and subsequently lost/outputted at rate βn. For every such photon, an atom transitions from N_2 to N_1. * $N_2 A_{21}$: This is the rate at which population arrives in the lower laser level $|1\rangle$ due to spontaneous emission from the upper laser level $|2\rangle$ (population N_2).

So, in steady state, the rate at which atoms *leave* N_1 ($N_1 R_1$) must exactly balance the rate at which they *arrive* in N_1 (due to net stimulated emission, βn, plus spontaneous emission, $N_2 A_{21}$). This makes perfect sense for a steady-state population.

The next bullet point provides the "Physical meaning" for the *other* power balance equation we derived, $P = \beta n + N_2(A_{21} + R_2)$:

"Physical meaning: pump power splits into useful photon output and unavoidable relaxation losses."

* "Pump power" (P): This is the energy per unit time per unit volume supplied to create the excited state population. * "useful photon output" (related to βn): A portion of the pump power is converted into coherent photons in the lasing mode, which are then extracted from the cavity as the laser beam. βn quantifies the rate of these photons. * "unavoidable relaxation losses" (related to $N_2(A_{21} + R_2)$): Another portion of the pump power is inevitably lost through processes that don't contribute to the laser output. These include: * Spontaneous emission ($N_2 A_{21}$) from the upper laser level in random directions. * Non-radiative decay ($N_2 R_2$) from the upper laser level (e.g., due to collisions or conversion to heat).

Understanding this split is crucial for optimizing laser efficiency – the goal is to maximize the fraction of pump power that goes into βn and minimize the losses.

The "---" (one at the top, one at the bottom) indicates the slide's content. These steady-state relations are powerful tools for analyzing CW laser performance.

Page 43:

Now we turn to "Slide 17: Stationary Inversion Expression." This slide aims to derive an explicit algebraic expression for the steady-state population inversion, ΔN_{stat} (where "stat" means stationary or steady-state), in terms of the pump power P and other laser parameters. This expression often reveals critical conditions for achieving and maintaining inversion.

The first bullet point describes the mathematical manipulation: "Multiply first rate equation by R_2 and second by R_1, add:" The "first rate equation" refers to $\frac{dN_1}{dt} = 0$, and the "second" to $\frac{dN_2}{dt} = 0$ from our steady-state set (page 41). Let $B' = B_{21} nh\nu$.

(1) $(N_2 - N_1)B' + N_2 A_{21} - N_1 R_1 = 0$ (Note: B' has 'n' in it from $B_{21}\, n\, h\nu$, so it's $B_{21} h\nu$)

Let's rewrite to be clearer: $(N_2 - N_1)(B_{21} h\nu)n + N_2 A_{21} - N_1 R_1 = 0$

(2) $P - (N_2 - N_1)(B_{21} h\nu)n - N_2 A_{21} - N_2 R_2 = 0$

This algebraic derivation to get the slide's expression for ΔN_{stat} is non-trivial and typically involves solving for N_1 and N_2 and then forming $\Delta N = N_2 - N_1$ (or a degeneracy-weighted version, though the rate equations here used N_1 and N_2 directly). The slide presents the result of such an algebraic solution (likely for $\Delta N = N_2 - N_1$): $\Delta N_{\text{stat}} = \frac{(R_1 - A_{21})P}{B_{12} nh\nu(R_1 + R_2) + A_{21} R_1 + R_1 R_2}$

Let's read this carefully: "Capital Delta N sub stat, equals, the product of, open parenthesis R_1 minus A_{21} close parenthesis, times P, all divided by, open parenthesis $B_{12} nh\nu$, times, open parenthesis R_1 plus R_2 close parenthesis, plus $A_{21} R_1$, plus $R_1 R_2$ close parenthesis."

Let's analyze this expression for the steady-state inversion ΔN_{stat}:

* Numerator: $(R_1 - A_{21})P$. This term is proportional to the pump rate P, which makes sense – more pumping should lead to more inversion. The factor $(R_1 - A_{21})$ is very significant. R_1 is the decay rate of the lower laser level $|1\rangle$, and A_{21} is the spontaneous emission rate from the upper level $|2\rangle$ to the lower level $|1\rangle$ (which populates $|1\rangle$). For ΔN_{stat} to be positive (i.e., for inversion to occur) when P is positive, we need $(R_1 - A_{21}) > 0$, or $R_1 > A_{21}$.

* Denominator: $B_{12}nh\nu(R_1 + R_2) + A_{21}R_1 + R_1R_2$. (Note: B_{12} is used here; assuming B_{12} relates to B_{21} through degeneracies or they are treated as similar for this context. Let's interpret $B_{12}nh\nu$ as a term proportional to the stimulated emission rate, similar to our $(B_{21}h\nu)n$.) * The term $B_{12}nh\nu(R_1 + R_2)$ involves the photon density n. As n increases (i.e., as the laser operates further above threshold), this term increases, which tends to decrease ΔN_{stat}. This is known as gain saturation: as the field intensity builds up, it depletes the inversion. * The terms $A_{21}R_1 + R_1R_2$ are combinations of relaxation rates.

This expression shows that ΔN_{stat} depends on the pump power, all the relevant decay rates (A_{21}, R_1, R_2), and also on the photon density n itself.

The second point highlights a critical condition derived from this expression: "Continuous inversion requires $R_1 > A_{21}$". As noted above, for ΔN_{stat} to be positive (assuming $P > 0$ and the denominator is positive), the numerator $(R_1 - A_{21})$ must be positive. This means $R_1 > A_{21}$.

The third point, indicated by an arrow, gives the physical interpretation: "→ lower laser level must empty faster than it is refilled by spontaneous decay." This is the crucial design principle for a four-level laser: * R_1 is the rate at which the lower laser level $|1\rangle$ empties (typically to the ground state). * A_{21} is the rate at which level $|1\rangle$ is populated by spontaneous emission from the upper laser level $|2\rangle$. If R_1 is not greater than A_{21}, then level $|1\rangle$ will fill up due to spontaneous emission (and also stimulated emission once lasing starts) faster than it can empty. This will cause N_1 to increase, which will reduce or destroy the population inversion $(N_2 - N_1)$.

Therefore, for efficient and continuous lasing, it's essential that the lower laser level has a very fast decay rate R_1.

Page 44:

Continuing with "Slide 17: Stationary Inversion Expression," this page presents a stricter criterion for maintaining inversion, especially in the presence of a strong lasing field.

The first bullet point states: "In presence of strong field, stricter criterion $R_1 > A_{21} + B_{21}\rho$".

Let's read this: "R one is greater than, A two one plus B two one rho."

Here: * R_1 is the decay rate of the lower laser level $|1\rangle$. * A_{21} is the spontaneous emission rate from upper level $|2\rangle$ to lower level $|1\rangle$. $B_{21}\rho$ (B two one times rho): This term represents the rate at which the lower level $|1\rangle$ is being populated due to stimulated emission* from level $|2\rangle$. * B_{21} is the Einstein B-coefficient for stimulated emission. * ρ (rho) represents the energy density of the lasing field (ρ is proportional to the photon density n times $h\nu$, so $B_{21}\rho$ is akin to our $(B_{21}h\nu)n$ term, the stimulated transition rate).

The second bullet point explains the importance of this: "ensuring inversion not quenched by stimulated emission into other modes."

This phrasing might be slightly misleading. The $B_{21}\rho$ term here primarily refers to stimulated emission *into the lasing mode itself*. As the laser output power increases, the photon density ρ within the lasing mode becomes very high. This high photon density drives strong stimulated emission from N_2 to N_1, which is what produces the laser light. However, this same process also populates N_1. If N_1 cannot empty fast enough (i.e., if R_1 is not large enough), its population N_1 will build up. As N_1 increases, the inversion $\Delta N = N_2 - N_1$ (or its degeneracy-weighted form) decreases. If ΔN drops below the threshold value ΔN_{thr}, lasing will cease – this is "quenching" of the inversion.

So, the condition $R_1 > A_{21} + B_{21}\rho$ ensures that even under strong lasing conditions, the lower level $|1\rangle$ is depopulated sufficiently rapidly to prevent this quenching and maintain the necessary population inversion.

The phrase "into other modes" might allude to a situation where, if the main lasing mode is very strong, there might also be significant amplified spontaneous emission (ASE) over a broader range of frequencies or directions that could also contribute to populating N_1, but the dominant effect for quenching via $B_{21}\rho$ is usually from the lasing mode itself.

The "---" indicates the end of the slide. This highlights a practical limitation in laser design: the lower laser level lifetime is critically important, especially for high-power operation.

Page 45:

We now proceed to "Slide 18: Threshold Inversion via Rate Equations." We previously derived the threshold inversion ΔN_{thr} using a simple gain- equals- loss argument. This slide shows how the same result can be obtained from the steady- state analysis of the rate equations, providing a consistency check.

Step 1: "Set $\frac{dn}{dt} = 0$ in photon rate equation:"

The photon rate equation (from page 38) is:

$$\frac{dn}{dt} = -\beta n + (N_2 - N_1)B_{21}nh\nu.$$

Setting $\frac{dn}{dt} = 0$ for steady state, we get:

$$0 = -\beta n + (N_2 - N_1)B_{21}nh\nu.$$

This can be rewritten as:

$$(N_2 - N_1)B_{21}nh\nu = \beta n.$$

This equation states that in steady state, the rate of photon generation due to net stimulated emission, $(N_2 - N_1)B_{21}nh\nu$, is exactly equal to the rate at which photons are lost from the cavity (right side), for a constant photon density n.

Step 2 on the slide is: "Solve for inversion:"

Assuming the laser is operating ($n \neq 0$), we can divide both sides of the equation by n:

$$(N_2 - N_1)B_{21}h\nu = \beta.$$

Now, we define the population inversion density ΔN as $(N_2 - N_1)$. At threshold, this ΔN is the threshold inversion density, ΔN_{thr}. So,

$$\Delta N_{\text{thr}}B_{21}h\nu = \beta.$$

Solving for ΔN_{thr} gives us:

$$\Delta N_{\text{thr}} = \frac{\beta}{B_{21}h\nu}.$$

(Capital Delta N equals beta, divided by the product $B_{21}h\nu$.)

This expression gives the threshold population inversion in terms of the photon loss rate β and the product $B_{21}h\nu$, which characterizes the strength of stimulated emission per unit inversion per photon.

Here, B_{21} is the Einstein B-coefficient (or a related coefficient specific to this rate equation formulation), h is Planck's constant, and ν is the photon frequency. The product $B_{21}h\nu$ effectively acts as a volume per unit time, such that when multiplied by ΔN (a density) and n (a density), it gives a rate of change of density.

Step 3 is: "Relate β to γ and σ (using $\sigma = \frac{h\nu}{c}B_{21}$):"

(σ equals $\frac{h\nu}{c}$ times B_{21}.)

This step aims to re-express the threshold inversion ΔN_{thr} in terms of more commonly used macroscopic parameters: γ (round-trip loss) and σ (stimulated emission cross-section).

The slide provides the relationship between the cross-section σ and the Einstein coefficient B_{21} as:

$$\sigma = \frac{h\nu}{c} B_{21}.$$

From this, we can express the product $B_{21}h\nu$:

$$B_{21}h\nu = \sigma c.$$

Now, substitute this expression for $B_{21}h\nu$ back into our equation for ΔN_{thr}:

$$\Delta N_{\text{thr}} = \frac{\beta}{B_{21}h\nu} \quad \text{becomes} \quad \Delta N_{\text{thr}} = \frac{\beta}{\sigma c}.$$

So, from the rate equations, we've found that the threshold inversion density is equal to the photon loss rate β, divided by the product of the stimulated emission cross-section σ and the speed of light c.

This is as far as the explicit steps on this particular slide take us. The final connection to γ and L will appear on the next slide, where this result is shown to be identical to our previous derivation.

Page 46:

Slide 18: Threshold Inversion via Rate Equations

This page continues "Slide 18: Threshold Inversion via Rate Equations" and presents the final form of the threshold inversion, demonstrating consistency.

The equation shown at the top of this page is:

$$\Delta N_{\text{thr}} = \frac{\gamma}{2\,L\sigma}$$

(Capital Delta N sub t h r, equals gamma, divided by the product of two, capital L, and sigma).

Let's analyze this.

- ΔN_{thr} is the threshold population inversion density. - γ (gamma) is the dimensionless total loss per round-trip in the cavity. - L (Capital L) is the length of the active gain medium. - σ (sigma) is the stimulated emission cross-section (at the lasing frequency ν).

The crucial point made next is: "Exactly identical to threshold derived from gain-loss balance → confirms internal consistency of rate-equation approach."

This is a very important conclusion.

Recall that on page 24, using a straightforward argument that round-trip gain must equal round-trip loss at threshold (specifically, $2\,L\Delta N_{\text{thr}}\sigma(\nu) = \gamma$), we derived precisely this same expression: $\Delta N_{\text{thr}} = \frac{\gamma}{2\,L\sigma}$.

Now, starting from the rate equations, we first found $\Delta N_{\text{thr}} = \frac{\beta}{\sigma c}$ (at the end of page 45).

For this to be equal to $\frac{\gamma}{2\,L\sigma}$, we must have:

$$\frac{\beta}{\sigma c} = \frac{\gamma}{2\,L\sigma}$$

Dividing both sides by σ (assuming $\sigma \neq 0$), we get:

$$\frac{\beta}{c} = \frac{\gamma}{2\,L}$$

Which means $\beta = \frac{\gamma c}{2\,L}$.

This relationship, $\beta = \frac{\gamma c}{2\,L}$, connects the photon loss rate β (from the rate equations) to the macroscopic round-trip loss γ, the speed of light c, and crucially, the *length of the active medium* L.

Previously, on page 40, we related β to γ using the cavity length 'd': $\beta = \frac{\gamma c}{2\,d}$.

The consistency between $\Delta N_{\text{thr}} = \frac{\gamma}{2\,L\sigma}$ and $\Delta N_{\text{thr}} = \frac{\beta}{\sigma c}$ requires that the β used in the rate equations (or its interpretation in the context of gain occurring over length L within a cavity of length d) effectively makes β behave as if it's $\frac{\gamma c}{2\,L}$ when considering the gain dynamics.

This often implies that the photon density n and the populations N_1, N_2 are considered uniform over the active medium length L, and the rate equations are normalized to this volume.

If $L < d$ (the active medium doesn't fill the cavity), then the factor $\frac{L}{d}$ (filling factor) implicitly gets absorbed into the definition of the effective β or the effective gain term in the rate equation for $\frac{dn}{dt}$ if n is averaged over the whole cavity.

However, the key message of this slide is that regardless of these definitional nuances, the rate equation approach, when taken to its steady-state threshold condition, yields the *exact same expression* for the required population inversion as the simpler, more intuitive gain-equals-loss model.

This gives us great confidence in the validity and internal consistency of both approaches for describing this fundamental laser parameter.

Page 47:

We now move to "Slide 19: He-Ne Laser — Detailed Power Budget Example." This slide will apply the concepts from the rate equations, particularly the steady-state power balance, to a Helium-Neon laser to understand how the pump power is distributed.

First, "Given:" values for a He-Ne laser:

* $N_2 = 1.0 \times 10^{10}$ cm^{-3} (N sub two equals one point zero times ten to the ten per centimeter cubed). This is the steady-state population density of the upper laser level $|2\rangle$ when the laser is operating. This value would itself be a result of the laser being above threshold and reaching a saturated gain condition.

* $A_{21} + R_2 = 2.0 \times 10^7$ s^{-1} (A sub two one, plus R sub two, equals two point zero times ten to the seven per second). This is the total decay rate from the upper laser level $|2\rangle$ due to spontaneous emission to level $|1\rangle$ (A_{21}) and all non-radiative decay processes out of $|2\rangle$ (R_2). This combined rate represents losses from the upper laser level that do not contribute to the useful laser output.

* Tube volume $V = 0.075$ cm^3 ($L = 10$ cm, diameter $= 1$ mm) (Capital V equals zero point zero seven five centimeters cubed, where capital L equals 10 centimeters, and diameter is 1 millimeter). The volume of the active gain medium is given. This can be calculated from the length L and diameter (Area $= \pi$(diameter$/2)^2$). Let's check: Area $=$ $\pi(0.05$ cm$)^2 \approx 3.14 \times 0.0025$ cm$^2 \approx 0.00785$ cm^2. Volume $=$ Area $\times L \approx$ 0.00785 cm$^2 \times 10$ cm ≈ 0.0785 cm^3, which is close to 0.075 cm^3. This volume will be used to convert densities to total numbers or rates.

Next, calculation of the "Incoherent loss rate (upper-level depletion):"

This is the total rate at which atoms are lost from the upper laser level N_2 throughout the entire volume V, due to the non-lasing decay paths A_{21} and R_2.

The rate per unit volume is $N_2(A_{21} + R_2)$.

The total rate for the whole volume is $N_2(A_{21} + R_2)V$.

Plugging in the given values:

$$N_2(A_{21} + R_2)V = (1.0 \times 10^{10}\ \text{cm}^{-3}) \times (2.0 \times 10^7\ \text{s}^{-1}) \times (0.075\ \text{cm}^{3)}$$

$$= (1.0 \times 2.0 \times 0.075) \times (10^{10} \times 10^7)\ \text{s}^{-1}$$

$$= 0.15 \times 10^{17}\ \text{s}^{-1}$$

$$= 1.5 \times 10^{16}\ \text{s}^{-1}$$

The slide shows: "N_2 ($A_{2\ 1}$ + R_2)V = 1.5 × 10^{16} s^{-1}" (N sub two times, open parenthesis A sub two one plus R sub two close parenthesis, times V, equals one point five times ten to the sixteen per second).

This means that 1.5×10^{16} atoms per second are "wasted" from the upper laser level via spontaneous emission and non-radiative decay within the entire active volume. Each of these represents a pumped atom that did not contribute to laser output.

Page 48:

Slide 19: He-Ne Laser — Detailed Power Budget Example

Continuing with "Slide 19: He-Ne Laser — Detailed Power Budget Example," this page calculates the required pump rate based on a target output power.

First, we have the "Target output power $P_{\text{L}} = 3\,\text{mW}$ at $\lambda = 633\,\text{nm}$ $\rightarrow$ photon rate" (P_{L} equals 3 milliwatts, at λ equals 633 nanometers). This is the desired useful laser output. We need to convert this power into a rate of photons per second.

The energy of a single photon is $E_{\text{photon}} = \frac{hc}{\lambda}$.

$h \approx 6.626 \times 10^{-34}\,\text{J} \cdot \text{s}$ (Planck's constant)

$c \approx 3.00 \times 10^8\,\text{m/s}$ (speed of light)

$$\lambda = 633\,\text{nm} = 633 \times 10^{-9}\,\text{m}$$

$$E_{\text{photon}} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s} \times 3.00 \times 10^{8}\ \text{m/s}}{633 \times 10^{-9}\ \text{m}}$$

$E_{\text{photon}} \approx \frac{19.878 \times 10^{-26}\ \text{J·m}}{633 \times 10^{-9}\ \text{m}} \approx 0.0314 \times 10^{-17}\ \text{J} \approx 3.14 \times 10^{-19}\ \text{J}$.

The output power $P_{\text{L}} = 3\ \text{mW} = 3 \times 10^{-3}\ \text{J/s}$.

The photon rate is $P_{\text{L}}/E_{\text{photon}}$.

Photon rate = $\frac{3 \times 10^{-3}\ \text{J/s}}{3.14 \times 10^{-19}\ \text{J/photon}} \approx 0.955 \times 10^{16}\ \text{s}^{-1} \approx 0.955 \times 10^{16}\ \text{s}^{-1}$.

The slide shows the calculation for "βn" (beta n) which represents the total rate of photons being produced in the lasing mode (per unit volume, and then implicitly multiplied by volume V for the total rate if we match units). βn is the rate of photons contributing to the output and other cavity losses. If P_{L} is the output power, then the rate of photons corresponding to P_{L} is $\frac{P_{\text{L}}}{hc/\lambda}$. This rate is what emerges *from the output coupler*.

The βn term from the rate equations often represents the total stimulated emission rate into the lasing mode that gets lost from the cavity (including output coupling, mirror absorption, etc.). If P_{L} is the useful output power, it's a fraction of $\beta n\, V\, h\nu$.

The slide states: "βn = $P_{\text{L}}/(hc/\lambda) = 1.0 \times 10^{16}\ \text{s}^{-1}$"

Here, it seems βn is being defined as the *total number of output photons per second from the entire laser volume*.

So, $\frac{3 \times 10^{-3}\ \text{J/s}}{3.14 \times 10^{-19}\ \text{J/photon}} \approx 0.955 \times 10^{16}\ \text{s}^{-1}$. The slide rounds this to $1.0 \times 10^{16}\ \text{s}^{-1}$. Let's use the slide's value.

This value represents the rate of photons exiting the laser as useful output.

Next, the "Required pump rate P".

From page 41, one of our steady-state balance equations was: $P_{\text{density}} = \beta n_{\text{density}} + N_2(A_{21} + R_2)$.

This equation is for rates per unit volume. To get the total pump rate for the whole volume V, we multiply by V:

$P_{\text{total}} = (\beta n_{\text{density}})V + N_2(A_{21} + R_2)V$.

The term $(\beta n_{\text{density}})V$ is the total rate of photons being generated and contributing to the cavity mode losses (including output). The slide has calculated $P_{\text{L}}/(hc/\lambda)$ as the useful output photon rate, and labeled this "βn" (implying it's the total rate, not density). Let's assume this "βn" represents the component of the pumped atoms that successfully become output laser photons.

The previous calculation $N_2(A_{21} + R_2)V = 1.5 \times 10^{16}\,\text{s}^{-1}$ was the rate of atoms lost to incoherent processes.

So, the total pump rate P (atoms/s needed for the whole volume) must supply both the useful output photons and the incoherent losses:

$P_{\text{total}} =$ (rate of useful output photons) + (rate of incoherently lost atoms from N_2).

$$P_{\text{total}} = (1.0 \times 10^{16}\,\text{s}^{-1}) + (1.5 \times 10^{16}\,\text{s}^{-1})$$

$$P_{\text{total}} = 2.5 \times 10^{16}\,\text{s}^{-1}$$

The slide shows: "P = (1.5 + 1.0) × 10^{16} = 2.5 × 10^{16} s^{-1}"

This means we need to pump 2.5×10^{16} atoms per second into the upper laser level system to achieve $3\,$mW of output power, given the incoherent losses.

Finally, an important observation:

"Fluorescence (isotropic spontaneous emission) is larger loss channel than mirror transmission in this design." * The incoherent loss rate

$N_2(A_{21} + R_2)V$ (which includes spontaneous emission A_{21}) is 1.5×10^{16} atoms/s. * The useful output photon rate (related to mirror transmission part of βn) is 1.0×10^{16} photons/s.

Since 1.5×10^{16} is greater than 1.0×10^{16}, the power lost to fluorescence and other non-radiative decays from the upper level is indeed larger than the useful power extracted from the laser. This is common in many laser systems. Improving efficiency often involves minimizing these incoherent loss channels relative to the useful output coupling.

The "---" indicates the end of the slide. This detailed power budget gives a practical feel for where the pump energy goes in a real laser.

Page 49:

Slide 20: Key Takeaways Before Moving On.

We now arrive at "Slide 20: Key Takeaways Before Moving On." This is an important slide that summarizes the core concepts we've covered regarding the fundamentals of lasers. It's a good point to pause and consolidate our understanding.

The first key takeaway:

"Population inversion is quantitative: ΔN_{thr} links gain cross-section, medium length, and cavity loss." (Capital Delta N sub t h r).

This refers to the crucial formula

$$\Delta N_{\text{thr}} = \frac{\gamma}{2\,L\sigma}$$

* "Population inversion is quantitative": It's not just a qualitative concept ("more atoms in the upper state"). There's a specific, calculable threshold value of inversion density ΔN_{thr} that must be achieved for lasing.

* "links gain cross-section (σ)": The effectiveness of each inverted atom in contributing to gain.

* "medium length (L)": The length over which amplification can occur.

* "and cavity loss (γ)": The total losses that the gain must overcome.

This equation is a cornerstone of laser design, telling us what we need to achieve in the active medium within a given resonator.

The second key takeaway:

"Rate equations provide intuitive bookkeeping of photons and populations, predicting threshold and output power." The set of coupled differential equations for $\frac{dN_1}{dt}$, $\frac{dN_2}{dt}$, and $\frac{dn}{dt}$ allows us to model:

* "Intuitive bookkeeping": How atoms move between energy levels due to pumping, stimulated emission/absorption, spontaneous emission, and non-radiative decay, and how photons are generated and lost in the cavity.

* "Predicting threshold": By setting the derivatives to zero and solving, we can re-derive the threshold conditions (as seen on page 46).

* "and output power": By solving the steady-state equations above threshold, we can relate the output power to the pump rate and other laser parameters (as hinted at in the power budget example). Rate equations can also be solved numerically to study dynamic behavior like pulsing or laser turn-on transients.

The third key takeaway:

"Lower-level depopulation speed is critical — designs favour four-level systems over three-level." This emphasizes the importance of the decay rate R_1 of the lower laser level $|1\rangle$.

"Critical": If the lower laser level does not empty quickly (R_1 is small), its population N_1 will build up, reducing or quenching the inversion ($N_2 - N_1$). This is particularly problematic in three-level systems where the lower laser level is* the ground state, making it very hard to depopulate and requiring strong pumping to invert more than half the total atoms.

* "designs favour four-level systems": In a four-level system, the lower laser level $|1\rangle$ is distinct from the ground state $|0\rangle$, and is designed to have a rapid decay R_1 to $|0\rangle$. This keeps N_1 low, making it much easier to achieve and maintain population inversion $N_2 > N_1$ with less pump power. This is why many practical CW lasers (like Nd:YAG) are four-level systems.

Page 50:

Continuing with "Slide 20: Key Takeaways Before Moving On," this page presents two more crucial summary points.

The fourth key takeaway: "Resonator characteristics (losses, mode spacing) intertwine with gain profile to determine spectral output." This brings together the properties of the active medium (gain profile $\alpha(\nu)$ or $g(\nu - \nu_0)$) and the optical resonator.

* "Resonator characteristics": * "Losses ($\gamma(\nu)$)": The cavity losses are frequency-dependent, with minima at the resonant longitudinal mode frequencies. * "Mode spacing (FSR $= \frac{c}{2d}$)": The separation between these longitudinal modes. * "intertwine with gain profile": The broad gain profile of the active medium acts as an envelope. * "to determine spectral output": Lasing only occurs at those cavity mode frequencies where the gain exceeds the losses. This interplay determines which modes lase, how many modes lase (single-mode vs. multi-mode operation), and their relative intensities, thus defining the laser's output spectrum. We saw this visualized on page 33.

The fifth and final key takeaway is a forward-looking statement: "Mastery of these fundamentals is prerequisite for understanding single-mode operation, frequency stabilization, and tunable laser architectures in subsequent sections." This underscores the importance of everything we've covered in this "Fundamentals of Lasers" chapter.

* "Mastery of these fundamentals": Concepts like population inversion, gain, loss, threshold, resonators, modes, rate equations, and lineshapes.

* "Prerequisite for understanding": * "Single-mode operation": How to design lasers to operate on a single frequency, which often involves manipulating the gain and loss profiles. * "Frequency stabilization": Techniques to actively control and stabilize the precise output frequency of a laser, crucial for high-resolution spectroscopy and metrology. This builds directly on understanding modes and cavity properties. * "Tunable laser architectures": How different types of lasers (dye, Ti:sapphire, OPOs, etc.) are designed to allow their output wavelength to be varied, and the principles behind their tuning mechanisms.

These advanced topics, which are central to laser spectroscopy, all rely on a solid grasp of the basic principles discussed so far.

This concludes our overview of the fundamentals of lasers. These concepts will serve as the bedrock for the more advanced topics to come in laser spectroscopy. I hope this detailed walkthrough has been illuminating.