Chapter

2.9

Page 1:

Alright everyone, welcome. Today we embark on a fascinating and absolutely crucial topic in laser spectroscopy, which is Chapter 2, Section 9, focusing on the **Coherence Properties of Radiation Fields**. Understanding coherence is fundamental, not just for appreciating how lasers work, but for comprehending a vast array of spectroscopic techniques that rely on the wave nature of light and its ability to interfere.

These lecture slides were prepared by Distinguished Professor Doctor M. A. Gondal for the Physics 608 Laser Spectroscopy course at King Fahd University of Petroleum and Minerals. We'll be diving deep into what makes a light source coherent or incoherent, and what the quantitative measures of these properties are. This understanding will underpin much of what we discuss later in the course when we look at specific laser systems and their applications.

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So, let's start with a **Big Picture Overview of Coherence Properties of Radiation Fields**. The first bullet point gives us a very good working definition: **Coherence is the ability of different portions of an electromagnetic, or E M, field to maintain a well-defined phase relationship.**

Now, what do we mean by "different portions" of an E M field? This is key. It can refer to the field at the *same point in space* but at *different times*, or it can refer to the field at *different points in space* at the *same time*. This distinction naturally leads us to two complementary aspects of coherence.

These **two complementary aspects** are:

First, **Temporal coherence**, which is also sometimes called **longitudinal coherence**. This describes the **correlation in time at a fixed point.** Imagine you're sitting at one specific location, and you're observing an electromagnetic wave passing by. Temporal coherence tells you how well you can predict the phase of the wave arriving now, based on the phase of the wave that arrived a short time ago. If the phase relationship is maintained over long durations, the source has high temporal coherence. If the phase jumps around randomly and unpredictably very quickly, it has low temporal coherence. Think of it as the wave's "memory" of its own phase at a given point.

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Continuing our overview, the second complementary aspect is: 2. **Spatial coherence**, which is also referred to as **transverse coherence**. This describes the **correlation in space at a fixed instant.** So, now, instead of looking at one point over time, we freeze time – take a snapshot – and look at the phase of the electromagnetic field at two different points in space, typically transverse to the direction of propagation. If the phase difference between these two points remains constant or varies in a predictable way across a wavefront, then the field has high spatial coherence over that region. If the phases at nearby points are completely random with respect to each other, it has low spatial coherence.

The next bullet point highlights the immense practical importance of these concepts:

* **Both aspects govern interference and diffraction phenomena, which are absolutely central to laser spectroscopy.** Why? Because many, if not most, spectroscopic techniques rely on making light interfere – whether it's in an interferometer to measure wavelength precisely, or in a diffraction grating to disperse light, or even in the interaction of multiple laser beams with a sample. Interference can only produce stable, observable patterns if the interfering waves possess some degree of coherence. Without it, the interference patterns would fluctuate wildly and average out to nothing.

To quantify these ideas, we will need to develop some **Key quantities**:

1. **Coherence time**, denoted as Δt_{c} (that's capital Delta, t, subscript c), measured in seconds [s]. This will give us a timescale over which the phase of the wave remains predictable.

2. **Coherence length**, denoted as Δs_{c} (capital Delta, s, subscript c). This is related to the coherence time by the simple equation

$$\Delta s_{\mathrm{c}} = c \times \Delta t_{\mathrm{c}}$$

where c is the speed of light. It's measured in meters [m]. Physically, this is the spatial extent, or the length of the "wave train," over which the wave maintains its phase predictability. If you try to make a wave interfere with a copy of itself that's been delayed by a path longer than the coherence length, you won't see fringes.

3. **Coherence surface**, denoted as S_{c} (capital S, subscript c), measured in square meters [m^2]. This quantity will relate to spatial coherence, defining an area over which the wavefront has a well-defined phase.

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And the fourth key quantity we will develop is:

4. **Coherence volume**, denoted as V_{c} (capital V, subscript c), measured in cubic meters [$m^{3]}$. As the name suggests, this will combine the concepts of coherence length (longitudinal) and coherence surface (transverse) to define a three-dimensional volume within which the electromagnetic field can be considered coherent.

Now, for our **Road-map**: Our approach will be to **derive each of these quantities from the fundamental superposition of partial waves emitted by an extended source.** This is a very physical and intuitive way to build up the concepts. Real light sources, unlike idealized point sources or perfect plane waves, always have some finite physical extent. Different points on an extended source, especially a thermal source like an incandescent bulb, can emit light independently. It's the summation of these many individual wavelets, each with potentially different initial phases, that determines the overall coherence properties of the light field observed at some distant point. So, we'll start by considering how these partial waves add up.

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This slide provides a wonderful visual illustration of the **Coherence Properties of Radiation Fields**, contrasting an incoherent source with a coherent one.

Let's first look at the top part, labeled **"Incoherent Source (e.g., Lamp)"**. We see a depiction of a **"Large Source"** on the left, visualized as a yellowish circular area. From this source, numerous wavy orange lines emanate, representing light waves. Notice how these wave trains look rather short and irregular.

There are two key annotations here:

First, **"Poor Temporal Coherence (Short phase-locked trains** Δt)". This is indicated by a double-headed blue arrow along one of the wiggly wave trains, implying that the length Δt (or rather, $c\Delta t$) over which the phase is predictable is short. For a typical lamp, light emission comes from many individual atoms undergoing spontaneous emission. Each emission event is independent and produces a short burst of light, a wave packet, with a random initial phase. So, the resulting field is a jumble of these short, phase-uncorrelated wave trains.

Second, **"Poor Spatial Coherence (Random phases across area)"**. This is indicated by a vertical double-headed blue arrow suggesting a comparison of phases across different wave trains emanating from different parts of the large source. Because the emission from different parts of an extended thermal source is uncorrelated, the phases at different points on an emergent wavefront will be random. You can't predict the phase at one point by knowing it at another nearby point.

Now, let's contrast this with the bottom part, labeled **"Coherent Source (e.g., Laser)"**. Here, we see a **"Narrow Beam"** originating from what looks like a laser aperture on the left. The emitted waves are depicted as very regular, long, sinusoidal red lines.

The annotations are:

First, **"Good Temporal Coherence (Long phase-locked trains** Δt)". The horizontal double-headed blue arrow spans a much longer distance along these wave trains, indicating that the phase remains predictable over a much longer duration or length. Lasers achieve this through stimulated emission, where emitted photons are in phase with the stimulating photons, leading to a continuous, long wave train with a well-defined phase.

Second, **"Good Spatial Coherence (Phases locked across beam** S)". The vertical double-headed blue arrow now indicates that across the entire transverse profile of the beam (denoted S), the phases are locked together. This results in a smooth, uniform wavefront. Lasers achieve this due to the resonant cavity and mode selection mechanisms, which ensure that only certain spatial field distributions can oscillate and be amplified.

So, this diagram gives us a very intuitive feel for the difference. Incoherent light is like a chaotic jumble of short, independent waves. Coherent light, particularly from a laser, is like a highly disciplined army of waves, all marching in step over long distances and wide fronts. And as we'll see, these properties are what make lasers such powerful tools for spectroscopy.

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Now, let's begin to formalize these ideas. We'll start with the **Superposition of Partial Waves – Setting Up the Problem**, which is the foundation of our road-map.

* **Consider an extended optical source** S with many infinitesimal surface elements dS. Imagine our light source, whatever it may be – a hot filament, a gas discharge, the surface of a star. We can think of this source as being made up of a vast number of tiny, independent emitting regions, each of which we'll call dS. This is very much in the spirit of Huygens' principle, where each point on a wavefront can be considered as a source of secondary spherical wavelets.

* **Each element emits a spherical elementary (partial) wave.** This is a key simplifying assumption. We're saying that each tiny bit dS of our source acts like a point source, sending out light in spherical waves.

* **The complex field amplitude at element** n, let's say the n-th infinitesimal element, can be written as: $A_{n0}\, e^{i\phi_{n0}(t)}$. That is, $A_{n0}\, e^{i\phi_{n0}(t)}$. Here, A_{n0} represents the amplitude of the wave emitted by the n-th element, and $\phi_{n0}(t)$ represents its phase at the source element itself, at time t. Using complex notation is incredibly convenient because it allows us to handle both amplitude and phase in a single quantity. The actual electric field would be the real part of this complex amplitude.

* A_{n0} is the real amplitude at the source surface, and it has units of Volts per meter $\mathrm{V\,m^{-1}}$. This is the strength of the electric field component of the light wave right at the surface of that little n-th element.

So, we're building a model where our extended source is a collection of tiny emitters, each launching its own spherical wave. The next step will be to see how these waves add up at some distant observation point.

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Continuing with our setup for the superposition of partial waves:

* The phase term from the previous page, **phi sub n zero of t ($\Phi_{n0}(t)$)**, is given by $\omega t + \Phi_{\mathrm{n}}(0)$. This represents the initial phase. Let's break this down: * 'omega t' (ωt) is the standard time-varying part of the phase for a wave of angular frequency ω. This describes the rapid oscillations of the field. * 'phi sub n of zero' ($\Phi_{\mathrm{n}}(0)$) is the initial phase of the n-th element at time $t = 0$. This term is absolutely critical. If these initial phases, $\phi_{\mathrm{n}}(0)$, are random and uncorrelated for different elements 'n' (as in a thermal source), the overall field will tend to be incoherent. If they are all the same, or have a fixed, well-defined relationship (as in a laser), then the field can be coherent.

* Next, we define an **Observation point P, which is at a distance** r_{n} from element n. So, each little wave from element 'n' has to travel a distance r_{n} to reach our detector or observation point P.

* Now, we can write the **Total complex field amplitude at point P.** This is where the superposition principle comes in. The total field $A(P)$ is the sum of the contributions from all the N elements (or, in the limit, an integral over the source surface). The equation is: $A(P)$ equals the sum, from $n = 1$ to infinity (or N for N elements), of: A_{n0}, times $\frac{1}{r_{\mathrm{n}}}$, times $e^{i\left[\Phi_{n0}(t)+\frac{2\pi r_{\mathrm{n}}}{\lambda}\right]}$.

$$A(P) = \sum_{n=1}^{\infty} A_{n0} \, \frac{1}{r_{\mathrm{n}}} \, e^{i\left[\Phi_{n0}(t)+\frac{2\pi r_{\mathrm{n}}}{\lambda}\right]}$$

Let's deconstruct this sum term by term: * **A sub n zero (A_{n0})**: This is the amplitude of the wave as it leaves the n-th source element. * **1 over r sub n ($\frac{1}{r_{\mathrm{n}}}$)**: This factor accounts for the decrease in amplitude of a spherical wave as it propagates. The amplitude falls off as one over the distance. * **e to the power of (i times [phi sub n zero of t + 2 pi r sub n / lambda])**: This is the complex phase factor. ***phi sub n zero of t ($\Phi_{n0}(t)$)***: This is the phase of the wave at the source element n *at time* t*. * **2 pi r sub n over lambda ($\frac{2\pi r_n}{\lambda}$)**: This is the additional phase accumulated by the wave as it travels the distance r_n from the source element to the observation point P. You'll recognize $\frac{2\pi}{\lambda}$ as the wave number k. So this term is just $k\, r_n$. It tells us how many wavelengths fit into the path r_n, and thus what the phase shift is.

Finally, some **Notation & units** to be clear:

* **omega (**ω) equals 2 pi nu ($2\pi\nu$), which is the angular frequency, and its units are radians per second (rad s^{-1}). 'nu' (ν) here is the linear frequency in Hertz.

This summation is the heart of understanding how extended sources produce fields with varying degrees of coherence. The properties of $\Phi_{n0}(t)$ and the variations in r_n will determine everything.

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Continuing with our notation and the setup:

* $\lambda = \frac{2\pi c}{\omega}$. This is the vacuum wavelength of the light, measured in meters [m]. Here, c is the speed of light in vacuum, and ω is the angular frequency we just defined. This is a standard relationship.

* The next point is important for transitioning from a conceptual sum over discrete elements to a more realistic continuous source: **The summation approximates an integral over the entire radiating surface.** If our source S is truly continuous, and the infinitesimal elements dS are indeed infinitesimal, then this sum becomes an integral. This is the basis of the Huygens-Fresnel principle, which is a very powerful tool for calculating wave propagation and diffraction. We would integrate the contributions from all dS elements over the entire surface of the source S.

Now, the slide indicates **"[IMAGE REQUIRED: Geometry diagram similar to Fig. 2.29 – source surface elements, distances** r_n, and phase increments illustrated.]" Since we don't have the image directly, let me describe what it would typically show. Imagine an extended light source, let's call it S, which could be a flat surface or a curved one. We would then pick an arbitrary infinitesimal element on this source, perhaps labeled dS or identified as originating from a point Q on the source. Then, we'd have our observation point P located some distance away. The crucial distance r_{n} (or r_{Q} if we use Q) would be a line segment drawn from this element dS (or point Q) on the source to the observation point P.

The diagram would likely show several such elements on the source, each with its own path r_{n} to the same observation point P. This immediately highlights that for an extended source, the distance r_{n} will generally be different for different parts of the source. This variation in r_{n} leads to different propagation phase shifts $\frac{2\pi r_{\mathrm{n}}}{\lambda}$, which is a key factor in determining spatial coherence. The "phase increments illustrated" would refer to how this $\frac{2\pi r_{\mathrm{n}}}{\lambda}$ term changes as the wave propagates. This kind of diagram is essential for visualizing the geometry that underlies our calculations of coherence.

The three hyphens below just indicate the end of this section of text on the slide.

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Alright, let's move on to **Phase Accumulation Along Propagation**. This concept is critical for understanding how interference patterns are formed and how coherence plays a role.

The slide states: For each element n, the full phase at the observation point P is given by

$$\Phi_{\mathrm{n}}(r_{\mathrm{n}}, t) = \Phi_{n0}(t) + \frac{2\pi r_{\mathrm{n}}}{\lambda}.$$

That is, $\Phi_{\mathrm{n}}(r_{\mathrm{n}}, t) = \Phi_{n0}(t) + \frac{2\pi r_{\mathrm{n}}}{\lambda}$.

Let's break this down carefully:

$\Phi_{\mathrm{n}}(r_{\mathrm{n}}, t)$ is the total phase of the wavelet originating from the n-th source element, measured at the observation point P (which is at distance r_{n}) at time t.

$\Phi_{n0}(t)$, which is labeled **"at source"**, is the phase of that wavelet *as it leaves the* n-th source element at time t. Remember, this $\Phi_{n0}(t)$ itself contains the initial phase $\phi_{\mathrm{n}}(0)$ plus ωt.

$\frac{2\pi r_{\mathrm{n}}}{\lambda}$, labeled the **"propagation term"**, is the phase acquired by the wavelet simply due to traveling the distance r_{n} from the source element to the observation point P. As we noted, this is essentially k times r_{n}, where k is the wave number.

Now for some **Important observations** stemming from this:

1. **Propagation adds a deterministic phase proportional to distance.**

The term $\frac{2\pi r_{\mathrm{n}}}{\lambda}$ is "deterministic" in the sense that if we know the wavelength λ and the path length r_{n}, we can calculate this phase shift precisely. It's purely a consequence of geometry and the wave nature of light. There's nothing random about this part of the phase accumulation, *given* r_{n} and λ. The randomness, if any, comes from the $\Phi_{n0}(t)$ term, specifically the initial phases $\phi_{\mathrm{n}}(0)$ contained within it.

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Continuing with our important observations about phase accumulation:

2. **Initial phases,** $\phi_{\mathrm{n}}(0)$, may be random (as in a thermal source) or constant (or highly correlated, as in a laser).

This is a point we've touched on before, but it's so fundamental it bears repeating.

* For a **thermal source**, like an incandescent bulb or a flame, the individual atoms or molecules emit light through spontaneous emission. These emission events are independent and uncoordinated. Thus, the initial phase $\phi_n(0)$ associated with each conceptual source element 'n' will be random and uncorrelated with the phases of other elements. This randomness is the root cause of the incoherence of thermal light.

* For a **laser**, on the other hand, the emission process is predominantly stimulated emission. Stimulated photons are in phase with the stimulating photons. This process, occurring within a resonant cavity, leads to a highly ordered state where the initial phases $\phi_n(0)$ from different parts of the emitting medium (or different effective source elements) are locked together, or at least have a very well-defined and stable relationship. This is the origin of the high coherence of laser light.

3. **Interference at point** P follows from the coherent sum of all such contributions.

"Coherent sum" means we add the complex amplitudes of all the wavelets arriving at P, taking their phases into account. The resultant intensity at P is then the squared magnitude of this total complex amplitude. If the phases add up constructively, we get high intensity. If they add up destructively, we get low intensity. This is interference.

* This leads to a crucial insight: **If phase relationships are stable across 'n' (meaning, across different source elements) AND stable over time, then interference fringes will persist.**

* "Stable across n": This refers to **spatial coherence**. If the relative phases of contributions from different parts of the source are well-defined and

unchanging as they arrive at the observation region, then stable spatial interference patterns (like Young's fringes) can form.

* "Stable over time": This refers to **temporal coherence**. If the phase of the wave at a given point is predictable from one moment to the next, then effects that depend on comparing the wave with a time-delayed version of itself (like in a Michelson interferometer) will produce stable fringes.

* And as a preview: **The next slides will focus on quantifying what we mean by "stable" through the development of formal criteria for temporal and spatial coherence.** We need mathematical conditions to define these properties.

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Now we begin to formalize these ideas, starting with **Defining Temporal Coherence – Phase Stability in Time**.

The **Key Criterion** for temporal coherence involves looking at how the phase of a wave, from a single source element, changes over time at a fixed observation point.

* So, we first state: **At a fixed position** P, consider two observation instants, t_1 and t_2. We are camping at one spot and watching the wave go by at two different moments.

* Next, we look at the **Phase change for element n** during this time interval: **Delta phi sub n equals capital Phi sub n of (P, comma t sub 1) minus capital Phi sub n of (P, comma t sub 2).**

$$\Delta\phi_{\mathrm{n}} = \Phi_{\mathrm{n}}(P, t_1) - \Phi_{\mathrm{n}}(P, t_2)$$

Here, $\Phi_{\mathrm{n}}(P, t)$ is the full phase of the wavelet from the n-th source element arriving at point P at time t, which we defined on page 9 as $\Phi_{n0}(t_{\text{arrival}}) + \frac{2\pi r_{\mathrm{n}}}{\lambda}$. Since P and r_{n} are fixed, the $\frac{2\pi r_{\mathrm{n}}}{\lambda}$ term is constant. So, $\Delta\phi_{\mathrm{n}}$ is really about the change in $\Phi_{n0}(t_{\text{arrival}})$ between the two arrival times corresponding to t_1 and t_2 at the observation point. If the light left the source at t'_1 and t'_2 to arrive at P at t_1 and t_2 respectively, then this $\Delta\phi_{\text{n}}$ is $\Phi_{n0}(t'_1) - \Phi_{n0}(t'_2)$.

* With this definition of phase change, we can now state the condition: **Temporal coherence exists when...** and this condition will be specified on the next page. The idea is that for the wave to be temporally coherent, this phase change $\Delta\phi_{\text{n}}$ should not be too large or too random over a certain time window.

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Continuing with our definition of temporal coherence, the condition is:

* **For all** n ($\forall n$): the absolute value of $\Delta\phi_{\text{n}}$ is less than π ($|\Delta\phi_{\text{n}}| < \pi$).

This condition must hold for all the partial waves contributing to the field at P, assuming we are considering the coherence of the individual wave trains. What does this mean? π radians is 180 degrees. So, we are saying that for the wave to be considered temporally coherent over the time interval $\Delta t = t_2 - t_1$, the phase of any contributing wavelet should not drift by more than 180 degrees. Why this specific value of π? If the phase drifts by, say, exactly π, then a part of the wave that would have interfered constructively now interferes destructively. If phase drifts are kept significantly less than π, the wave maintains a more predictable character, allowing for stable interference when combined with a time-shifted version of itself (as in a Michelson interferometer). A drift of much more than π would mean the phase has become essentially random relative to its earlier value.

* So, the **Meaning** is: **all partial waves drift in phase by less than 180 degrees (**π radians) over the time window $\Delta t = t_2 - t_1$. (Delta t equals t two minus t one).

From this criterion, we derive some important **Derived Quantities**:

* **The maximum** Δt that satisfies the above condition ($|\Delta\phi_n| < \pi$) defines the coherence time, denoted Δt_c (Δt_c), in seconds [s].

So, Δt_c is the longest duration over which the wave's phase, on average, remains predictable enough (doesn't change by more than π) to allow for interference.

* **The corresponding distance that light travels in this coherence time is the coherence length,** Δs_c (Δs_c).

The formula is $\Delta s_c = c\Delta t_c$ ($\Delta s_c = c\Delta t_c$), where 'c' is the speed of light. This Δs_c is often visualized as the average length of the "wave trains" that constitute the light. Within one such wave train of length Δs_c, the phase is well-behaved. If you try to interfere parts of the wave separated by a distance greater than Δs_c (by introducing a path difference larger than Δs_c in an interferometer), the phase relationship will be lost, and interference fringes will have poor visibility or disappear altogether.

* And a crucial relationship: **A smaller spectral bandwidth leads to a slower phase drift, which in turn means a larger coherence length** Δs_c.

This is a manifestation of the time-frequency uncertainty principle (or more accurately, the relationship between the duration of a signal and its bandwidth via the Fourier transform). A perfectly monochromatic wave (zero bandwidth) would have an infinitely slow phase drift (its phase evolves perfectly predictably as ωt) and thus an infinite coherence time and length. Conversely, a wave with a broad range of frequencies (large bandwidth) will have its different frequency components drifting out of phase with each other very quickly, leading to a short coherence time and length. We will see this quantified soon.

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Now let's turn our attention to the other aspect of coherence, with **Defining Spatial Coherence – Phase Stability in Space**.

The approach is analogous to temporal coherence, but now we look at phase relationships between different points in space at the same instant.

* First, we **Fix the time** t; and then inspect two different spatial points, P_1 and P_2. Imagine taking a snapshot of the wave field and looking at the phase at these two locations.

* Next, we consider the **Phase difference of the total field** between these two points: **Delta phi equals phi of (**P_1, t) minus phi of (P_2, t).

$$\Delta\phi = \phi(P_1, t) - \phi(P_2, t)$$

Here, $\phi(P, t)$ refers to the phase of the *total* electromagnetic field at point P and time t, which is the result of the superposition of all partial waves from the source. We're asking: at a given moment, how does the phase at P_1 relate to the phase at P_2?

* With this, we state the condition for spatial coherence: **Spatial coherence exists when the absolute value of this phase difference, Delta phi, is less than pi (** $|\Delta\phi| < \pi$) for all times t. The logic is similar to the temporal case: if the phase difference between these two points is less than $180°$, then light from these two regions can interfere effectively (e.g., if P_1 and P_2 were two slits in a Young's experiment). The condition "for all t" is important. It means this phase relationship must be stable over time. If $\Delta\phi$ randomly fluctuated between 0 and 2π over time, even if at some instants it was less than π, we wouldn't observe stable interference fringes. So, spatial coherence implies a persistent, well-defined phase relationship across a region of space.

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Continuing our discussion on spatial coherence:

A set of points satisfying the above inequality ($|\Delta\phi| < \pi$) for a given source forms what we call the "coherence volume".

Actually, if we fix the distance from the source and look at a transverse plane, the set of points (P_2, P_3, etc.) that are spatially coherent with a reference point P_1 would define a **coherence area** or **coherence surface** S_c on that plane. If we then consider the extent along the propagation direction over which temporal coherence is maintained (the coherence length Δs_c), the combination of this coherence area and coherence length defines the **coherence volume** V_c. Within this volume, the field's phase is, in a sense, well-behaved and predictable both spatially and temporally.

Now, let's consider the practical implications:

Practically, spatial coherence dictates whether Young-type interference can be observed.

This is the quintessential experiment for demonstrating spatial coherence. In Young's double-slit experiment, light passes through two closely spaced pinholes or slits. If the light illuminating these two slits is spatially coherent (meaning the phase difference between the light at slit 1 and slit 2 is stable and not too large), then clear interference fringes will be observed on a screen placed beyond the slits. If the illumination is spatially incoherent over the distance separating the slits, no fringes will be seen.

And critically: **Spatial coherence is strongly linked to the angular size of the source and the propagation distance.**

Angular size of the source: Generally, for a source of a given physical size, the smaller its angular size as seen from the observation plane (i.e., the farther away it is, or the smaller its physical extent), the higher the spatial coherence of its radiation will be in that plane. This is described by the Van

Cittert-Zernike theorem, which we'll touch upon. Think of a very distant star: even though it's physically enormous, its angular size is tiny, and its light is spatially coherent over considerable distances on Earth. Conversely, a large, nearby source (like a frosted light bulb) will produce light with very limited spatial coherence.

Propagation distance: As light propagates away from an extended incoherent source, its spatial coherence tends to increase. The wavefronts become smoother and more correlated over larger transverse distances. This is why, even for a source that is not perfectly point-like, at a sufficient distance, a significant degree of spatial coherence can develop.

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Let's do a **Quick Contrast between Temporal and Spatial Coherence** to solidify our understanding of these two distinct yet related concepts.

First, let's focus on **Temporal Coherence**:

* One key aspect is that it **Relates to spectral purity: a narrow angular frequency spread,** $\Delta\omega$, implies a long coherence time, Δt_{c}. This is perhaps the most important takeaway for temporal coherence. A light source that is highly monochromatic (very pure color, very narrow range of frequencies) will have a very long coherence time and coherence length. Think of it this way: if all the frequency components are almost identical, they will march in step for a very long time before their tiny frequency differences cause them to drift out of phase. This is intrinsically linked to the Fourier transform relationship between the time domain representation of the wave (its duration or coherence time) and its frequency domain representation (its spectral bandwidth). A perfectly sinusoidal wave of a single frequency (zero bandwidth) lasts forever and has infinite coherence time.

* Temporal coherence is typically **Measured with path-difference devices, for example, a Michelson interferometer.** In a Michelson interferometer, a beam of light is split into two paths, and then these two paths are recombined. By changing the length of one path relative to the other, a time delay (equal to $\frac{\text{path difference}}{c}$) is introduced between the two recombined beams. The visibility of the interference fringes observed as this path difference is varied gives a direct measure of the temporal coherence of the source for that corresponding time delay.

Now, for **Spatial Coherence**:

* It **Relates to the geometric size and the distance of the source.** As we discussed, a smaller (or more distant) source generally leads to higher spatial coherence at an observation plane. It's about how "point-like" the source appears from the region where coherence is being assessed.

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Continuing our contrast, specifically for Spatial Coherence:

* Spatial coherence is **Measured with double-slit or pinhole experiments, such as Young's experiment.** In these experiments, we take two samples of the light field at two spatially separated points (the slits or pinholes) and observe whether they can produce interference fringes. The visibility of these fringes directly indicates the degree of spatial coherence between those two points for the incident light. If you can see clear fringes, the light at the two slits is spatially coherent.

And a very important unifying point:

* **Both temporal and spatial coherence can be treated within a unified correlation framework using what's called the mutual coherence function. We'll introduce this in later slides.** While we've defined them somewhat separately for clarity, they are not entirely independent concepts.

The mutual coherence function, often denoted $\Gamma_{12}(\tau)$ (Gamma sub one two of tau), is a more general quantity that describes the correlation between the field at a point $\mathbf{r}_1$ at time t and the field at another point $\mathbf{r}_2$ at time $t + \tau$.

* If $\mathbf{r}_1 = \mathbf{r}_2$, then $\Gamma_{11}(\tau)$ describes temporal coherence (correlation at the same point but different times).

* If $\tau = 0$, then $\Gamma_{12}(0)$ describes spatial coherence (correlation at different points at the same time).

So, this function provides a comprehensive way to characterize the coherence of a light field.

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Let's delve into one of the classic instruments for studying coherence: the **Michelson Interferometer**, focusing on its **Fundamental Geometry**.

* As noted earlier, this is the **Classic apparatus for probing temporal coherence.** Its invention by Albert A. Michelson in the 1880s was a landmark, enabling precise measurements of wavelengths and, through Fourier transform spectroscopy, the study of spectral line shapes, all of which are deeply connected to temporal coherence.

* We need to define some **Components & notation**, and the slide says to refer to the next figure, which we'll see shortly. The main components are:

* A **Beam splitter (BS)**, which **divides the incoming beam** of light into two separate beams, ideally with equal intensity. This is typically a partially silvered mirror or a dielectric coating designed for, say, 50% transmission and 50% reflection.

* Two **Mirrors, M sub 1 (**M_1) and M sub 2 (M_2), which **retro-reflect the partial beams.** This means they send the beams back along (or parallel to) their

incident paths. One of these mirrors is usually fixed, while the other is mounted on a precision translation stage, allowing its position to be varied.

* An **Observation plane (**B), which **receives the recombined fields.** After the two beams are reflected by M_1 and M_2, they travel back to the beam splitter, where they are again partially transmitted and reflected. A portion of each beam will then overlap and travel towards the observation plane (which could be a screen, a photodetector, or the input of a spectrometer). It is here, at B, that interference between the two beams is observed.

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Continuing with the Michelson interferometer's geometry and defining the path lengths:

We need to consider the **Optical path lengths** for the two arms of the interferometer. Let S be the point on the beam splitter where the beam is divided and also where the returning beams are recombined to go to the detector B.

Arm 1: The path length is $SM_1 + M_1\, B$. This represents the distance from the beam splitter (S) to mirror M_1 and then from M_1 back to the beam splitter (effectively to point B for the recombined beam going to the detector). If L_1 is the distance from BS to M_1, then this round trip path is $2\, L_1$.

Arm 2: Similarly, the path length is $SM_2 + M_2\, B$. If L_2 is the distance from BS to M_2, this round trip path is $2\, L_2$.

The slide notation "$SM_1 + M_1B$" usually simplifies if we consider L_1 as the BS-M_1 distance, then the light travels L_1 to M_1, and L_1 back. So the path length in arm 1 is $2\, L_1$, and in arm 2 is $2\, L_2$.

The crucial quantity is the **Path difference** between these two arms. If one mirror, say M_2, is movable, changing its position will change L_2 and thus the path difference. The path difference, Δs (delta s), is often defined as:

$$\Delta s = 2(SM_1 - SM_2)$$

If SM_1 is L_1 (the distance from beamsplitter to mirror M_1) and SM_2 is L_2 (the distance from beamsplitter to mirror M_2), then the path difference is $\Delta s = 2(L_1 - L_2)$.

The factor of 2 is critical: if you move mirror M_2 by a distance x, the path length of arm 2 changes by $2x$ because the light has to travel to the mirror *and back*. This path difference Δs directly corresponds to a time delay $\tau = \frac{\Delta s}{c}$ between the two wave trains when they recombine. By varying this Δs (and thus τ), we can probe the temporal coherence of the light.

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Here we have the diagram illustrating the **Michelson Interferometer: Fundamental Geometry**.

Let's walk through it:

* On the far left, we have **S (Light Source)**, emitting a beam of light that travels horizontally to the right. * This beam encounters the **BS (Beam Splitter)**, which is shown as a tilted, partially transmissive plate (often a cube in practice, but a plate is fine for illustration). * At the beam splitter, the light is divided: * A portion is **Transmitted (to M_1)**, traveling vertically upwards to M_1 (Fixed Mirror). The distance from the center of BS to M_1 is labeled L_1. The light reflects off M_1 and travels back down to BS. * A portion is **Reflected (to M_2)**, traveling horizontally to the right to M_2 (Movable Mirror). The distance from the center of BS to M_2 is labeled L_2. M_2 is shown with a double-headed arrow beneath it, indicating it can be moved horizontally, thus changing L_2. Light reflects off M_2 and travels back to the left, towards BS. * The two beams, one

returning from M_1 and one from M_2, meet again at the beam splitter. Here, a portion of the beam from M_1 is reflected downwards, and a portion of the beam from M_2 is transmitted downwards. These two portions are now **Recombined Beams** and travel downwards together. * These recombined beams then fall on **B (Screen/Detector)**, where interference can be observed.

At the bottom of the slide, the crucial formula is reiterated:

Optical Path Difference: Δs equals 2 times (L_1 minus L_2). ($\Delta s = 2(L_1 - L_2)$).

$$\Delta s = 2(L_1 - L_2)$$

This Δs is the difference in the total distance traveled by light in arm 1 ($2\,L_1$) versus arm 2 ($2\,L_2$). It is this path difference that determines the relative phase of the two beams when they recombine at the detector B, and thus whether they interfere constructively or destructively. By systematically varying L_2 (by moving M_2), we vary Δs and can map out the fringe visibility, which, as we'll see, relates directly to the temporal coherence of the source S.

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Now that we have the Michelson interferometer setup and the path difference Δs, let's look at the **Interference Condition & Visibility in the Michelson**.

* **For a given wavelength lambda (**λ) of the light:

* **Constructive interference** occurs when the two recombined beams are in phase. This happens when the path difference Δs is an integer multiple of the wavelength: $\Delta s = m\lambda$, where m is an integer ($m \in \mathbb{Z}$, the set of all integers: ..., -2, -1, 0, 1, 2, ...). When this condition is met, crests align with crests, troughs with troughs, and we get maximum intensity.

* **Destructive interference** occurs when the two beams are out of phase by 180 degrees (or π radians). This happens when the path difference Δs is a

half-integer multiple of the wavelength: $\Delta s = \frac{(2m+1)\lambda}{2}$. Here, $2m + 1$ ensures an odd number, so we have $\frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$, etc. Crests align with troughs, leading to minimum intensity (ideally zero if the amplitudes are equal).

* The **Fringe contrast** (how distinct the bright and dark fringes are) is quantified by a very important parameter called **visibility, V (capital Vee).** It's defined as: $V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$. ($V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$). * $I_{\max}$ is the intensity at the peak of a bright fringe (constructive interference). * $I_{\min}$ is the intensity at the minimum of a dark fringe (destructive interference). * If $I_{\min}$ is zero (perfect destructive interference) and $I_{\max}$ is some positive value, then $V = \frac{I_{\max}}{I_{\max}} = 1$. This represents perfect, 100% contrast. * If $I_{\min} = I_{\max}$ (no variation in intensity as Δs changes), then $V = 0$. This means no fringes are visible. The visibility V is not just an arbitrary measure; it is directly related to the degree of temporal coherence of the light source for the specific path difference Δs (which corresponds to a time delay $\tau = \frac{\Delta s}{c}$). If the source is highly coherent over that path difference, V will be close to 1. If it's incoherent, V will be close to 0.

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Continuing with interference and visibility in the Michelson:

* A key **Experimental observation** is that the visibility V decays as the absolute value of the path difference, $|\Delta s|$, exceeds the coherence length, Δs_c. This is the practical manifestation of temporal coherence. When the path difference Δs is small (much less than Δs_c), the two recombined wave trains are still well-correlated in phase, and we get high-contrast fringes ($V \approx 1$). However, as we increase the path difference Δs (by moving one of the mirrors), we are trying to interfere a wave train with a version of itself that originated much earlier in time. If Δs becomes comparable to or greater than the coherence length Δs_c, the phase relationship between these two "older"

and "newer" segments of the wave is lost, and the fringe visibility V drops, eventually to zero. The coherence length Δs_{c} is essentially the maximum path difference over which discernible interference can be observed.

* There's an **Empirical relation connecting the spectral width,** $\Delta\omega$, of the light source to its coherence length, Δs_{c}. This is given in the box: Δs_{c} is approximately equal to $\frac{c}{\Delta\omega}$, which is also equal to $\frac{c}{2\pi\Delta\nu}$.

$$\Delta s_{\text{c}} \approx \frac{c}{\Delta\omega} = \frac{c}{2\pi\Delta\nu}.$$

* c is the speed of light. * $\Delta\omega$ is the bandwidth of the source in terms of angular frequency (radians per second). * $\Delta\nu$ is the bandwidth of the source in terms of linear frequency (Hertz).

This relationship is profoundly important. It tells us that sources with a very narrow spectral width (small $\Delta\omega$ or $\Delta\nu$, i.e., very monochromatic light) will have a very long coherence length Δs_{c}. Conversely, sources with a broad spectral width (large $\Delta\omega$ or $\Delta\nu$, i.e., "whiter" or less monochromatic light) will have a short coherence length. This is a direct consequence of the Fourier relationship between the temporal characteristics of a wave (like its coherence time $\Delta t_{\text{c}} = \frac{\Delta s_{\text{c}}}{c}$) and its spectral characteristics (its bandwidth $\Delta\omega$).

* And just to clarify the notation: $\Delta\nu$ equals $\frac{\Delta\omega}{2\pi}$ – this is the bandwidth in Hertz.

The slight inconsistency in the use of 2π, which I noted in my thoughts when preparing, often arises from different definitions of bandwidth (e.g., Full Width at Half Maximum vs. $1/e$ width) and the precise form of the Fourier uncertainty relation (e.g., $\Delta\omega\Delta t \sim 1$ or $\Delta\omega\Delta t \sim 2\pi$). For our purposes, the inverse relationship between coherence length and bandwidth is the crucial concept.

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Let's explore the **Physical Interpretation of this relationship,** $\Delta s_c = \frac{c}{\Delta\omega}$. Why does this connection between coherence length and spectral bandwidth arise?

* The first point is that a **Finite bandwidth source is actually a superposition of many quasi-monochromatic components.** No real light source is perfectly monochromatic (except an ideal laser, perhaps). A source with a spectral bandwidth $\Delta\omega$ is emitting light not just at a single frequency ω_0, but over a range of frequencies from roughly $\omega_0 - \frac{\Delta\omega}{2}$ to $\omega_0 + \frac{\Delta\omega}{2}$. We can think of the light field as a sum (or integral) of many pure sinusoidal waves, each with a slightly different frequency ω_n within this band $\Delta\omega$.

* Now, **Each individual frequency component,** ω_n, produces a perfect sinusoid. However, because these components have slightly different frequencies, they will drift out of phase with respect to each other over time. Imagine two waves starting in phase, but one has a frequency ω_1 and the other ω_2. The rate at which their phase difference changes is $\omega_1 - \omega_2$. Over a time interval Δt, their relative phase will shift by $(\omega_1 - \omega_2)\Delta t$. The slide states that these **components drift out of phase over a time** Δt which is approximately 1 divided by $\Delta\omega$ ($\Delta t \approx \frac{1}{\Delta\omega}$). This Δt is essentially our coherence time, Δt_c. It's the time it takes for the collection of different frequency components that make up the wave packet to become significantly dephased from each other, typically by about one radian or so, such that their coherent superposition starts to break down.

* The **Result** of this is that the light can be thought of as a **wave train (or wave packet) of finite spatial extent.** This extent is the coherence length: $\Delta s_c = c\Delta t$, which is approximately $\frac{c}{\Delta\omega}$. ($\Delta s_c = c\Delta t \approx \frac{c}{\Delta\omega}$). So, the light from a source with bandwidth $\Delta\omega$ behaves as if it's composed of a series of these wave packets, each of length Δs_c. Within each packet, the phase is reasonably well-defined. But the phase relationship between one packet and the next can

be random, especially for thermal sources. This "packet" picture helps visualize why interference is lost when path differences exceed Δs_c.

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Building on this physical interpretation of wave trains:

* **When the interferometer arm difference,** Δs, is greater than the coherence length, Δs_c ($\Delta s > \Delta s_\mathrm{c}$): This means we are trying to make a wave packet interfere with a version of itself that has been delayed by a distance longer than the length of the packet itself.

* In this scenario, the **Split wave packets do not overlap upon recombination.** Imagine you have two identical short pulses of light (our wave packets). If you send them down two paths in a Michelson interferometer, and the path difference is larger than the length of the pulses, then when they arrive at the detector, one pulse will have already passed through before the other one arrives. They won't be at the detector at the same time to interfere.

* Consequently, the **Interference term averages to zero, and the visibility collapses.** When the wave packets don't overlap, you just get the sum of their individual intensities at the detector, averaged over time. The cross-term in the intensity calculation, which gives rise to interference, will average to zero because there's no consistent phase relationship between the non-overlapping packets arriving from the two arms. Thus, you see no fringes, and the visibility V becomes zero. This is the fundamental reason why the coherence length limits the path differences over which interference can be observed.

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Let's look at some **Numerical Examples to Contrast Different Sources** and get a feel for typical coherence lengths.

Example 1 – A Mercury 546 nanometer (nm) line:

This is a common line from a mercury discharge lamp, often used in labs. The wavelength is 546×10^{-9} meters.

- Such a line from a thermal discharge source is not perfectly monochromatic. It will be broadened, primarily by the Doppler effect due to the thermal motion of the emitting mercury atoms. The slide gives a typical **Doppler Full Width at Half Maximum (FWHM) bandwidth as** $\Delta\nu_D = 4 \times 10^9$ Hz, or 4 Gigahertz. This is a substantial frequency spread.

- Now, let's calculate the **Coherence length,** Δs_c: The formula used here seems to be Δs_c is approximately c divided by $2\pi\Delta\nu_D$ $\left(c/(2\pi\Delta\nu_D)\right)$, and the result is given as approximately 8 centimeters (≈ 8 cm). Let's verify this: c (speed of light) is about 3×10^8 meters per second.

$$\Delta s_c \approx \frac{3 \times 10^8 \text{ m/s}}{2\pi \times 4 \times 10^9 \text{ Hz}}$$

$$\Delta s_c \approx \frac{3 \times 10^8 \text{ m/s}}{8\pi \times 10^9 \text{ s}^{-1}}$$

$$\Delta s_c \approx \left(\frac{3}{8\pi}\right) \times 10^{-1} \text{ meters}$$

Since 8π is roughly $8 \times 3.14159 = 25.13$,

$$\Delta s_c \approx \frac{3}{25.13} \times 0.1 \text{ meters} \approx 0.119 \times 0.1 \text{ meters} \approx 0.0119 \text{ meters},$$

which is about 1.2 centimeters.

The slide says "≈ 8 cm". This value of 8 cm would be obtained if the formula used was $\Delta s_c \approx \frac{c}{\Delta\nu_D}$ (without the 2π factor), as

$$\frac{3 \times 10^8 \text{ m/s}}{4 \times 10^9 \text{ Hz}} = 0.075 \text{ m} = 7.5 \text{ cm}.$$

It's common to see slight variations in the exact formula (presence or absence of 2π) depending on how bandwidth and coherence time are precisely defined (e.g., 1/e point vs. FWHM). The key point is the order of magnitude. For a typical atomic emission line from a lamp, the coherence length is on the order of centimeters. This means if you set up a Michelson interferometer with this mercury lamp, you'd only see clear fringes if the path difference between the arms is kept within a few centimeters.

Example 2 – A Single-mode Helium-Neon (He-Ne) laser:

This is a very common type of laser, often emitting red light at 632.8 nm. "Single-mode" means it's designed to lase on a single longitudinal and transverse mode, which makes its output highly coherent.

- The **Bandwidth,** $\Delta\nu$, for such a laser is vastly smaller, typically around 1×10^6 Hz, or 1 Megahertz. Compare this to the 4 Gigahertz for the mercury lamp – that's a factor of 4000 narrower!

The calculation for the He-Ne laser's coherence length is on the next page.

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Continuing with the numerical examples, for the single-mode Helium-Neon laser with a bandwidth $\Delta\nu = 1 \times 10^6$ Hz:

* The **Coherence length,** Δs_c (Δs_c), is calculated using the formula: Δs_c is approximately $\frac{c}{2\pi\Delta\nu}$. With $\Delta\nu = 1 \times 10^6$ Hz, this gives:

$$\Delta s_{\mathrm{c}} \approx \frac{3 \times 10^8 \text{ m/s}}{2\pi \times 1 \times 10^6 \text{ Hz}}$$

$$\Delta s_{\mathrm{c}} \approx \frac{3 \times 10^8 \text{ m/s}}{6.283 \times 10^6 \text{ s}^{-1}}$$

$$\Delta s_{\mathrm{c}} \approx 47.7 \text{ meters}$$

The slide gives the result as **approximately 50 meters (**≈ 50 m). This is consistent. The contrast is dramatic: the mercury lamp had a coherence length of a few centimeters (let's say 1.2 cm if using the 2π factor consistently, or 7.5 cm if not). The single-mode He-Ne laser has a coherence length of about 50 *meters*! This is thousands of times longer. This is a direct consequence of the laser's extremely narrow spectral bandwidth.

* The **Key takeaway** from these examples is profoundly important: **coherence length increases inversely with bandwidth.** A spectrally purer source (smaller bandwidth) is more temporally coherent (longer coherence length). Lasers excel at producing light with extremely small bandwidths.

Now, the slide indicates **"[IMAGE REQUIRED: Graph of fringe visibility vs.** Δs depicting rapid decay for lamp, slow decay for laser.]"

Let me describe what this graph would show:

* The horizontal axis would be the path difference, Δs, in an interferometer.

* The vertical axis would be the fringe visibility, V, ranging from 0 to 1.

* There would be two curves plotted: 1. **For the lamp (like the mercury lamp):** This curve would start at $V \approx 1$ when $\Delta s = 0$. However, it would decay very rapidly as Δs increases, perhaps falling to near zero for Δs values of just a few centimeters. The "width" of this visibility curve along the Δs axis would correspond to the short coherence length (e.g., ~ 1.2 cm if using the 2π factor consistently, or ~ 7.5 cm if not) of the lamp. 2. **For the laser (like the He-Ne laser):** This curve would also start at $V \approx 1$ when $\Delta s = 0$. But, it would decay

much, much more slowly as Δs increases. You would have to extend the Δs axis out to tens of meters before the visibility drops significantly. The width of this curve would represent the very long coherence length (e.g., ~ 50 m) of the laser.

This visual comparison would powerfully illustrate the vastly superior temporal coherence of a typical laser compared to a thermal lamp source.

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Alright, we've spent a good deal of time on temporal coherence. Now let's shift our focus to spatial coherence, and the classic experiment to test it: **Young's Double-Slit experiment**, which serves as a **Basic Spatial Coherence Test.**

Let's describe the **Setup**:

- We have an **extended light source of width** b. This is not an ideal point source, but a source with some physical dimension.
- This source **illuminates two slits,** S_1 (S_1) and S_2 (S_2), which are separated by a distance d. These slits are typically narrow and parallel, acting as secondary sources.
- Beyond the slits, there's an **Observation screen placed at a distance** r from the plane of the slits. Interference fringes may form on this screen.

Now, a key concept for understanding how the extended nature of the source affects fringe visibility is the **Path difference for light from a generic source point** Q.

Let Q be any arbitrary point on the surface of our extended source. Light from Q travels to slit S_1 and to slit S_2. If the light reaching S_1 and S_2 from the source is spatially coherent, then S_1 and S_2 will act as coherent secondary sources, producing interference fringes on the screen.

The slide mentions: $\Delta s_Q = QS_1 - QS_2$.

This Δs_Q represents the difference in path lengths from a single point Q on the source to the two slits S_1 and S_2. The variation of this quantity for different points Q across the source is what determines the degree of spatial coherence of the illumination at the slits. If all points Q on the source produce waves that arrive at S_1 and S_2 with a stable, well-defined phase relationship (meaning Δs_Q varies in a controlled way, or is nearly constant), then the illumination is spatially coherent.

However, the context for Δs_{max} on the next slide usually refers to path differences related to how different parts of the source contribute to the interference pattern on the screen. Let's clarify that. The critical factor for spatial coherence in Young's experiment is whether the fields *at* S_1 and *at* S_2 maintain a constant phase difference over time. This depends on the angular size of the source b as seen from the slits at distance $r_{source\text{\_to\_slits}}$.

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Continuing with Young's double-slit experiment and the effect of an extended source:

- The first bullet says: **Largest** Δs_Q (Δs_Q) occurs for extreme points R_1, R_2 at source edges. This Δs_Q was $QS_1 - QS_2$. This refers to the path difference from a point on the source to the two slits. The variation in this quantity, as Q moves from one edge of the source (say R_1) to the other edge (R_2), is what matters. If this variation is too large (comparable to a wavelength), then different parts of the source will cause the phase relationship between S_1 and S_2 to vary too much, washing out the fringes.

- The next bullet introduces an approximation: **Approximate geometry,** $b \ll r$ ($b \ll r$), leads to Delta s sub max approximately b sine theta ($\Delta s_{\text{max}} \approx b\sin\theta$). Here: • '$b$' is the width of the source. • 'r' is typically the distance from the source to the slits. • 'θ' (theta) is the angle subtended by the slit separation 'd' at the source. So, $\sin\theta \approx \theta \approx \frac{d}{r}$ (if r is source-to-slit distance). • So, $\Delta s_{\max} \approx b\left(\frac{d}{r}\right)$. This $\Delta s_{\max}$ represents the maximum difference in path lengths for light rays coming from the opposite edges of the source (width b) and passing through the two slits (separation d) on their way to form the central interference fringe on the screen. More precisely, it's the extra path that light from one edge of the source travels compared to light from the other edge, in reaching the two slits in such a way that they would constructively interfere at the screen center.

- The crucial condition for observing clear interference fringes is then: **If this** $\Delta S_{\max}$ (Delta S sub max) is greater than lambda over 2 ($\lambda/2$), then the phase differences introduced by different parts of the extended source exceed pi (π) radians, and as a result, the fringes wash out. If $\Delta s_{\max} > \frac{\lambda}{2}$, it means that light from one edge of the source might be trying to produce a bright fringe at a certain location on the screen, while light from the other edge of the source is trying to produce a dark fringe (or something significantly phase-shifted) at the same location. These contributions from different parts of the incoherent source will average out, leading to a loss of fringe visibility. So, for good fringes, we need $\Delta s_{\max} < \frac{\lambda}{2}$, which means $b\left(\frac{d}{r}\right) < \frac{\lambda}{2}$, or more commonly, the condition is relaxed to $b\left(\frac{d}{r}\right) < \lambda$. This is the famous condition for spatial coherence in Young's experiment with an extended incoherent source.

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This brings us to **Slide 12: Criterion for Spatial Coherence Between Two Apertures.** This formalizes the condition we just discussed.

* The **Coherent illumination condition** is given first as:

$$b\sin(\theta/2) < \lambda/2$$

Here, 'b' is the source width. θ (theta) is the angular separation of the two apertures (slits) as viewed from the source. So, $\theta \approx \frac{d}{r}$, where 'd' is the slit separation and 'r' is the distance from source to slits.

For small angles, $\sin(\theta/2) \approx \theta/2$. So the condition becomes $b(\theta/2) < \lambda/2$, or $b\theta < \lambda$.

If $\theta = \frac{d}{r}$, then $b\left(\frac{d}{r}\right) < \lambda$. This is consistent with our previous discussion. This form, $b\sin(\theta/2) < \lambda/2$, is related to Zernike's precise formulation for the visibility.

* The slide then says: **Using the small-angle relation** $2\sin\theta \approx \frac{d}{r}$:

This seems a bit unorthodox as a "small-angle relation." Usually, $\sin\theta \approx \theta$. If they mean $\theta \approx \frac{d}{r}$, and are using the θ from the previous formula (where it was angular separation of apertures), this is slightly confusing.

However, the important resulting condition is presented in the box, and it's the one most commonly used:

$$\frac{bd}{r} < \lambda$$

Let's clearly define the terms in this critical formula: * **b**: The width of the (assumed incoherent) light source. * **d**: The separation distance between the two apertures (e.g., the slits in Young's experiment). * **r**: The distance from the source to the plane containing the two apertures. * **λ**: The wavelength of the light.

This inequality, $\frac{bd}{r} < \lambda$, tells us that for the illumination at the two apertures to be spatially coherent enough to produce good interference fringes, the product of the source width 'b' and the slit separation 'd', divided by the source-to-slit distance 'r', must be less than the wavelength λ.

Alternatively, $\frac{b}{r}$ is the angular size of the source as seen from the slits. Let's call this $\theta_{\text{source}} \approx \frac{b}{r}$. Then the condition is $\theta_{\text{source}} \cdot d < \lambda$. This is a very standard and useful form.

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Let's look at the **Interpretation** of this crucial spatial coherence criterion, $\frac{bd}{r} < \lambda$.

* **Larger source size 'b'** or larger slit separation 'd' reduces spatial coherence. This is evident from the formula. If you increase 'b' (a physically wider source) or 'd' (move the slits further apart), the left side of the inequality $\frac{bd}{r}$ increases. To maintain the condition $\frac{bd}{r} < \lambda$, you would then need to compensate, for example, by increasing 'r' or using a shorter wavelength. If 'b' or 'd' becomes too large, the condition will be violated, and spatial coherence will be lost, meaning interference fringes will disappear.

* **Increasing the distance** 'r' from the source to the apertures increases the coherence area at the observation plane (the plane of the slits). If 'r' increases, then for a fixed source width 'b' and wavelength λ, the permissible slit separation 'd' (from $d < \frac{\lambda r}{b}$) can be larger. This means the light is spatially coherent over a larger transverse distance 'd'. The "coherence area" (roughly d^2) over which the field is coherent grows as r increases. This is why very distant sources, like stars, can exhibit significant spatial coherence over large areas on Earth, even if the stars themselves are physically enormous.

The slide then indicates: **"[IMAGE REQUIRED: Adapted Fig. 2.32 showing path differences from central vs. edge source points.]"**

Let me describe what such an image would typically illustrate to make this concept clearer:

* Imagine an extended source of width 'b'.

* At some distance 'r' away, you have two slits, S_1 and S_2, separated by 'd'.

* Consider light rays originating from the very center of the source. One ray goes to S_1, another to S_2. These will have some relative phase when they arrive at S_1 and S_2.

* Now consider rays from one extreme edge of the source (say, the "top" edge). Again, one ray goes to S_1, another to S_2. These will also have a relative phase at S_1 and S_2, but this relative phase might be different from that produced by the center of the source, due to the different path lengths involved from the source edge to S_1 versus S_2.

The condition $\frac{bd}{r} < \lambda$ essentially ensures that the maximum change* in this relative phase (between S_1 and S_2) as you consider contributions from all points across the source (from one edge, through the center, to the other edge) does not exceed roughly 2π radians (or that the path difference variation doesn't exceed λ).

* If this change is too large, the contributions from different parts of the source will effectively "smear out" any consistent phase relationship between S_1 and S_2, destroying the interference.

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Now we move to the **Coherence Surface & Solid Angle Formulation**, which provides a more general and elegant way to express spatial coherence.

* First, let's **Define the source area, A sub s (**A_s), as b squared ($A_\text{s} = b^2$). We're simplifying here by assuming a square source of side b. For a circular source of radius R, A_s would be πR^2. The exact shape factor isn't crucial for the principle.

* Next, we define the **Coherence surface at the observation plane (the plane of the slits, for instance) as A sub c (**A_c) equals d squared ($A_c = d^2$). Here, d is the maximum separation between two points in that plane for which the light is still considered spatially coherent. From our previous condition $\frac{bd}{r} < \lambda$, we have $d < \frac{\lambda r}{b}$. So, the maximum d is roughly $\frac{\lambda r}{b}$. Therefore, the coherence area A_c would be approximately $\left(\frac{\lambda r}{b}\right)^2$.

* The slide then says: **From the previous inequality: b squared d squared, all divided by r squared, is less than or equal to lambda squared (**$\frac{b^2 d^2}{r^2} \leq \lambda^2$). This is obtained by squaring our condition $\frac{bd}{r} < \lambda$ and taking the limit as equality. We can rewrite this using $A_s = b^2$ and $A_c = d^2$: $\frac{A_s A_c}{r^2} \leq \lambda^2$.

* Now, a key step: **Recognize the solid angle subtended by** A_c. The solid angle, let's call it $d\Omega$ (d Omega or delta Omega), subtended by the coherence area A_c when viewed from the source (at distance r) would be

$$d\Omega \approx \frac{A_c}{r^2}$$

Alternatively, and perhaps more standardly for this formulation, $d\Omega$ is the solid angle into which the source radiates, or the solid angle "seen" by the detector system from a point on the source. If we are interested in coherence over an area $A_c = d^2$ at distance r, this area subtends a solid angle

$$d\Omega = \frac{A_c}{r^2} = \frac{d^2}{r^2}$$

from the source.

Let's see how the next slide uses this.

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Continuing with the solid angle formulation:

* The slide defines **d Omega (**$d\Omega$) equals d squared divided by r squared ($d\Omega = \frac{d^2}{r^2}$). This $d\Omega$ is the solid angle subtended by an area d^2 (which is our coherence area A_c) at a distance 'r'. So, this is the solid angle associated with the region over which we are demanding coherence.

* With this, the **Spatial coherence condition can be expressed very compactly as:** $A_s d\Omega \leq \lambda^2$. Let's check this. We have $A_s = b^2$ and $d\Omega = \frac{d^2}{r^2}$. So, $A_s d\Omega = b^2 \left(\frac{d^2}{r^2}\right) = \left(\frac{bd}{r}\right)^2$. The condition $\left(\frac{bd}{r}\right)^2 \leq \lambda^2$ is what we had on the previous page. So, this compact form $A_s d\Omega \leq \lambda^2$ is indeed equivalent. This is a very beautiful and general result. It states that the product of the source area (A_s) and the solid angle ($d\Omega$) over which coherence is required (or into which the light is collected/observed) must be less than or on the order of the wavelength squared. This product $A_s d\Omega$ is related to the "étendue" or "throughput" of an optical system. For coherent operations, this étendue is limited by λ^2.

* From this, we can find the **Limiting solid angle (**$d\Omega_{\text{max}}$) within which radiation from a source of area A_s remains spatially coherent: **d Omega max (**$d\Omega_{\text{max}}$) equals lambda squared divided by A sub s ($d\Omega_{\text{max}} = \frac{\lambda^2}{A_s}$). This tells us that for a source of a given area A_s and wavelength λ, there's a maximum solid angle $d\Omega_{\text{max}}$. If you collect light from this source only within this solid angle, the light will be spatially coherent. This is extremely important for applications like coupling light into a single-mode optical fiber, which can only accept light from a very small solid angle (related to its numerical aperture) and requires spatially coherent input. If $d\Omega_{\text{source\_to\_fiber}} > \frac{\lambda^2}{A_{s,\text{source}}}$, you won't efficiently couple coherent light.

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Continuing our discussion on the solid angle formulation of spatial coherence:

* A very important limiting case is that of a **Point source.**

If the source area A_s approaches zero ($A_\text{s} \to 0$), then what happens to our limiting solid angle $d\Omega_\text{max} = \frac{\lambda^2}{A_\text{s}}$?

As A_s goes to zero, $d\Omega_\text{max}$ goes to infinity.

This means that for an ideal point source, **coherence extends over the full** 4π steradians (4π sr) solid angle.

In other words, a true point source emits perfectly spatially coherent light in all directions. This is, of course, an idealization, as no real source is a perfect mathematical point. However, if a source is physically very small compared to the wavelength and observation distances, it can approximate a point source and exhibit very high spatial coherence.

The two hyphens just denote the end of this thought on the slide. This concept underscores why sources that are "effectively point-like" (either physically small or very far away, making their angular size tiny) are good for experiments requiring high spatial coherence.

Page 33:

Slide 14: Distance Dependence – Astronomical Relevance.

* The **Coherence surface (or coherence area),** A_c, grows quadratically with distance r. We had the limiting solid angle for coherence $d\Omega_{\max} = \frac{\lambda^2}{A_\text{s}}$. The coherence area A_c at a distance 'r' from the source is simply this solid angle multiplied by r^2: A_c equals $d\Omega_{\max}$ times r^2, which equals $\frac{\lambda^2}{A_\text{s}} r^2$.

$$A_\mathrm{c} = d\Omega_\mathrm{max}\, r^2 = \frac{\lambda^2}{A_\mathrm{s}}\, r^2$$

This clearly shows that A_c is proportional to r^2. As you move further away from the source, the area over which the light maintains spatial coherence increases as the square of the distance.

* This has profound **Astronomical Relevance**, particularly for **Stars**: **Stars, despite having a large physical diameter (meaning** A_s, their actual surface area, is enormous), are at such an enormous distance 'r' from Earth that the r^2 factor in the A_c equation dominates. This makes the coherence area A_c of starlight on Earth very large, often exceeding the aperture (diameter) of typical telescopes. For example, the Sun is huge, but if it were much further away (like other stars), the r^2 term would make its light appear spatially coherent over large areas on Earth.

* The **Consequence** of this is that **starlight is spatially coherent across a telescope mirror.** This allows for **stellar interferometry.** Because the incoming starlight wavefront has a well-defined phase relationship across the entire diameter of a telescope's primary mirror (or across even larger baselines if multiple telescopes are used as an interferometer), astronomers can make these wavefronts interfere. Albert A. Michelson, along with Francis Pease, famously used this principle with the stellar interferometer at Mount Wilson Observatory to make the first direct measurements of stellar diameters (for Betelgeuse in 1920). By varying the separation of two apertures collecting starlight and observing the visibility of the interference fringes, they could deduce the angular size of the star, which, combined with its (estimated) distance, gave its physical diameter. This was a triumph of understanding and applying coherence principles.

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Now, let's synthesize our understanding of temporal and spatial coherence by **Putting Temporal & Spatial Together** to define the **Coherence Volume** on **Slide 15**.

- We need to **Combine the longitudinal coherence length,** Δs_{c}, with the transverse coherence surface, S_{c}.

(Note: the slide uses S_{c} here for coherence surface, whereas previously it used A_{c}. They represent the same concept: the transverse area over which the field is spatially coherent.)

- Recall, $\Delta s_{\mathrm{c}} \approx \frac{c}{\Delta\omega}$, which characterizes coherence along the direction of propagation (temporal coherence).

- And $S_{\mathrm{c}} \approx \frac{\lambda^2 r^2}{A_{\mathrm{s}}}$, which characterizes coherence in a plane transverse to the propagation direction (spatial coherence).

- The **Resulting coherence volume,** V_{c}, is simply the product of these two:

The formula in the box is:

$$V_{\mathrm{c}} = S_{\mathrm{c}} \Delta s_{\mathrm{c}} = \frac{\lambda^2 r^2 c}{\Delta\omega A_{\mathrm{s}}}$$

$V_{\mathrm{c}} = S_{\mathrm{c}} \Delta s_{\mathrm{c}} = \frac{\lambda^2 r^2 c}{\Delta\omega A_{\mathrm{s}}}$.

Let's verify this product:

$S_{\mathrm{c}} = \frac{\lambda^2 r^2}{A_{\mathrm{s}}}$ (using the limiting case for spatial coherence area).

$\Delta s_{\mathrm{c}} = \frac{c}{\Delta\omega}$ (temporal coherence length).

Multiplying them gives $\left(\frac{\lambda^2 r^2}{A_{\mathrm{s}}}\right) \times \left(\frac{c}{\Delta\omega}\right) = \frac{\lambda^2 r^2 c}{A_{\mathrm{s}} \Delta\omega}$. This matches the slide.

This V_c represents a three-dimensional volume in space. Within this "coherence volume," the electromagnetic field can be thought of as having well-defined phase correlations, both in the direction of propagation (over length Δs_c) and in the transverse plane (over area S_c). It's a region where the wave behaves coherently.

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Let's elaborate on the significance of this coherence volume, V_c.

* The **Physical meaning** of the coherence volume is that it's a **region of space-time where the electromagnetic (EM) field maintains fixed phase correlations.** "Space-time" is appropriate here because V_c involves both spatial dimensions (through S_c) and a temporal dimension (through $\Delta s_c = c\Delta t_c$, where Δt_c is the coherence time). If you pick any two points within this coherence volume, their phases will have a predictable relationship. If you consider the field at one point within V_c at two different times (separated by less than Δt_c), their phases will be correlated.

* The second bullet point is a very important teaser for later concepts: **This coherence volume will later connect to quantized "modes" of the electromagnetic field and to photon statistics.** This is a profound connection. In quantum optics, the electromagnetic field can be quantized into modes. Each mode can be thought of as a sort of "container" for photons. The coherence volume V_c turns out to be closely related to the volume occupied by a single mode of the radiation field. Furthermore, the average number of photons found within one coherence volume (or per mode) is a crucial parameter called the "degeneracy parameter" or "occupation number." This parameter, often denoted $\bar{n}$ (n̄), tells us whether the light is "classical-like" (many photons per mode, $\bar{n} \gg 1$) or "quantum-like" (few photons per mode, $\bar{n} \leq 1$). Lasers can achieve very high $\bar{n}$, while thermal

sources at optical frequencies typically have $\bar{n} \ll 1$. We'll likely explore this in more detail soon.

Page 36:

Slide 16: Photon Statistics Inside a Coherence Volume.

Now we delve into **Slide 16: Photon Statistics Inside a Coherence Volume.** This builds directly on the previous point.

* First, let's define **Spectral radiance,** L_ω (with units of $\mathrm{W\,m^{-2}\,sr^{-1}\,rad^{-1}}$). This L_ω describes the power (Watts) emitted from a unit area of the source (per m^2) into a unit solid angle (per sr) per unit of angular frequency bandwidth (per rad s^{-1}). It's a measure of the "brightness" of the source at a particular angular frequency ω.

* Next, we want the **Number flux density of photons per Hertz per steradian from unit area.** The slide gives this as L_ω divided by $h\nu$ ($\frac{L_\omega}{h\nu}$). Let's be careful with units and terms here: L_ω is radiance per unit angular* frequency ω. $h\nu$ is the energy of a photon, where ν is the linear* frequency $\left(\nu = \frac{\omega}{2\pi}\right)$ and h is Planck's constant. Alternatively, photon energy is $\hbar\omega$. * If L_ω is energy/(time × area × solid_angle × angular_frequency_bandwidth), then to get photon number, we should divide by photon energy, $\hbar\omega$. * So, number flux density would be $\frac{L_\omega}{\hbar\omega}$. * If $\frac{L_\omega}{h\nu}$ is used, it implies either L_ω was actually $\frac{L_\nu}{2\pi}$ (radiance per linear frequency, converted), or some factors of 2π are being absorbed. * Let's assume L_ω refers to the spectral radiance in terms of angular frequency, and $h\nu$ is simply shorthand for the photon energy E_{photon}. For consistency, if L is in terms of ω, the photon energy should be $\hbar\omega$. If we use $h\nu$, then L should be L_ν. * Given $\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$ later for thermal sources, it seems calculations

are often done with ν. Let's proceed assuming the formula $\frac{L_\omega}{h\nu}$ yields the correct photon number density for the L_ω defined by the subsequent math.

* Now, the **Total average number of photons in a coherence volume,** $\bar{n}$, is given by: $\bar{n}$ equals $\frac{L_\omega}{h\nu}$ times A_s, times $\Delta\Omega$, times $\Delta\omega$, times Δt_c. $\bar{n} = \frac{L_\omega}{h\nu} \cdot A_\text{s} \cdot \Delta\Omega \cdot \Delta\omega \cdot \Delta t_\text{c}$. Let's break this down: * $\frac{L_\omega}{h\nu}$: Photons per unit time, per unit source area, per unit solid angle, per unit angular frequency bandwidth. * A_s: The area of the source. So $\frac{L_\omega}{h\nu} \cdot A_\text{s}$ is photons per unit time, per unit solid angle, per unit angular frequency bandwidth, from the whole source. * $\Delta\Omega$: The solid angle of coherence ($d\Omega_\text{max}$ from page 31, which was $\frac{\lambda^2}{A_\text{s}}$). * $\Delta\omega$: The spectral bandwidth over which coherence is considered. * Δt_c: The coherence time $\left(\Delta t_\text{c} \approx \frac{1}{\Delta\omega}\right)$. The product $A_\text{s} \cdot \Delta\Omega \cdot \Delta\omega \cdot \Delta t_\text{c}$ effectively defines the "number of available slots" or "modes" within the coherence volume originating from source A_s, within solid angle $\Delta\Omega$, bandwidth $\Delta\omega$, and coherence time Δt_c. So, $\bar{n}$ is indeed the average number of photons found in such a spatio-temporal-spectral cell, i.e., per mode or per coherence volume. This is the degeneracy parameter.

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Let's continue with the calculation of $\bar{n}$, the average number of photons in a coherence volume.

* We need to **Substitute** $\Delta\Omega = \frac{\lambda^2}{A_\text{s}}$ and $\Delta t_\text{c} = \frac{1}{\Delta\omega}$. * $\Delta\Omega = \frac{\lambda^2}{A_\text{s}}$ (capital Delta Omega equals lambda squared over A_s) is the limiting solid angle for spatial coherence derived earlier. * $\Delta t_\text{c} = \frac{1}{\Delta\omega}$ (Delta t_c equals 1 over Delta ω) is the coherence time related to the spectral bandwidth $\Delta\omega$ (this is an order-of-magnitude relationship; sometimes a 2π factor appears depending on definitions, but it often cancels).

Plugging these into the expression for $\bar{n}$ from the previous page:

$$\bar{n} = \left(\frac{L_\omega}{h\nu}\right) A_s \, \Delta\Omega \, \Delta\omega \, \Delta t_c$$

$$\bar{n} = \left(\frac{L_\omega}{h\nu}\right) A_s \left(\frac{\lambda^2}{A_s}\right) \Delta\omega \left(\frac{1}{\Delta\omega}\right)$$

We can see some nice cancellations: * The source area A_s in the numerator cancels with A_s in the denominator (from the $\Delta\Omega$ substitution). * The spectral bandwidth $\Delta\omega$ in the numerator cancels with $\Delta\omega$ in the denominator (from the Δt_c substitution).

This leaves us with a remarkably simple expression, shown in the box: $\bar{n}$ equals $\left(\frac{L_\omega}{h\nu}\right) \lambda^2$. (i.e., $\bar{n} = \left(\frac{L_\omega}{h\nu}\right) \lambda^2$).

* The slide then makes a crucial point: **Remarkably,** $\bar{n}$ is independent of $\Delta\omega$ or r. ***Independence from*** $\Delta\omega$ (spectral bandwidth): This might seem surprising at first, but it's because the definitions of coherence time ($\Delta t_c \sim \frac{1}{\Delta\omega}$) and coherence volume (which involves Δt_c) scale inversely with $\Delta\omega$. So, while a larger bandwidth means a smaller coherence volume, the number of photons per coherence volume* remains characterized by this simpler formula. * **Independence from** r (distance from the source): This also seems remarkable. L_ω is the spectral radiance of the source. $h\nu$ and λ are properties of the light. This implies that $\bar{n}$, the number of photons per coherence volume (or per mode), is an intrinsic property of the radiation field generated by the source, not dependent on how far away you observe it. This is true as long as L_ω is understood as the source radiance and the coherence volume is correctly defined at distance r (recall S_c or A_c depended on r, but $\Delta\Omega = \frac{S_c}{r^2}$ and $A_s \Delta\Omega = \lambda^2$ were key relations).

This $\bar{n}$ is a fundamental quantity in radiation physics, known as the photon degeneracy parameter, or the mean occupation number of a radiation mode.

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Now, let's apply this to a very important case: **Slide 17: Degeneracy Parameter for Thermal Radiation.** We'll use Planck's law.

* First, the **Planck spectral radiance (for a single polarization state)** is given as:

$$L_\nu = \frac{h\nu^3}{c^2 \left(\exp\left(\frac{h\nu}{kT} \right) - 1 \right)}$$

($L_\nu = \frac{h\nu^3}{c^2}$ / $\left(\exp\left(\frac{h\nu}{kT} \right) - 1 \right)$).

L_ν here is the spectral radiance per unit linear* frequency ν (in Hz). Units: Watts per meter squared per steradian per Hertz ($\mathrm{W\,m^{-2}\,sr^{-1}\,Hz^{-1}}$). * h is Planck's constant. * ν is the linear frequency. * c is the speed of light. * k is Boltzmann's constant. * T is the absolute temperature of the thermal source (a black body). * $\exp(\quad)$ is the exponential function.

This formula describes the emitted power spectrum of an ideal black body. For a single polarization, we take half the value of the unpolarized Planck's law.

* Next, we need to **Convert** L_ν to L_ω because our formula for $\bar{n}$ used L_ω. The relationship is based on energy conservation: the power in a frequency interval $d\nu$ must equal the power in the corresponding angular frequency interval $d\omega$.

So,

$$L_\omega \, d\omega = L_\nu \, d\nu.$$

Since $\omega = 2\pi\nu$, we have $d\omega = 2\pi \, d\nu$.

Therefore,

$$L_\omega = L_\nu \frac{d\nu}{d\omega} = \frac{L_\nu}{2\pi}.$$

* Now, we **Insert this into our formula for** $\bar{n}$, which was

$$\bar{n} = \left(\frac{L_\omega}{h\nu}\right)\lambda^2.$$

Let's do this step-by-step: 1. $L_\omega = \frac{1}{2\pi} \cdot L_\nu = \frac{1}{2\pi} \cdot \frac{h\nu^3}{c^2\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)}$. 2.

$$\bar{n} = \left[\frac{1}{2\pi} \cdot \frac{h\nu^3}{c^2\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)}\right] \cdot \frac{1}{h\nu} \cdot \lambda^2.$$

3. The $h\nu$ term in the denominator cancels one $h\nu$ from $h\nu^3$ in the numerator, leaving $\frac{\nu^2}{c^2}$.

$$\bar{n} = \left[\frac{1}{2\pi} \cdot \frac{\nu^2}{c^2\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)}\right] \cdot \lambda^2.$$

4. We know that wavelength $\lambda = \frac{c}{\nu}$, so $\lambda^2 = \frac{c^2}{\nu^2}$. 5. Substitute λ^2:

$$\bar{n} = \left[\frac{1}{2\pi} \cdot \frac{\nu^2}{c^2\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)}\right] \cdot \frac{c^2}{\nu^2}.$$

6. The terms $\frac{\nu^2}{c^2}$ and $\frac{c^2}{\nu^2}$ cancel out perfectly! This leaves:

$$\bar{n} = \frac{1}{2\pi} \cdot \frac{1}{\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)}.$$

However, the slide gives the result for $\bar{n}$ directly as: $\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$. This implies that the L_ω in the formula $\bar{n} = \left(\frac{L_\omega}{h\nu}\right)\lambda^2$ on page 37 was actually

$$L_\omega = \frac{h\nu}{2\pi\lambda^2} \cdot \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

for a thermal source, or that the 2π factor was absorbed differently. If we assume the final $\bar{n}$ result is correct (which it is, it's the Bose-Einstein distribution), then the L_ω used to derive it must have been

$$L_\omega = \frac{h\nu}{\lambda^2} \cdot \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}.$$

Let's re-check the derivation of $\bar{n}$ from page 36-37 using standard definition of L_ω from Planck's law. Planck's law for spectral radiance per unit angular frequency, for a single polarization, is:

$$L_\omega(T) = \frac{\hbar\omega^3}{4\pi^2 c^2 \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)} \quad \text{(where } \hbar = \frac{h}{2\pi}\text{)}.$$

The formula for $\bar{n}$ was $\bar{n} = \left(\frac{L_\omega}{\hbar\omega}\right)\lambda^2$ (using $\hbar\omega$ as photon energy if L is L_ω). So,

$$\bar{n} = \left[\frac{\hbar\omega^3}{4\pi^2 c^2 \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)}\right] \cdot \frac{1}{\hbar\omega} \cdot \lambda^2.$$

$$\bar{n} = \left[\frac{\omega^2}{4\pi^2 c^2 \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)}\right] \cdot \lambda^2.$$

Since $\omega = 2\pi\nu$ and $\lambda = \frac{c}{\nu}$, then $\omega = \frac{2\pi c}{\lambda}$. So,

$$\omega^2 = \frac{4\pi^2 c^2}{\lambda^2}.$$

Substitute this into the expression for $\bar{n}$:

$$\bar{n} = \left[\frac{\frac{4\pi^2 c^2}{\lambda^2}}{4\pi^2 c^2 \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1 \right)} \right] \cdot \lambda^2.$$

$$\bar{n} = \left[\frac{1}{\lambda^2 \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1 \right)} \right] \cdot \lambda^2.$$

$$\bar{n} = \frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}.$$

And since $\hbar\omega = h\nu$, this becomes

$$\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}.$$

This matches the slide's final result exactly! So the derivation is sound if we consistently use L_ω and $\hbar\omega$ (or L_ν and $h\nu$). The formula on page 37, $\bar{n} = \left(\frac{L_\omega}{h\nu}\right)\lambda^2$, was a slight notational mix if L_ω is radiance per angular frequency and $h\nu$ is used for energy. However, the derived result here for thermal radiation is correct and fundamental.

Page 39:

Now for the **Interpretation** of this degeneracy parameter $\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$ for thermal radiation.

* This n-bar ($\bar{n}$) represents the **mean photon number per electromagnetic mode.** It is also known as the **degeneracy parameter** or the **photon occupation number.** For thermal radiation in equilibrium at temperature T, this formula tells us, on average, how many photons occupy each distinct electromagnetic mode (each "slot" in phase space, or each coherence

volume) that has a frequency ν. This is precisely the Bose-Einstein distribution function for a gas of photons, which are bosons.

Let's look at two important limits:

* **Low-frequency (Rayleigh-Jeans) limit:** This occurs when $h\nu$ is much less than kT ($h\nu \ll kT$), meaning the photon energy is small compared to the characteristic thermal energy. In this case, the exponential term $\exp\left(\frac{h\nu}{kT}\right)$ can be approximated as $1 + \frac{h\nu}{kT}$. So, $\bar{n} \approx \frac{1}{\left(1+\frac{h\nu}{kT}\right)-1} = \frac{1}{\frac{h\nu}{kT}} = \frac{kT}{h\nu}$. Since $h\nu \ll kT$, then $\frac{kT}{h\nu}$ is much greater than 1 ($\bar{n} \gg 1$). This means that in the low-frequency (long wavelength, e.g., radio waves or far-infrared at room temperature) limit, there are many photons per mode. The modes are highly populated. This is the regime where classical wave descriptions of electromagnetism (like the Rayleigh-Jeans law itself) work well because the quantum discreteness of photons is less apparent due to their sheer number.

* **High-frequency (Wien) limit:** This occurs when $h\nu$ is much greater than kT ($h\nu \gg kT$), meaning the photon energy is large compared to the thermal energy. In this case, $\exp\left(\frac{h\nu}{kT}\right)$ is much greater than 1. So, $\bar{n} \approx \frac{1}{\exp\left(\frac{h\nu}{kT}\right)} = \exp\left(-\frac{h\nu}{kT}\right)$. Since $h\nu \gg kT$, the negative exponent is large and negative, so $\bar{n}$ is much less than 1 ($\bar{n} \ll 1$). This means that in the high-frequency (short wavelength, e.g., visible or UV light at room temperature) limit, there are very few photons per mode, typically much less than one on average. The modes are sparsely populated. This is a distinctly quantum regime where the particle nature of light (photons) becomes dominant. It's hard to "build up" a classical wave if you don't even have one photon per mode on average.

This parameter $\bar{n}$ is crucial for understanding lasers. A laser achieves its special properties because it can generate a light field where $\bar{n}$ is extremely large ($\bar{n} \gg 1$) even at optical frequencies where thermal $\bar{n}$ would be vanishingly small. This means lasers produce highly populated, non-thermal states of light.

Page 40:

Now we explore a fascinating connection on **Slide 18: Coherence Volume ↔ Elementary Phase-Space Cell.** This links coherence to one of the most fundamental concepts in quantum mechanics, the Heisenberg Uncertainty Principle.

* First, let's recall the **Heisenberg uncertainty principle in 3-Dimensions (3-D):**

$\Delta p_x \Delta p_y \Delta p_z \Delta x \Delta y \Delta z \geq \hbar^3$.

* Δx, Δy, Δz are the uncertainties in the position coordinates of a particle. * Δp_x, Δp_y, Δp_z are the uncertainties in the corresponding momentum components. * $\hbar$ (h-bar) is the reduced Planck constant $(h/2\pi)$.

This principle states that the volume occupied by a quantum state in 6-dimensional phase space (3 position, 3 momentum coordinates) cannot be smaller than approximately $\hbar^3$. Each such minimal volume, $\hbar^3$, represents one "elementary cell" or one quantum state. (Sometimes h^3 is used, depending on convention; $\hbar$ is more common in this context).

* Now, let's consider **photons escaping an aperture of size 'b'.** For simplicity, let's assume a square aperture of side length 'b' in the x-y plane.

* The **Position uncertainties** in the transverse directions, Δx and Δy, are determined by the size of the aperture:

$\Delta x \approx b$, and $\Delta y \approx b$.

The photon, as it passes through the aperture, is localized within this transverse extent.

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Continuing our connection between coherence volume and phase-space cells:

- When light passes through an aperture, it undergoes diffraction. The **Diffraction-limited divergence angle, theta (**θ), is approximately lambda divided by b ($\theta \approx \lambda/b$). This divergence gives rise to a spread in the transverse momentum components of the photon. The transverse momentum spread, for example Δp_x, can be estimated. The momentum of a photon is $p = \frac{h}{\lambda} = \hbar k$. The uncertainty in its direction by an angle θ leads to a transverse momentum component $p_x \approx p\theta$. So, $\Delta p_x \approx p\,(\lambda/b) = \left(\frac{h}{\lambda}\right)(\lambda/b) = h/b$. Or, using $\hbar$: $\Delta p_x \approx \hbar k\,(\lambda/b) = \hbar(2\pi/\lambda)(\lambda/b) = \hbar(2\pi/b) = h/b$. The slide gives an intermediate step: Δp_x is approximately $(h/\lambda)\big(\lambda/(2\pi b)\big)$, which equals $\hbar/b$ ($\Delta p_x \approx (h/\lambda)\big(\lambda/(2\pi b)\big) = \hbar/b$). This result $\Delta p_x \approx \hbar/b$ is what we get if the characteristic angular spread is taken as $\lambda/(2\pi b)$, which is related to the 1/e width for a Gaussian beam rather than the first minimum of a slit. However, the product $\Delta x \Delta p_x \approx b\,(\hbar/b) = \hbar$. This is consistent with the uncertainty principle for one dimension. Similarly, $\Delta y \Delta p_y \approx \hbar$.

- Next, the **Spectral width, Delta omega (**$\Delta\omega$), of the light adds a longitudinal momentum spread, Δp_z. The longitudinal momentum of a photon is $p_z = \hbar k_z = \hbar(\omega/c)$. So, the uncertainty in p_z due to an uncertainty $\Delta\omega$ in angular frequency is: $\Delta p_z = \hbar\Delta\omega/c$. This is correct.

- Now, the crucial step: **Insert these uncertainties into the uncertainty product. The claim is that the spatial cell volume reproduces** V_c (the coherence volume). Let's look at the spatial volume element defined by our uncertainties: $\Delta V_{\text{spatial}} = \Delta x\, \Delta y\, \Delta z$.

  - $\Delta x \approx b$
  - $\Delta y \approx b$
  - Δz is the uncertainty in the photon's longitudinal position. This is precisely the coherence length Δs_c, which we

found to be related to the coherence time $\Delta t_c \approx 1/\Delta\omega$ (or more precisely, $\Delta\omega\Delta t_c \approx 2\pi$, or related to spectral line shape). If we use $\Delta s_c \approx c/\Delta\omega$, then $\Delta z \approx c/\Delta\omega$.

So, the spatial cell volume is

$$\Delta V_{\text{spatial}} \approx b \cdot b \cdot \left(\frac{c}{\Delta\omega}\right) = \frac{b^2 c}{\Delta\omega}.$$

Let's recall our coherence volume V_c from page 34: $V_c = S_c \Delta s_c$. If the transverse coherence area S_c is determined by the aperture size b, then $S_c \approx b^2$. And the longitudinal coherence length $\Delta s_c \approx c/\Delta\omega$. So, $V_c \approx b^2\,(c/\Delta\omega)$. Indeed, the spatial cell volume $\Delta x\,\Delta y\,\Delta z$, as derived from uncertainty considerations related to diffraction and spectral bandwidth, is identical to our expression for the coherence volume V_c!

The full phase-space volume is

$$(\Delta x\,\Delta p_x)(\Delta y\,\Delta p_y)(\Delta z\,\Delta p_z) \approx (\hbar)(\hbar)\left(\Delta z \cdot \frac{\hbar\Delta\omega}{c}\right).$$

If $\Delta z \approx c/\Delta\omega$, then

$$\Delta z\,\Delta p_z \approx \left(\frac{c}{\Delta\omega}\right)\left(\frac{\hbar\Delta\omega}{c}\right) = \hbar.$$

So the total phase space volume is indeed $\hbar^3$. The "spatial cell volume reproduces V_c" means that the $\Delta x\,\Delta y\,\Delta z$ part of the $\hbar^3$ phase space cell is exactly the coherence volume.

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This leads to the profound **Conclusion** on this slide:

* **The coherence volume equals one elementary photon phase-space cell.**

To be perfectly precise, the coherence volume V_c is a *real-space volume* ($\Delta x \Delta y \Delta z$). The elementary phase-space cell has a *phase-space volume* of $\hbar^3$ (or h^3 by some conventions).

The connection is that the photons occupying one coherence volume $V_c = \Delta x \Delta y \Delta z$ also simultaneously occupy a corresponding volume in momentum space, $\Delta P_{vol} = \Delta p_x \Delta p_y \Delta p_z$, such that the product $V_c \cdot \Delta P_{vol} \approx \hbar^3$.

So, one coherence volume V_c can be associated with a single quantum state, or a single "mode" of the electromagnetic field.

This is an incredibly deep and unifying concept. It tells us that coherence, which we initially defined in terms of classical wave interference, has a fundamental quantum underpinning related to the discrete nature of phase space.

The degeneracy parameter $\bar{n}$ we discussed earlier is then the average number of photons occupying this single phase-space cell, or mode, defined by the coherence volume.

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This slide presents a diagram beautifully illustrating **Diffraction, Coherence Volume, and the Phase-Space Cell.**

Let's break down the diagram:

On the **Left Side: "Real Space: Diffraction"**

- We see an **Aperture (size $b \times b$)**, depicted as a slit in a gray barrier. The dimension "b" is indicated as the height of the aperture.
- A beam of light (yellow arrows) is incident on this aperture from the left.
- As the light passes through, it diffracts, spreading out into a cone. The angular spread is indicated by $\theta \approx \frac{\lambda}{b}$.

- The longitudinal position uncertainty, or coherence length, Δz, would be along the propagation direction (labeled "z"). The transverse position uncertainty $\Delta y \approx b$ is shown. (And similarly $\Delta x \approx b$ into the page).

- The shaded blue region represents the diffracted beam. The coherence volume V_{c} would be like a "sausage" or "cylinder" with transverse area roughly $b \times b$ (or more precisely, the coherence area S_{c}) and length Δz (the coherence length Δs_{c}).

On the **Right Side: "Momentum Space (p-space)"**

- This depicts the corresponding uncertainties in momentum.

- A small green cuboid represents the volume element in momentum space, $\Delta p_{\mathrm{x}}\, \Delta p_{\mathrm{y}}\, \Delta p_{\mathrm{z}}$. • The transverse momentum spreads are labeled: $\Delta p_{\mathrm{y}} \approx \frac{h}{b}$ and $\Delta p_{\mathrm{x}} \approx \frac{h}{b}$. (Note: the diagram uses h, not $\hbar$. If so, then $\Delta x\, \Delta p_{\mathrm{x}} \approx h$, etc.) • The longitudinal momentum spread is labeled: $\Delta p_{\mathrm{z}} \approx \frac{h}{c} \Delta\omega$.

- The axes are p_{x}, p_{y}, and p_{z}.

At the bottom, a crucial statement: **"Heisenberg Uncertainty Principle links real & momentum space volumes."**

And the formula:

$$\Delta x\, \Delta p_{\mathrm{x}}\, \Delta y\, \Delta p_{\mathrm{y}}\, \Delta z\, \Delta p_{\mathrm{z}} \geq \hbar^3$$

(using $\hbar$ here).

Let's reconcile the h vs $\hbar$.

If we use the diagram's

$$\Delta p_{\mathrm{x}} = \frac{h}{b}, \quad \Delta p_{\mathrm{y}} = \frac{h}{b}, \quad \Delta p_{\mathrm{z}} = \frac{h}{c} \Delta\omega:$$

and

$$\Delta x = b, \quad \Delta y = b, \quad \Delta z = \frac{c}{\Delta\omega} \quad \left(\text{coherence length as } c \times \text{coherence time, where } \Delta t \approx \frac{1}{\Delta\omega}\right),$$

then

$$(\Delta x\, \Delta p_{\mathrm{x}}) = b \times \frac{h}{b} = h.$$

$$(\Delta y\, \Delta p_{\mathrm{y}}) = b \times \frac{h}{b} = h.$$

$$(\Delta z\, \Delta p_{\mathrm{z}}) = \frac{c}{\Delta\omega} \times \frac{h}{c} \Delta\omega = h.$$

So the product

$$(\Delta x\, \Delta p_{\mathrm{x}})(\Delta y\, \Delta p_{\mathrm{y}})(\Delta z\, \Delta p_{\mathrm{z}}) = h \times h \times h = h^3.$$

The coherence volume

$$V_{\mathrm{c}} = \Delta x\, \Delta y\, \Delta z = b \times b \times \frac{c}{\Delta\omega}.$$

The momentum volume

$$\Delta V_{\mathrm{p}} = \Delta p_{\mathrm{x}}\, \Delta p_{\mathrm{y}}\, \Delta p_{\mathrm{z}} = \frac{h}{b} \times \frac{h}{b} \times \frac{h}{c} \Delta\omega = \frac{h^3}{b^2\, c} \Delta\omega.$$

So

$$V_{\mathrm{c}} \times \Delta V_{\mathrm{p}} = \left[\frac{b^2\, c}{\Delta\omega}\right] \times \left[\frac{h^3}{b^2\, c} \Delta\omega\right] = h^3.$$

This is perfectly consistent. The coherence volume V_{c} is the real-space volume associated with a single h^3 cell in phase space. This diagram elegantly ties together diffraction (which gives transverse momentum spread from spatial confinement) and spectral bandwidth (which gives longitudinal momentum spread and coherence length/time).

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Slide 19: Mutual Coherence Function – Formal Definition.

We now transition to a more formal mathematical description with **Slide 19: Mutual Coherence Function – Formal Definition.** This framework, developed by Emil Wolf and others, allows for a rigorous treatment of partial coherence.

* First, we represent the **Complex electric field at position $\mathbf{r}$** (vector r) and time t as: $E(\mathbf{r}, t)$ equals A_0 times e to the power of i times $(\omega t - \mathbf{k} \cdot \mathbf{r})$, plus c.c.

$$E(\mathbf{r}, t) = A_0 e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})} + \text{c.c.}$$

* This is a representation of a monochromatic plane wave. A_0 is its amplitude. ω is angular frequency, $\mathbf{k}$ is the wave vector. * "c.c." stands for "complex conjugate." The physical electric field is real. Often, in coherence theory, we work with the "analytic signal," which is a complex representation whose real part is the physical field. For the analytic signal, the c.c. term is often omitted, and $E(\mathbf{r}, t)$ itself is complex. Let's assume $E(\mathbf{r}, t)$ here is the analytic signal.

* We then consider **Two spatial points,** S_1 (S_1) and S_2 (S_2), and introduce a time delay, τ (τ). These could be, for example, the locations of two pinholes in an interference experiment.

* The **Mutual coherence function**, which describes **first-order correlation**, is defined as: **Capital Gamma sub 1 2 of** τ, equals the angle brackets of, $E_1(t + \tau)$ times $E_2^*(t)$, close angle brackets.

$$\Gamma_{12}(\tau) = \langle E_1(t + \tau)\, E_2^*(t) \rangle.$$

Let's parse this: * $\Gamma_{12}(\tau)$ is the mutual coherence function between points 1 and 2, for a time delay τ. * $E_1(t+\tau)$ represents the complex electric field at point S_1 at time $(t+\tau)$. (The point S_1 is implicit in E_1.) \(*E_2^*(t)\) represents the complex conjugate of the electric field at point S_2 at time t. (Point S_2 is implicit in E_2.) * The angle brackets $\langle \ldots \rangle$ denote a temporal average. This function $\Gamma_{12}(\tau)$ quantifies the correlation between the field at S_1 (at a certain time) and the field at S_2 (at a time τ earlier, or later depending on convention if τ can be negative). It's a measure of how similar the fields are at these two space-time points, averaged over time.

* Crucially: **Angle brackets denote temporal average over an interval much much greater than Δt_c** (Δt_c), the coherence time of the source. This averaging is essential for statistically stationary fields, which is often assumed for partially coherent light. The averaging time needs to be long enough to capture the representative statistical behavior of the field fluctuations. If the field is perfectly coherent ($\Delta t_c \to \infty$), then the averaging may not be strictly necessary as the product $E_1(t+\tau)\,E_2^*(t)$ would be deterministic.

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Continuing with the Mutual Coherence Function, $\Gamma_{12}(\tau)$:

* A key property is that it **Encodes simultaneous spatial and temporal correlations.** Let's see how: * **Spatial correlation:** This is encoded because E_1 and E_2 refer to the fields at two generally different spatial points, S_1 and S_2. If S_1 is different from S_2, $\Gamma_{12}(\tau)$ tells us about the relationship between fields at these distinct locations. ***Temporal correlation:*** *This is encoded by the time delay* τ. $\Gamma_{12}(\tau)$ measures how the field at S_1 is related to the field at S_2 after a time delay τ has been introduced in one of them*.

Special cases help illustrate this:

If $S_1 = S_2$ (so $E_1 = E_2 = E$), then \(\Gamma_{11}(\tau) = \langle E(t+\tau)E^(t) \rangle\). This is called the **auto-coherence function** (or temporal coherence function) and it describes the temporal coherence of the field at a single point. Its behavior with τ directly relates to the coherence time Δt_{c}.

If $\tau = 0$, then \(\Gamma_{12}(0) = \langle E_1(t)E_2^(t) \rangle\). This is called the **mutual intensity** and it describes the spatial coherence between points S_1 and S_2 at the same instant in time.

So, the mutual coherence function $\Gamma_{12}(\tau)$ is a very powerful and general tool, capturing both aspects of coherence in a single mathematical object.

The two hyphens on the slide just indicate the end of the text for this point.

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Slide 20: Normalized Degree of Coherence

Now we introduce **Slide 20: Normalized Degree of Coherence.** While the mutual coherence function $\Gamma_{12}(\tau)$ contains all the first-order coherence information, its magnitude depends on the intensities of the fields. It's often more convenient to work with a normalized quantity.

* First, let's define the **Self-coherence functions at each aperture (or point** S_1 and S_2), evaluated at zero time delay ($\tau = 0$): * **Capital Gamma sub 1 1 of zero (**$\Gamma_{11}(0)$) equals the angle brackets of the magnitude of $E_1(t)$, squared ($\langle |E_1(t)|^2 \rangle$). This $\Gamma_{11}(0)$ is proportional to the average intensity, I_1, of the light at point S_1.

* **Capital Gamma sub 2 2 of zero (**$\Gamma_{22}(0)$) equals the angle brackets of the magnitude of $E_2(t)$, squared ($\langle |E_2(t)|^2 \rangle$). This $\Gamma_{22}(0)$ is proportional to the average intensity, I_2, of the light at point S_2.

* Now, we can **Define the normalized complex degree of coherence, little gamma sub 1 2 of tau (**$\gamma_{12}(\tau)$), as shown in the box: $\gamma_{12}(\tau)$ equals Capital

Gamma sub 1 2 of tau ($\Gamma_{12}(\tau)$) divided by the square root of (Capital Gamma sub 1 1 of zero, times Capital Gamma sub 2 2 of zero). $\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}}$.

This $\gamma_{12}(\tau)$ is a dimensionless complex number. * The numerator, $\Gamma_{12}(\tau)$, is the mutual coherence function we just defined. *The denominator,* $\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}$, is the geometric mean of the intensities (or quantities proportional to them) at the two points. This normalization removes the dependence on the absolute intensities I_1 and I_2, making $\gamma_{12}(\tau)$ a measure of the degree* of correlation, ranging from 0 to 1 in magnitude.

* We will now look at the **Properties** of this normalized complex degree of coherence.

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Here are the key **Properties** of the normalized complex degree of coherence, $\gamma_{12}(\tau)$:

* **The magnitude of little gamma sub 1 2 (**$|\gamma_{12}|$) is less than or equal to 1 ($\leq$ 1). This is a consequence of the Cauchy-Schwarz inequality applied to the definition of $\Gamma_{12}(\tau)$ and the intensities. This means $|\gamma_{12}|$ is bounded, typically varying between 0 and 1.

* **If the magnitude** $|\gamma_{12}|$ equals 1, this signifies perfect coherence between the fields $\mathrm{E}_1(t+\tau)$ and $\mathrm{E}_2(t)$. The two fields are perfectly correlated (or anti-correlated if the phase of γ_{12} is π).

* **If the magnitude** $|\gamma_{12}|$ equals 0, this signifies complete incoherence. There is no statistical correlation whatsoever between the fields $\mathrm{E}_1(t+\tau)$ and $\mathrm{E}_2(t)$.

* The **Phase of** γ_{12} (the argument of the complex number $\gamma_{12}(\tau)$) equals the average effective phase difference between the fields $\mathrm{E}_1(t+\tau)$ and $\mathrm{E}_2(t)$.

This phase information is crucial for determining the position of interference fringes.

So, $\gamma_{12}(\tau)$ is a very convenient quantity: its magnitude tells us "how much" coherence there is, and its phase tells us "what the phase relationship" is.

The three hyphens indicate the end of this list on the slide. This $\gamma_{12}(\tau)$ will appear directly in the interference law for partially coherent light.

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Let's now look at the **Irradiance at an Observation Point** P when fields from two sources or apertures interfere. This will give us the general **Interference Law** for partially coherent light, as shown on **Slide 21**.

First, we write the **Superposed field at point** P at time t. Let's say light from two points (or slits) S_1 and S_2 propagates to P.

$$E(P,t) = k_1\, E_1\left(t - \frac{r_1}{c}\right) + k_2\, E_2\left(t - \frac{r_2}{c}\right).$$

Let's break this down:

$E_1\left(t - \frac{r_1}{c}\right)$ is the field that was at S_1 at an earlier time $\left(t - \frac{r_1}{c}\right)$, now arriving at P. Here r_1 is the distance from S_1 to P.

$E_2\left(t - \frac{r_2}{c}\right)$ is the field that was at S_2 at an earlier time $\left(t - \frac{r_2}{c}\right)$, now arriving at P. Here r_2 is the distance from S_2 to P.

The terms k_1 and k_2 are **complex transfer factors.** They are generally dimensionless and account for things like the efficiency of transmission from

S_1 to P (e.g., slit area, diffraction effects, attenuation) and any phase shifts introduced by the propagation path itself, beyond the simple $\left(t - \frac{r}{c}\right)$ retardation.

Next, we need the **Time-averaged irradiance** at point P. Irradiance I_{P} is proportional to the time average of the squared magnitude of the total electric field. In SI units, this proportionality involves $\varepsilon_0 c$ (epsilon naught times c). So,

$$I_{\text{P}} = \varepsilon_0 c \, \langle |E(P,t)|^2 \rangle.$$

The explicit formula for this irradiance in terms of the coherence functions is on the next page.

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Here's the result for the time-averaged irradiance I_{P} at point P:

* $I_{\text{P}} = \varepsilon_0 c \langle |E(P,t)|^2 \rangle = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{Re}[\gamma_{12}(\tau)]$ ($I_{\text{P}} = \varepsilon_0 c \langle |E(P,t)|^2 \rangle = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\text{Re}[\gamma_{12}(\tau)]$).

Let's carefully define the terms in this crucial formula:

* I_{P}: The time-averaged irradiance observed at point P. * I_1: The time-averaged irradiance that would be observed at P if only source/slit 1 were open (i.e., if k_2 were zero). $I_1 = \varepsilon_0 c \langle |k_1 \, E_1(t - r_1/c)|^2 \rangle$. * I_2: The time-averaged irradiance that would be observed at P if only source/slit 2 were open (i.e., if k_1 were zero). $I_2 = \varepsilon_0 c \langle |k_2 \, E_2(t - r_2/c)|^2 \rangle$. τ (tau): This is the crucial time delay. It is $\frac{r_2}{c} - \frac{r_1}{c}$ if γ_{12} is defined for fields $E_1(t)$ and $E_2(t)$ at the sources S_1 and S_2. *Or, if E_1 and E_2 in γ_{12} are fields at P from path 1 and path 2 respectively without the other*, then τ is the relative delay introduced by the paths.

More precisely, if $E_{S1}(t')$ and $E_{S2}(t')$ are fields at the sources, then $E_1(t - r_1/c)$ and $E_2(t - r_2/c)$ arrive at P. The cross term involves $\langle E_{S1}(t - r_1/c) E_{S2}^*(t - r_2/c) \rangle$. Let $t' = t - r_2/c$. Then this becomes $\langle E_{S1}\left(t' +\right.$

$\frac{r_2-r_1}{c}\Big) E_{S2}^*(t')\rangle$. So, $\tau = \frac{r_2-r_1}{c}$, the difference in propagation times from S_1 and S_2 to P. * $\gamma_{12}(\tau)$: This is the normalized complex degree of coherence between the field from S_1 and the field from S_2, evaluated for the time difference τ. * $\text{Re}[\gamma_{12}(\tau)]$: The real part of this complex degree of coherence.

* This equation is rightfully called the **Master interference formula for partially coherent light.** It's incredibly general. * The first two terms, $I_1 + I_2$, represent the sum of intensities you'd get if there were no interference (e.g., if the light were completely incoherent, $\gamma_{12} = 0$). * The third term, $2\sqrt{I_1 I_2}\,\text{Re}[\gamma_{12}(\tau)]$, is the **interference term.** Its magnitude and sign depend directly on the degree of coherence γ_{12} and the time delay τ.

* Let's consider the **Visibility when** $I_1 = I_2$. Let $I_1 = I_2 = I_0$. Then

$$I_{\text{P}} = 2\,I_0 + 2\,I_0\,\text{Re}[\gamma_{12}(\tau)] = 2\,I_0(1 + \text{Re}[\gamma_{12}(\tau)]).$$

The maximum intensity I_{max} occurs when $\text{Re}[\gamma_{12}(\tau)]$ is maximal. Since $\gamma_{12}(\tau) = |\gamma_{12}(\tau)|e^{i\alpha(\tau)}$ (where α is the phase of γ_{12}), $\text{Re}[\gamma_{12}(\tau)] = |\gamma_{12}(\tau)|\cos\big(\alpha(\tau)\big)$. So

$$I_{\text{max}} = 2\,I_0(1 + |\gamma_{12}(\tau)|) \quad \text{(assuming } \alpha \text{ can be such that } \cos(\alpha) = 1\text{, by adjusting overall path).}$$

And

$$I_{\text{min}} = 2\,I_0(1 - |\gamma_{12}(\tau)|) \quad \text{(assuming } \cos(\alpha) = -1 \text{ can be achieved).}$$

The visibility

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

becomes

$$V = \frac{2\,I_0(1 + |\gamma_{12}(\tau)|) - 2\,I_0(1 - |\gamma_{12}(\tau)|)}{2\,I_0(1 + |\gamma_{12}(\tau)|) + 2\,I_0(1 - |\gamma_{12}(\tau)|)} = \frac{4\,I_0|\gamma_{12}(\tau)|}{4\,I_0} = |\gamma_{12}(\tau)|.$$

So, as the slide states: V equals the magnitude of little gamma sub 1 2 of tau ($|\gamma_{12}(\tau)|$).

* **Thus, the fringe contrast (visibility** V) directly measures the magnitude of the degree of coherence, $|\gamma_{12}(\tau)|$, when the interfering beams have equal intensities. This provides a direct experimental way to measure $|\gamma_{12}|$.

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Slide 22:

Let's apply this formalism to an **Example – The Michelson Interferometer Revisited**, on **Slide 22**.

* Assume we have **Equal amplitudes in both arms** of the Michelson. Let the complex field incident on the beam splitter be $E(t) = E_0 e^{i\phi(t)}$. E_0 is a constant real amplitude. $\phi(t)$ is the phase, which may fluctuate for a partially coherent source.

* We are interested in the **Degree of temporal coherence.** In a Michelson, the beam is split, and one part is delayed by $\tau = \Delta s/c$ relative to the other before they are recombined. So we are correlating the field $E(t)$ with $E(t-\tau)$ (or $E(t+\tau)$ with $E(t)$). This is described by the normalized auto-coherence function, which we can denote as $\gamma_{11}(\tau)$.

$\gamma_{11}(\tau)$ equals the angle brackets of $e^{i[\phi(t+\tau)-\phi(t)]}$.

This arises because

$$\Gamma_{11}(\tau) = \langle E(t+\tau)E^*(t)\rangle = \langle E_0 e^{i\phi(t+\tau)}\, E_0 e^{-i\phi(t)}\rangle = E_0^2 \langle e^{i[\phi(t+\tau)-\phi(t)]}\rangle,$$

and

$$\Gamma_{11}(0) = \langle |E(t)|^2\rangle = E_0^2.$$

So, $\gamma_{11}(\tau) = \frac{\Gamma_{11}(\tau)}{\Gamma_{11}(0)}$ gives the expression on the slide.

* Now, consider the case **If the source is strictly monochromatic.** This means the frequency ω is perfectly defined, so the phase $\phi(t) = \omega t + \phi_0$ (where ϕ_0 is a constant initial phase). Then the phase difference $\phi(t+\tau) - \phi(t) = [\omega(t+\tau) + \phi_0] - [\omega t + \phi_0] = \omega\tau$. So, $\gamma_{11}(\tau) = \langle e^{i\omega\tau} \rangle$. Since $\omega\tau$ is a constant for a given τ, the average is just $e^{i\omega\tau}$.

$\gamma_{11}(\tau)$ equals $e^{i\omega\tau}$.

And consequently, **the magnitude of gamma sub one one,** $|\gamma_{11}|$, equals 1. ($|e^{i\omega\tau}| = 1$).

* Therefore, for a strictly monochromatic source, the **Visibility** $V = |\gamma_{11}(\tau)| = 1$, and this is independent of the time delay τ. You get perfect fringes no matter how large the path difference in the Michelson, which makes sense because a perfectly monochromatic wave has infinite coherence time and length.

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Continuing with the Michelson interferometer example:

* Now consider the opposite extreme: a **Broadband source, where the time delay** τ is much greater than $\frac{1}{\Delta\omega}$ ($\tau \gg \frac{1}{\Delta\omega}$).

Recall that $\Delta t_c \approx \frac{1}{\Delta\omega}$ is the coherence time of the source. So, this condition means $\tau \gg \Delta t_c$; the introduced time delay is much larger than the source's coherence time.

In this situation, the **phases randomize.** The term inside the expectation for $\gamma_{11}(\tau)$ was $e^{i[\phi(t+\tau)-\phi(t)]}$. If τ is much larger than the coherence time, then the phase $\phi(t+\tau)$ has no correlation with the phase $\phi(t)$. Their difference, $[\phi(t+\tau) - \phi(t)]$, will fluctuate randomly over many multiples of 2π as we average over 't'. When you average $e^{i(\text{a random phase that's uniformly distributed})}$, the result is 0.

Therefore, **gamma sub one one (γ_{11}**) approaches zero ($\rightarrow 0$).

Since the visibility $V = |\gamma_{11}(\tau)|$, this means V also approaches zero. The interference fringes disappear when the path difference in the Michelson (which determines τ) significantly exceeds the coherence length of the broadband source. This is exactly what we discussed earlier in a more qualitative way, and now we see it emerge from the formalism of the coherence function.

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This slide presents a graph showing the **Magnitude of Temporal Coherence, absolute value of** $\gamma_{11}(\tau)$ ($|\gamma_{11}(\tau)|$), versus Time Delay τ. This visually summarizes what happens in a Michelson interferometer.

Let's analyze the graph:

- The **horizontal axis is Time Delay (**τ), with points 0, τ (generic), 2τ, 3τ marked. This τ on the axis label might represent a characteristic coherence time, Δt_{c}.

- The **vertical axis is the magnitude of the temporal coherence,** $|\gamma(\tau)|$ (specifically $|\gamma_{11}(\tau)|$), ranging from 0.0 to 1.0. Values 0.5 and $1/e$ (which is about 0.368) are marked.

Two curves are shown:

1. The **blue dashed line represents a Monochromatic source (**$\Delta\omega = 0$). For such a source, $|\gamma_{11}(\tau)| = 1$ for all τ. So, this is a horizontal line at a height of 1.0. This means perfect coherence and constant visibility of 1, regardless of the time delay.

2. The **red solid curve represents a source with Finite Bandwidth (**$\Delta\omega >$ 0). • At $\tau = 0$ (zero time delay), $|\gamma_{11}(0)| = 1$. This means the wave is perfectly correlated with itself at the same instant. • As τ increases,

$|\gamma_{11}(\tau)|$ decreases from 1. The curve shows a decaying behavior, often exponential for sources with certain spectral shapes (e.g., a Lorentzian spectrum gives an exponential decay of γ_{11}). • The annotation on the curve "$|\gamma(\tau)| \sim e$" seems incomplete. It likely intends to show an exponential decay, like $|\gamma(\tau)| \sim e^{-\tau/\tau_c}$, where τ_c is the coherence time. The point where the curve drops to $1/e$ of its initial value (i.e., to 0.368) occurs at $\tau = \tau_c$. The x-axis labeling of τ, 2τ, 3τ might be in units of this coherence time.

This graph vividly illustrates that for any real source with finite spectral bandwidth, the temporal coherence (and thus fringe visibility in a Michelson) diminishes as the time delay (or path difference) increases. The rate of this decay is determined by the coherence time τ_c, which in turn is inversely related to the bandwidth $\Delta\omega$.

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Slide 23: Example – Young's Double-Slit Spatial Coherence.

Now let's consider another example: We'll apply the coherence formalism here.

* Assume we have a **Plane quasi-monochromatic wave** incident on the double slits. The field is given by $E = E_0 e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$. $E = E_0 e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$.

* "Plane wave" implies that the incident wavefront is perfectly spatially coherent. That is, at a given time t, the phase is the same for all points $\mathbf{r}$ lying on a plane perpendicular to $\mathbf{k}$.

* "Quasi-monochromatic" means the light has a narrow spectral bandwidth $\Delta\omega$ around the central frequency ω, so its temporal coherence is good. We are primarily interested in spatial effects here.

* Light from this plane wave passes through two slits, S_1 and S_2, and then travels to an observation screen. Let r_1 be the distance from S_1 to a point P on

the screen, and r_2 be the distance from S_2 to P. The **Optical path difference,** $\Delta r = r_2 - r_1$, between the paths from the two slits to point P **induces a phase difference.** This phase difference at P is $\phi_{12} = k\Delta r$. * $k = \frac{2\pi}{\lambda}$ is the wave number. *This* ϕ_{12} is the phase difference that arises purely from the geometry of propagation after *the slits, assuming the fields* at the slits themselves* were in phase due to the incident plane wave.

*The **Irradiance pattern** observed on the screen will then depend on this phase difference* ϕ_{12} and the degree of spatial coherence of the illumination at the slits*, which for an incident plane wave is perfect.

The formula for the irradiance pattern is on the next page.

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Continuing with the Young's double-slit example:

The irradiance pattern I_{P} at a point P on the screen is given by:

$$I_{\text{P}} = 2\, I_0[1 + |\gamma_{12}(0)|\cos(k\Delta r)]$$

($I_{\text{P}} = 2\, I_0[1 + |\gamma_{12}(0)|\cos(k\Delta r)]$).

Let's break this down using our master interference formula:

$$I_{\text{P}} = I_1 + I_2 + 2\sqrt{I_1\, I_2}\, \text{Re}[\gamma_{12}(\tau)]$$

* Here, I_0 is the irradiance from a single slit if the other were closed (assuming $I_1 = I_2 = I_0$).

$\gamma_{12}(0)$ is the normalized complex degree of spatial *coherence between the fields at slit* S_1 and slit S_2 at the same instant* (hence $\tau = 0$ in γ_{12}).

* $k\Delta r$ is the phase difference introduced by the path difference $\Delta r = r_2 - r_1$ from the slits to the point P on the screen.

* The formula effectively assumes that the phase of $\gamma_{12}(0)$ itself is zero (or has been absorbed into the definition of Δr). If $\gamma_{12}(0) = |\gamma_{12}(0)|e^{i\alpha}$, then the term would be $|\gamma_{12}(0)|\cos(k\Delta r - \alpha)$. For simplicity, if the illumination of the slits is symmetric, α might be zero.

So, the interference term is modulated by $|\gamma_{12}(0)|$, the degree of spatial coherence between the light at the two slits.

* **If the magnitude of gamma sub 1 2 of zero,** $|\gamma_{12}(0)|$, equals 1, this implies perfect spatial coherence between the slits. In this case, we get high-contrast fringes, as the interference term becomes $2\,I_0\cos(k\Delta r)$, leading to I_{P} varying between 0 (if $\cos = -1$) and $4\,I_0$ (if $\cos = +1$), giving visibility $V = 1$. This would be the case for our assumed incident plane wave.

* However, if the source illuminating the slits is not a perfect plane wave but an extended source, then $|\gamma_{12}(0)|$ might be less than 1.

Specifically, **Increasing the slit separation** d (which is implicit in the definition of S_1 and S_2) reduces $|\gamma_{12}(0)|$ according to the spatial coherence criterion we discussed earlier (e.g., related to $\frac{bd}{r} < \lambda$, where b is source width, d is slit separation, r is source-to-slit distance).

As d increases, the light at the two slits becomes less spatially correlated, $|\gamma_{12}(0)|$ decreases, and the visibility of the interference fringes diminishes. This is how Young's experiment probes spatial coherence.

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This slide shows a graph of **Fringe Visibility vs. Slit Separation** for a Young's Double-Slit Experiment, Illustrating Spatial Coherence.

Let's analyze the graph:

- The **horizontal axis is Slit Separation (**d) in arbitrary units. It's marked with 0, 2.5, L_{c}, 7.5, and $2\,L_{\mathrm{c}}$. Here, L_{c} represents a characteristic **transverse**

coherence length – it's the slit separation at which the visibility of fringes drops significantly (often to zero for the first time). This L_c is determined by the source size b, wavelength λ, and source-to-slit distance r ($L_c \approx \frac{\lambda r}{b}$).

- The **vertical axis is labeled "Visibility (V), Degree of Coherence** $|\gamma_{12}(d, \tau = 0)|$". It ranges from 0.0 to 1.0. This confirms that for equal intensity slits, $V = |\gamma_{12}(0)|$, where $\gamma_{12}(0)$ now depends on slit separation d.

- The **blue solid curve labeled "Theoretical** $|\gamma_{12}(d, \tau = 0)|$": • When the slit separation $d = 0$ (conceptually), the coherence is perfect, $|\gamma_{12}| = 1$.

  - As d increases, $|\gamma_{12}|$ decreases. For a uniformly illuminated incoherent slit source of width b, the theoretical form of $|\gamma_{12}(d)|$ is a sinc function: $\left|\frac{\sin\left(\frac{\pi b d}{\lambda r}\right)}{\frac{\pi b d}{\lambda r}}\right|$. This function has its first zero when $\frac{\pi b d}{\lambda r} = \pi$, which means $d = \frac{\lambda r}{b}$. This value of d corresponds to L_c on the graph.

  - The graph shows this main lobe of the sinc-like function, decreasing from 1 at $d = 0$ to 0 at $d = L_c$.

  - It also shows a small secondary lobe (sidelobe of the sinc function) where visibility becomes slightly positive again before decaying. Real sources might have smoother profiles, leading to a more Gaussian-like decay of $|\gamma_{12}|$.

- Red dots labeled "Experimental Visibility (V)": These points generally follow the theoretical curve, showing that the fringe visibility measured in an experiment indeed drops as the slit separation d increases, confirming the loss of spatial coherence over larger distances.

This graph powerfully demonstrates how increasing the distance d between the points being probed (the slits) leads to a decrease in the spatial coherence of the light arriving from an extended source, and consequently, a reduction in interference fringe visibility.

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Let's look at **Slide 24: Measuring** γ_{12} Experimentally. How can we actually determine this complex degree of coherence from measurements?

* The general approach involves recording **three intensities at an observation point** P (e.g., a point on the screen in Young's experiment, or the output of a Michelson).

These intensities are:

1. **Capital** I (I): This is the intensity measured when **both apertures (or paths) are open** and light from both contributes to the field at P. This I corresponds to

$$I_{\mathrm{P}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

from our master interference formula.

2. **Capital** I_1 (I_1): This is the intensity measured when **only aperture** S_1 (or path 1) is open, and S_2 is blocked. This directly gives us the I_1 term.

3. **Capital** I_2 (I_2): This is the intensity measured when **only aperture** S_2 (or path 2) is open, and S_1 is blocked. This directly gives us the I_2 term.

* Once we have these three measured intensities (I, I_1, and I_2), we can **Compute the real part of the degree of coherence,** $\mathrm{Re}[\gamma_{12}(\tau)]$, using the formula on the next page.

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Continuing with measuring γ_{12} experimentally:

- The formula to compute the real part of $\gamma_{12}(\tau)$ from the measured intensities I, I_1, and I_2 is:

Real part of $\gamma_{12}(\tau)$ equals $\frac{I-I_1-I_2}{2\sqrt{I_1 I_2}}$. (Re[$\gamma_{12}(\tau)$] = $\frac{I-I_1-I_2}{2\sqrt{I_1 I_2}}$). This formula comes directly from rearranging our master interference law:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

Solving for Re[$\gamma_{12}(\tau)$] gives the expression above. So, by making these three simple intensity measurements, we can determine the real part of the complex degree of coherence.

- A special case we've encountered: **When** $I_1 = I_2$ (the intensities from the two paths are equal), the visibility V simplifies to: $V = \frac{I_{\text{max}}-I_{\text{min}}}{I_{\text{max}}+I_{\text{min}}}$, which equals the magnitude of γ_{12} ($|\gamma_{12}|$). In this case, measuring I_{max} (by varying τ or position to find a bright fringe peak) and I_{min} (a dark fringe minimum) directly gives us the *magnitude* of γ_{12}. This doesn't give the phase of γ_{12}, however.

- To get the full information, including the phase: **The full complex** $\gamma_{12}(\tau)$ is recoverable via additional phase-shifting techniques. These techniques involve introducing known, controlled phase shifts into one of the interfering beams (e.g., by precisely moving a mirror in one arm of an interferometer, or using an electro-optic modulator). By recording the intensity I for several (typically 3, 4, or 5) different known phase shifts, one can solve a system of equations to extract both the magnitude $|\gamma_{12}(\tau)|$ and the phase of $\gamma_{12}(\tau)$. This is common in phase-shifting interferometry.

The three hyphens mark the end of the text on this slide.

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Now, let's bring this all together and consider the **Slide 25: Practical Consequences for Laser Spectroscopy.** Why have we spent so much time on coherence? Because it's what makes lasers such extraordinary tools for spectroscopy.

The **High temporal coherence of single-mode lasers enables** several critical capabilities:

Precise heterodyne frequency measurements.

Heterodyning involves mixing two waves of slightly different frequencies to produce a beat frequency signal at their difference frequency. For this to work effectively and for the beat frequency to be stable and accurately measurable, both original waves must have very stable phases over the measurement period. This means they need high temporal coherence (long coherence times, narrow linewidths). Lasers provide this, allowing for extremely precise frequency comparisons and measurements.

High-resolution Doppler-free spectroscopy. Examples include **two-photon spectroscopy** and **saturation spectroscopy.**

Many atomic and molecular transitions are broadened by the Doppler effect in gaseous samples. Doppler-free techniques are designed to overcome this limitation and resolve the true, underlying narrow spectral features. These techniques inherently rely on the laser having a linewidth (which is inversely related to coherence time) that is much narrower than the Doppler width, and often narrower than the natural linewidth of the transition being probed. High temporal coherence is an absolute prerequisite. For example, in saturation spectroscopy, a strong pump beam and a weak probe beam interact with the same atoms; their ability to do so in a frequency-selective way depends on the laser's narrow linewidth.

Similarly, the **High spatial coherence of lasers enables:**

Tight focusing to diffraction-limited spots.

A laser beam, especially one in a fundamental transverse mode like TEM_{00}, has a very uniform and well-defined wavefront. Such a spatially coherent beam can be focused by a lens down to a very small spot, ideally limited only by diffraction (the spot size being on the order of the wavelength). This ability to concentrate light into a tiny volume achieves very high irradiance (power per unit area), which is essential for many spectroscopic techniques, including nonlinear spectroscopy, Raman microscopy, and material processing. Light from an incoherent source, with its jumbled wavefronts, cannot be focused so tightly.

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Continuing with the practical consequences for laser spectroscopy:

(High spatial coherence also enables:) **Long-baseline interferometry & holography.**

Long-baseline interferometry: Whether in astronomy (using separated telescopes to synthesize a larger aperture) or in laboratory settings for precision measurements, maintaining phase coherence across the extended baseline is crucial. Lasers, with their excellent spatial coherence, are ideal sources or references for such applications.

Holography: This technique records and reconstructs wavefronts. It relies on the interference between a wave scattered from an object and a coherent reference wave (usually from the same laser). The formation of a stable, high-contrast interference pattern (the hologram) over the entire recording medium requires both high spatial and temporal coherence of the light source.

A very practical point: **Understanding coherence boundaries is critical when mixing laser light with incoherent backgrounds or when designing interferometers with finite arm differences.**

In many experiments, a laser signal might be accompanied by incoherent background light (e.g., stray room light, fluorescence from the sample at different wavelengths, or thermal emission). Knowing the coherence properties helps in distinguishing or filtering the desired coherent laser signal from the incoherent background.

When designing an interferometer (like a Michelson or Mach-Zehnder), it's essential to know the coherence length Δs_{c} of your laser source. If the path difference between the arms of the interferometer exceeds Δs_{c}, you will lose fringe visibility. Therefore, arms must be matched to within the coherence length for effective interference. This is especially true for lasers that might not be perfectly single-mode or might have some residual bandwidth.

These considerations are paramount for successful experimental design and data interpretation in laser spectroscopy.

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This slide presents a very useful diagram summarizing the **Impact of Laser Coherence Properties on Spectroscopy Techniques.** It's a concept map showing how the characteristics of temporal and spatial coherence lead to various applications.

Let's trace the paths:

Starting from the top left: **High Temporal Coherence**

* Its **Characteristics** are listed as: * Spectral Purity (Narrow $\Delta\omega$) * Long Coherence Time (Δt_c) * Long Coherence Length (Δs_c) * This enables several techniques: * Directly: **Precise Heterodyne Frequency Measurements.** * Also: **High-Resolution Doppler-Free Spectroscopy.** This then branches into specific examples like **Two-Photon Spectroscopy** and **Saturation Spectroscopy.** * Another application is **Coherent LIDAR (e.g., Doppler LIDAR).** (LIDAR stands for Light Detection and Ranging). Doppler LIDAR relies on the coherence of the backscattered light to measure frequency shifts and thus velocities.

Now, from the top right: **High Spatial Coherence**

* Its **Characteristics** are: * Wavefront Uniformity * High Beam Quality * Enables Tight Focusing

* This enables: * Directly: **Tight Focusing to Diffraction-Limited Spots.** This, in turn, is crucial for techniques like **Confocal Raman Spectroscopy** (which requires tight focusing for spatial resolution and signal enhancement). * Also: **Holography.** *And **Long-Baseline Interferometry.** There's an asterisk here with a note: "*Also requires high temporal coherence*." This is true; for fringes to be stable over long path differences, both are needed. *Another application is **Optical Coherence Tomography (OCT).** A double asterisk notes: "High spatial coherence for beam quality/focusing; temporal coherence needs depend on OCT type*." OCT often uses broadband (low temporal coherence) sources to achieve high axial resolution, but good spatial coherence is still needed for beam delivery and collection.

Finally, both the temporal and spatial coherence branches converge at the bottom to the **Overall Impact:** High coherence **Enables Precise Diagnostics, High-Resolution Spectroscopy, & Advanced Imaging.**

This diagram provides an excellent overview of why coherence is not just an abstract physical property but a cornerstone of modern optical science and technology, particularly in the field of laser spectroscopy.

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We're nearing the end of our discussion on coherence. **Slide 26 provides a Summary & Key Formulae to Remember.** These are the essential takeaways you should have a firm grasp of.

1. **Temporal coherence length,** Δs_{c} (Δs_{c}):

$$\Delta s_{\mathrm{c}} = \frac{c}{\Delta\omega}$$

($\Delta s_{\mathrm{c}} = \frac{c}{\Delta\omega}$.)

This relates the spatial extent of temporal coherence to the speed of light c and the angular frequency bandwidth $\Delta\omega$ of the source. Remember, a smaller bandwidth means a longer coherence length. Variants exist with $\Delta\nu$ (linear frequency bandwidth), sometimes involving a 2π factor depending on precise definitions.

2. **Spatial coherence condition between two points on a wavefront:**

$$A_{\mathrm{s}}\, d\Omega \leq \lambda^2$$

This is the compact and general form. A_{s} is the source area, $d\Omega$ is the solid angle over which coherence is considered (or into which light is collected), and λ is the wavelength. This tells us that the product $A_s d\Omega$ (related to étendue) must be on the order of λ^2 for spatial coherence.

An alternative, more direct form for Young's slits (source width b, slit separation d, source-to-slit distance r) is

$$\frac{bd}{r} < \lambda$$

3. **Coherence volume,** V_c (V_c):

The formula is on the next page.

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Continuing with the Summary & Key Formulae:

The formula for **Coherence volume,** V_c (V_c), is:

$$V_c = \frac{\lambda^2 r^2 c}{\Delta\omega A_s}$$

($V_c = \frac{\lambda^2 r^2 c}{\Delta\omega A_s}$).

This coherence volume $V_c = S_c \Delta s_c$, where S_c is the coherence area ($\approx \frac{\lambda^2 r^2}{A_s}$) and Δs_c is the coherence length ($\approx \frac{c}{\Delta\omega}$). It represents the region of space-time where the field maintains fixed phase correlations and corresponds to a single electromagnetic mode or phase-space cell (of volume h^3).

4. **Degree of coherence (normalized correlation), little gamma sub 1 2 of tau (**$\gamma_{12}(\tau)$):

$\gamma_{12}(\tau)$ equals the angle brackets of (E_1 of (t plus tau) times E_2^* of (t)) divided by the square root of (angle brackets of $|E_1|^2$ times angle brackets of $|E_2|^2$).

$$\gamma_{12}(\tau) = \frac{\langle E_1(t+\tau) E_2^*(t) \rangle}{\sqrt{\langle |E_1|^2 \rangle \langle |E_2|^2 \rangle}}$$

Where E_1 and E_2 are the complex fields at points 1 and 2 respectively. This normalized complex degree of coherence ranges from 0 (incoherent) to 1 (perfectly coherent) in magnitude, and its phase gives the average phase difference.

5. **Fringe visibility V (capital Vee) is related to the magnitude of gamma sub 1 2 (**$|\gamma_{12}|$) for equal intensities.

Specifically, if the intensities of the two interfering beams are equal, then $V = |\gamma_{12}(\tau)|$. This provides a direct experimental measure of the magnitude of the degree of coherence.

The slide concludes with excellent advice:

Master these relations to analyze and design any optical interference experiment in laser spectroscopy.

Indeed, a solid understanding of coherence and these quantitative relationships is indispensable for anyone working seriously with lasers and spectroscopic techniques that rely on interference. It allows you to predict behavior, optimize setups, and correctly interpret experimental results.

This concludes our detailed look into the coherence properties of radiation fields. Thank you.