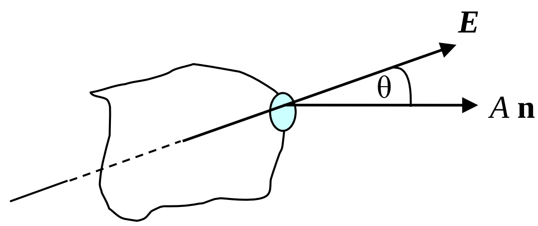
# GAUSS' LAW

**Introduction**: The electric field of a given charge distribution can in principle be calculated using Coulomb's law. The examples discussed in electric field showed however, that the actual calculations can become quit complicated.

**The electric flux (**to flow**)** ($\Phi$, $[\Phi]$ = N.m²/C) is represented by the number of electric field lines that penetrate a surface. If the electric field, ***E***, is uniform and makes an angle $\theta$ with the normal to the surface area *A*, the electric flux through the surface is




$$\Phi = \boldsymbol{E}.\boldsymbol{A} = EA\cos\theta$$

In general case

$$\Phi = \oiint_{\text{surface}} \boldsymbol{E}.d\boldsymbol{A}$$

**Ex.-** A uniform electric field ***E*** = a **i** + b **j** intersects a surface of area A. Calculate the flux through this area if the surface lies:

a) in the XZ-plane. [Answer: $\Phi_{XZ}$ = (a **i** + b **j**).(A **j**) = bA.]
b) in the YZ-plane. [Answer: $\Phi_{YZ}$= (a **i** + b **j**).(A **i**) = aA.]
c) in the XY-plane. [Answer: $\Phi_{XY}$= (a **i** + b **j**).(A **j**) = 0.]

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**Gauss' law** is a very powerful theorem which relates any charge distribution, having a high degree of symmetry, to the resulting electric field at any point in the vicinity of the charge. It states that "*the net electric flux, $\Phi_c$ , through any closed surface* is equal to the net charge inside the surface, $q_{in}$, divided by $\varepsilon_o$*". In symbols

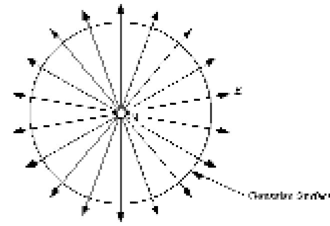
$$\Phi = \oiint_{\text{surface}} \boldsymbol{E}.d\boldsymbol{A} = \frac{q_{in}}{\varepsilon_o}, \qquad \varepsilon_o \approx 9\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2},$$

*usually called Gaussian surface, which has the exact symmetry as the charge distribution.

If the charge has symmetry, such as spherical, cylindrical, etc. one can use

$$\Phi = \oiint_{\text{surface}} \boldsymbol{E}.d\boldsymbol{A} = EA = \begin{cases} E\times 4\pi r^2 & \text{Spherical case} \\ E\times 4\pi rh & \text{Cylinderical case} \end{cases}$$

**Example 1: Field of point charge.** Calculate the electric field due to a point charge $q$.



The field generated by a point charge q is spherical symmetric, and its magnitude will depend only on the distance r from the point charge. The direction of the field is along the direction (see Figure 24.2). Consider a spherical surface centered around the point charge q (see Figure). The direction of the electric field at any point on its surface is perpendicular to the surface and its magnitude is constant. Using Gauss's law we obtain the following expression

$$\Phi = \oint_{\text{surface}} \boldsymbol{E}.d\boldsymbol{A} = E\,(4\pi r^2) = \frac{q}{\varepsilon_o} \quad \Rightarrow \quad \boxed{E = \frac{q}{4\pi\varepsilon_o r^2} = k\,\frac{q}{r^2}}$$

which is Coulomb's law.

For a continuous charge distribution, the electric field is given by

$$\boldsymbol{E} = k\sum_i \frac{q_i}{r_i^2}\boldsymbol{r}_i = k\int \frac{dq}{r^2}\boldsymbol{r}$$

and $dq$ could be calculated using the charge density as follows:

$$\overbrace{\text{Volume} \qquad \text{Surface} \qquad \text{Linear}}^{\text{charge density}}$$

$$\rho = \frac{dq}{dV} \qquad \sigma = \frac{dq}{dA} \qquad \lambda = \frac{dq}{dl}$$

**Ex. 2-** Calculate the electric field due to an insulating sphere of radius R. The sphere has a uniform charge density ρ and total charge $Q$, where $\rho = \dfrac{Q}{\frac{4}{3}\pi r^3}$

$$E = \frac{q_{in}}{r^2} = \begin{cases} k\,\dfrac{\rho(\frac{4}{3}\pi r^3)}{r^2} = k\,\dfrac{Q}{R^3}r & r < R \\[2ex] k\,\dfrac{\rho(\frac{4}{3}\pi R^3)}{r^2} = k\,\dfrac{Q}{r^2} & r > R \end{cases}$$

**Ex. 3-** Calculate the electric field due to a nonconducting plane sheet of charge Q and has a surface area $A$ , $\sigma = \dfrac{Q}{A}$.

$$\Phi = \oiint_{\text{surface}} \boldsymbol{E.dA} = E(2A) = \frac{Q}{\varepsilon_o} \quad \Rightarrow \quad E = \frac{\sigma}{2\varepsilon_o}$$

**Conductors and Insulators:**

1. **Conductors** are material in which electric charges move quite freely. Conductors are essentially metals, e.g. copper, aluminum, they contain **free** electrons which move freely in the conductor and can transfer the electric charge.
2. **Insulators** are materials that do not readily transport charge, e.g. glass, rubber, wood. Their electrons are tightly bound to their atoms and can not move freely.

**Properties of a conductor in electrostatic equilibrium:**

1. The excess charge resides entirely on its surface.
2. The electric field is zero everywhere inside it.
3. The electric field just outside it is perpendicular to its surface and has a magnitude ($\frac{\sigma}{\varepsilon_o}$), where σ is the charge per unit area at that point.
4. On an irregularly shaped conductor, the concentration of charge on it is greatest where the surface is most sharply curved (radius of curvature of the surface is the smallest).

**EXAMPLES**

**Example 1**. The electric field everywhere on the surface of a hollow sphere of radius $r$ =0.7 m is measured to be equal to $9.80\times10^2$ (N/C) and points radially toward the center of the sphere.

a- What is the net charge within the sphere's surface? [ $\boxed{E = k\,\frac{Q}{r^2}}$ ]

Since the electric field points radially inward towards the center of the sphere, then one can expect a **negative charge inside**. Using Gauss' law, we can have

**Ans**: $E = k\,\frac{-Q}{r^2} \quad \Rightarrow \quad Q = -\frac{(9.8\times10^2)(0.7)^2}{(9.0\times10^9)} = \underline{-5.3\times10^{-8}\text{ C}}$

b- What can you conclude about the nature and distribution of the charge inside the sphere?

Negative charge has spherically symmetric distribution.

**Example 2**.The electric field everywhere on the surface of a hollow sphere of radius r =11 cm is measured to be equal to $3.8\times10^4$ N/C and points radially inward towards the center of the sphere. How much charge is enclosed by this surface?

$$\Phi = EA = E(4\pi r^2) = 3.8\times10^4\times(4\pi)\times(0.11)^2 = \underline{5.78\times10^3\ \frac{\text{N.m}^2}{\text{C}}}$$

then

$$Q_{in} = -\varepsilon_o \Phi = \underline{-5.11\times10^{-8}\ \text{C}}$$

It is a **negative charge inside** because the electric field points radially inward towards the center of the sphere.

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Question 1

A uniform electric field $\vec{E} = \left(a\ \hat{\text{i}} + b\ \hat{\text{j}}\right)$N/C intersects a surface of area A. If the surface of the area A lies in YZ-plane, the flux through the area will be:

(a) b×A.
(b) Zero.
(c)@ a×A.
(d) a×b.
(e) A×A.

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Question 2

When a piece of paper is held with one face perpendicular to a uniform electric field, the electric flux is 48 N×m$^2$/C. When the plane of the paper makes 30 degrees with the direction of the electric field the electric flux through it is:

(a)@ 24 N×m$^2$/C
(b) 44 N×m$^2$/C
(c) 21 N×m$^2$/C
(d) 48 N×m$^2$/C
(e) 32 N×m$^2$/C

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Question 3

Calculate the electric flux (phi) through the curved surface of a cone of base radius R and height h. The electric field E is uniform and perpendicular to the base of the cone, and the field lines enter through the base. The cone has no charge enclosed in it, as seen in figure (2).

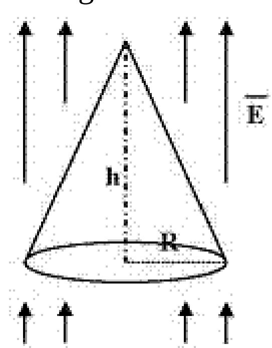


Figure 2

(a) π×R×h×E.
(b) -2×π ×R×E.
(c) 2× π ×R×E.

(d) $-\pi \times (R^2) \times E$.

(e)@ $\pi \times (R^2) \times E$.

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Question 4

Two concentric shells, one with radius R and the other with radius 2R, surround an isolated point charge. The ratio of the number of field lines through the larger shell to the number of field lines through the smaller is:

(a) 1/4.

(b)@ 1.

(c) 1/2.

(d) 4.

(e) 2.

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Question 5

For the electric field:

$$\vec{E} = \left(24\,\hat{i} + 30\,\hat{j} + 16\,\hat{k}\right)\text{N/C}$$

where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the unit vectors in the directions of x, y, and z, respectively, the electric flux through a 2.0 $m^2$ portion of the yz-plane is:

(a) 60 $N\times m^2/C$.

(b) 92 $N\times m^2/C$.

(c)@ 48 $N\times m^2/C$.

(d) 32 $N\times m^2/C$.

(e) 80 $N\times m^2/C$.

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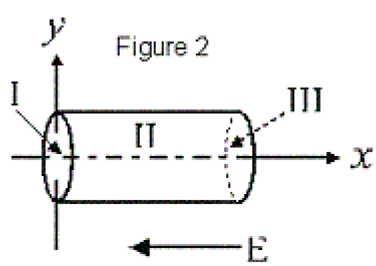
Question 6 0.51-34%

A closed cylinder whose main axis is along the x-axis is shown in figure 2. It is placed in a uniform electric field of magnitude 200 N/C pointing in the negative x-axis. The cylinder has a cross sectional area of 12.5 $cm^2$ and a length of 6.0 cm. The fluxes through faces I, II and III are respectively:




(a) zero , 0.25 , zero $N\cdot m^2/C$

(b) zero , -0.25 , zero $N\cdot m^2/C$

(c) - 0.25 , zero , 0.25 $N\cdot m^2/C$

(d) - 0.25 , 0.25 , - 0.25 $N\cdot m^2/C$

(e)@ 0.25 , zero , - 0.25 $N\cdot m^2/C$

$\varphi_I = +EA = 200\times 12.5$

$\varphi_{II} = 0,\ \text{since } E \perp A$

$\varphi_{III} = -EA = -200\times 12.5\times 10^{-4} = -0.25\ N\cdot m^2/C$

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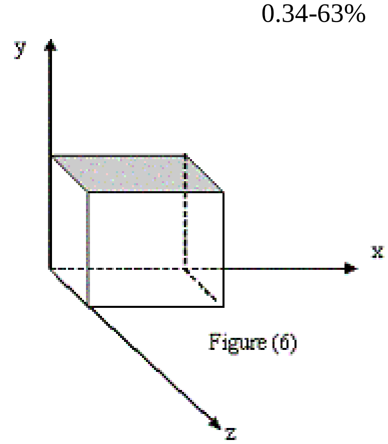
Question 7 0.34-63%

A cube, as in figure (6), has an edge length of 3.00 m in a region of a uniform electric field given by the equation: $\vec{E} = \left(-5\,\hat{j} + 6\,\hat{k}\right)\text{N/C}$,

where i, j, and k are the unit vectors in the directions of x, y, and z respectively. Find the electric flux through the top face (shaded).




$$\varphi = \vec{E}.\vec{A} = \left(-5\,\hat{j} + 6\,\hat{k}\right).9\,\hat{j} = -5\times 9 = \underline{-45\ N\cdot m^2/C}$$

(a) Zero.

(b) 45 $N\times m^2/C$.

(c) - 30 N×m$^2$/C.
(d) 30 N×m$^2$/C.
(e)@ - 45 N×m$^2$/C.

24-3 flux of an Electric field

Question 8
An infinitely long line has a charge density of 7.6 nano-C/m. Calculate the electric flux through a spherical surface of radius R = 7.7 cm whose center, C, lies on the line charge as shown in Figure 3.

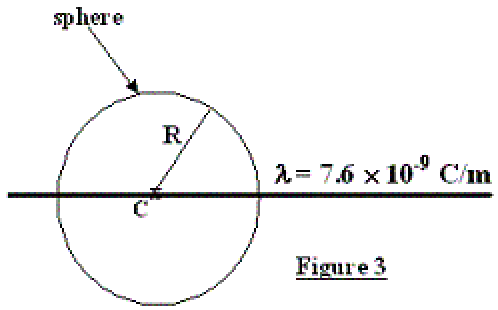


Figure 3

(a) zero
(b)@ 132 N×m$^2$/C
(c) 92.0 N×m$^2$/C
(d) 415 N×m$^2$/C
(e) 610 N×m$^2$/C

Question 9
A total charge of 5.00×10(-6) C is uniformly distributed inside an irregular insulator. The volume of the insulator is 2.50 m××3. Now, imagine a cube of volume 0.50 m××3 inside the insulator. What is the total electric flux through the surface of the cube?
(a) 4.53×105 N×m$^2$/C.
(b) Zero.
(c)@ 1.13×105 N×m$^2$/C.
(d) 8.10×105 N×m$^2$/C.
(e) 2.51×106 N×m$^2$/C.

Question 10
A point charge of 2.0 micro-C is placed at the center of a cube 50 cm on edge. What is the flux through the bottom surface?
(a) 1.7×104 N×m$^2$/C.
(b) -2.8×104 N× m$^2$/C.
(c) -5.6×104 N× m$^2$/C.
(d) 1.1×105 N× m$^2$/C.
(e)@ 3.8×104 N× m$^2$/C.

Question 11 0.50-43%
The cube in figure 3 has edge lengths of 2.00 m and is oriented as shown in a region in which a uniform electric field exists. The electric field is given by: $\vec{E} = \left(-5.00\ \hat{\mathrm{i}} + 8.00\ \hat{\mathrm{k}}\right)$ N/C, where i

and k are unit vectors parallel to the x-axis and z-axis respectively. Find the electric flux through the right face (shaded) of the box.

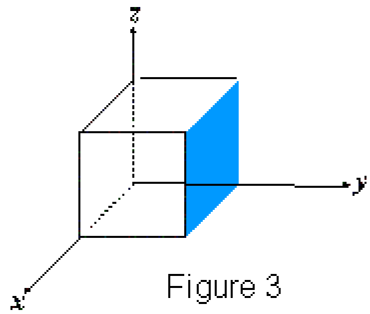



Figure 3

(a)@ zero
(b) +26.0 N× $m^2$/C
(c) -10.0 N× $m^2$/C
(d) +16.0 N× $m^2$/C
(e) +6.00 N× $m^2$/C

Question 12 0.39-74%

A spherical conducting shell has a radius of 20 cm. Point A is a distance of 30 cm from the center of the sphere. The electric field at point A is 500 N/C and is directed radially outward. An additional charge Q is introduced at the center of the shell. The electric field at point A decreases to 100 N/C. What is Q ?

(a) - 5 nano-Coulombs
(b) + 5 nano-Coulombs
(c)@ - 4 nano-Coulombs
(d) + 1 nano-Coulombs
(e) - 1 nano-Coulombs

major2-033-Q07 24-03

Question 13 0.58-47%

The electric field in the region of space shown in figure 3 is given by: E = (8.0 i + 2y j) (N/C), where y is in meters. What is the magnitude of the electric flux, in units of N×$m^2$/C, through the top face of the cube? (i and j are the unit vectors in the x and y directions, respectively)

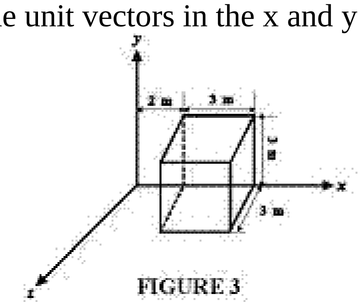



FIGURE 3

(a) 12
(b) 6.0
(c) 90
(d) 130
(e)@ 54

final-033-Q11 24-03

Question 14 0.51-63%

A charged conducting spherical shell with an outer radius of 2.0 m has a point charge of +3.0 micro-Coulomb at its center. The electric field at a distance of 3.0 from the center has a magnitude of 1.2×104 N/C and is radially outward. What is the charge on the outer surface of the shell?

(a)@ +12 micro-Coulomb
(b) -12 micro-Coulomb
(c) +6 micro-Coulomb
(d) -3 micro-Coulomb
(e) +3 micro-Coulomb

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final-042-Q09 24-03

<u>Question 15</u> 0.54-50%

If a rectangular area is turned in a uniform electric field from a position where the maximum electric flux goes through it to a position where only half the maximum flux goes through it, what is the turned angle?

(a) 45 degrees
(b) 30 degrees
(c) 90 degrees
(d) 23 degrees
(e)@ 60 degrees

| 24-4 Gauss' Law |
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major2-993-Q09 24-04

<u>Question 16</u> 0.41-45%

A point charge Q= 6 micro-C is placed at the center a rectangular box with dimensions a=b= 0.4 m and c= 0.6 m. Find the total electric flux through the surface of the box.

(a)@ 6.78×105 N×m2/C
(b) 10.9×104 N×m2/C
(c) 3.21×105 N×m2/C
(d) Zero
(e) 9.30×105 N×m2/C

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final-993-Q09 24-04

<u>Question 17</u> 0.14-30%

Charges q and Q are placed on the x axis at x= 0 and x= 2.0 m, respectively. If q= -40 pico-C and Q= +30 pico-C, determine the net electric flux through a spherical surface of radius 1.0 m centered on the origin.

(a) -8.5 N×m2/C
(b) -1.1 N×m2/C
(c)@ -4.5 N×m2/C
(d) -9.6 N×m2/C
(e) -6.8 N×m2/C

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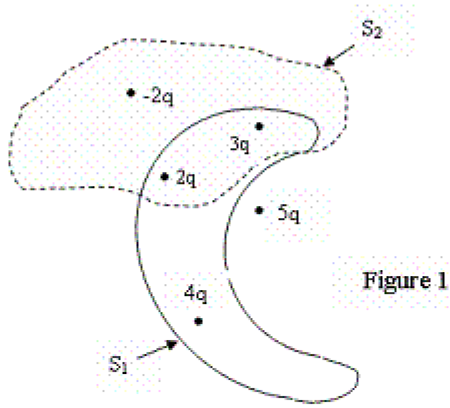
final-001-Q13 24-04

<u>Question 18</u>

In figure (1), what is the ratio of the electric flux that penetrates surface S1 to that penetrates surface S2? (Note that S1 and S2 are closed surfaces and q is a charge.)




(a) 2.
(b) 1.3.
(c) 0.
(d)@ 3.
(e) 1.0.

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major2-032-Q11 24-04

Question 19 0.44-48%

A point charge, q1 = -2.0×10(-6) C, is placed inside a cube of side 5.0 cm, and another point charge q2 = 3.0×10(-6) C is placed outside the cube. Find the net electric flux through the surfaces of the cube.

(a) +3.4×105 N m××2/C
(b)@ -2.3×105 N m××2/C
(c) 2.3×105 N m××2/C
(d) 1.1×107 N m××2/C
(e) -1.1×105 N m××2/C

$$\varphi_{cube} = \frac{q_1}{\varepsilon_o} = \left(\frac{-2.0\times10^{-6}}{9\times10^{-12}}\right) = \underline{-2.3\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C}}$$

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major2-032-Q14 24-04

Question 20 0.37-71%

A total charge of $5.0\times10^{-6}$ C is uniformly distributed inside an irregularly-shaped insulator. The volume of the insulator is 3.0 $m^3$. Now, imagine a cube of volume 0.5 $m^3$ inside the insulator. What is the total electric flux through the surfaces of the cube?

(a) $2.5\times10^3$ N×$m^2$/C.
(b) Zero.
(c) $4.5\times10^5$ N×$m^2$/C.
(d) $8.1\times10^5$ N×$m^2$/C.
(e)@ $9.4\times10^4$ N×$m^2$/C.

$$\rho = \frac{Q}{V} \Rightarrow q_{in} = \rho V_{in} = \left(\frac{5.0\times10^{-6}}{3}\right)\times0.5 = 1.67\times10^{-6}\ \text{C},$$

$$\varphi = \frac{q_{in}}{\varepsilon_o} = \frac{1.67\times10^{-6}}{8.85\times10^{-12}} = 9.4\times10^{4}\,\text{N}\cdot\text{m}^2/\text{C}$$

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major2-033-Q08 24-04

Question 21 0.09-83%

Charge is uniformly distributed over the entire xy plane with a surface charge density of 20 micro-C/$m^2$. A sphere has a radius of 1.0 m, and is centered at the origin. What is the net electric flux through the surface of the sphere?

(a) $2.8\times10^{7}\ \mathrm{N\cdot m^2/C}$
(b) $1.4\times10^{7}\ \mathrm{N\cdot m^2/C}$
(c) zero
(d)@ $7.1\times10^{6}\ \mathrm{N\cdot m^2/C}$
(e) $2.2\times10^{6}\ \mathrm{N\cdot m^2/C}$

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major2-041-Q04 24-04

Question 22 0.35-74%

An imaginary closed spherical surface S of radius R is centered on the origin. A positive charge is originally at the origin, and the flux through the surface is Phi. The positive charge is slowly moved from the origin to a point 2×R away from the origin. In doing so the flux through S

(a) decreases to Phi/4.
(b) remains the same Phi.
(c) increases to 2×Phi.
(d)@ decreases to zero.
(e) increases to 4×Phi.

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major2-041-Q06 24-04

Question 23 0.30-40%

Figure 1 shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicates the directions (in $\mathrm{N\times m^2/C}$)of the flux through the six sides of each cube. In which situations does the cube enclose, a positive net charge, a negative net charges and zero net charge? respectively.

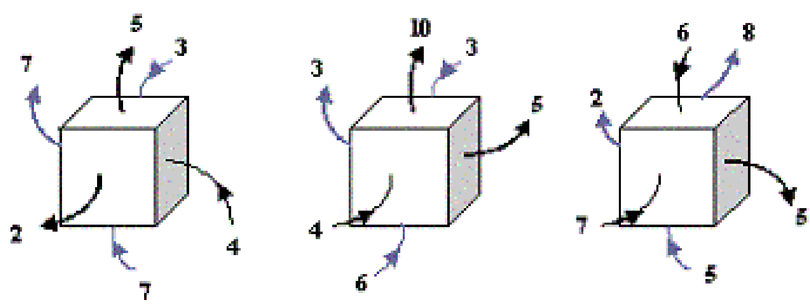

Figure 1

(a) 1,3 and 2.
(b) 2,1 and 3.
(c)@ 2,3 and 1.
(d) 3,2 and 1.
(e) 1,2 and 3.

$(2): -13+18=5$

$(1): 14-14=0$

$(3): -18+15=-2$

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major2-042-Q08 24-04

Question 24 0.40-29%

The net electric flux passing through a closed surface is -4.00×102 N×m2/C. What is net electric charge contained inside the surface if the surface is a cylinder of height 3.52 cm and radius 1.12 cm.

(a)@ -3.54×10(-9) C.
(b) 1.00×10(-2) C.
(c) -1.00×10(-2) C.
(d) zero.

(e) 3.54×10(-9) C.

## 24-5 Gauss' Law and Coulomb's Law

final-961-Q11 24-05

Question 25

The electric field everywhere on the surface of a hollow sphere of radius 11 cm is measured to be equal 3.8×104 N/C and points radially inward towards the center of the sphere. How much charge is enclosed by this surface?

(a) 3.7×10-8 C.
(b)@ -5.1×10-8 C.
(c) -3.3×10-8 C.
(d) -3.7×10-8 C.
(e) 5.1×10-8 C.

major2-042-Q09 24-05

Question 26 0.49-54%

A positive point charge q sits at the center of a hollow spherical shell. The shell, with radius R and negligible thickness, has net charge -2q. The electric field strength outside the spherical shell (at r>R) will be:

(a) $3k\times q/r^2$ radially inwards.
(b)@ $k\times q/r^2$ radially inwards.
(c) $3k\times q/r^2$ radially outwards.
(d) $k\times q/r^2$ radially outwards.
(e) zero.

## 24-6 A Charged Isolated Conductor

major2-992-Q11 24-06

Question 27

A spherical conducting shell of inner radius r1 and outer radius r2 has a net charge of 2 $\mu$C. If a point charge of -4.0 $\mu$C is placed at the geometrical center of the spherical shell, what is the charge on the outer surface of the spherical shell?

(a) -4.0 $\mu$C
(b) +4.0 $\mu$C
(c)@ -2.0 $\mu$C
(d) zero
(e) +2. $\mu$C

major2-001-Q07 24-06

Question 28

An isolated conducting spherical shell has an inner radius of 4.0 cm and outer radius of 5.0 cm. A charge $8.0\times10^{-6}$ C is puted on the shell. What is the ratio of the charge on the inner surface of the shell to the charge on the outer surface?

(a) 7/10.
(b) 5/4.
(c) 8/5.
(d)@ Zero.

(e) 1.

major2-011-Q11 24-06

Question 29 0.49-43%

A point charge of $q_{center} = -50e$ lies at the center of a hollow spherical metal shell that has a net charge of $q_{net} = -100e$, as seen in figure (4). Calculate the charge on the (a) shell's inner surface, and (b) on its outer surface. [e is the magnitude of the charge on the electron.]

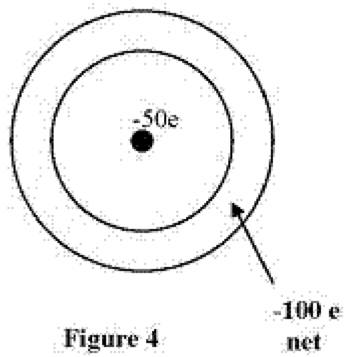



Figure 4

$$q_{inner} = -q_{center} = 50e,$$
$$q_{out} = q_{inner} + q_{net} = -50e + (-100e) = -150e$$

(a)@ (a) 50e (b) -150e.
(b) (a) 50e (b) -100e.
(c) (a) -50e (b) -100e.
(d) (a) Zero (b) -150e.
(e) (a) -50e (b) 150e.

major2-031-Q11 24-06

Question 30 0.29-78%

A point charge of +4.0 micro-C lies at the center of a hollow spherical conducting shell that has a net charge of -13.0 micro-C. If the inner radius of the shell is 2.0 cm and the outer radius is 3.0 cm, then the ratio between the charge density on the inner surface to the charge density on the outer surface is:

(a) 4 : 1.
(b) 1 : 2.
(c) -1 : 1.
(d) -1 : 2.
(e)@ 1 : 1.

major2-033-Q10 24-06

Question 31 0.10-67%

A +20 micro-Coulomb point charge is at the center of a conducting spherical shell that has an outer radius of 1.0 m and an inner radius of 0.50 m. The net charge of the spherical shell is zero. What is the surface charge density on the outer surface of the shell?

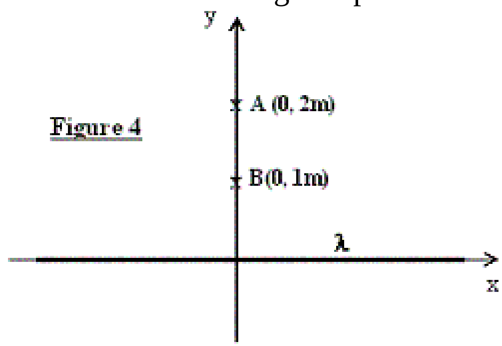
(a) +6.4 $\mu$C/m$^2$
(b) -6.4 $\mu$C/m$^2$
(c)@ +1.6 $\mu$C/m$^2$
(d) -1.6 $\mu$C/m$^2$
(e) zero

## 24-7 Applying Gauss' Law: Cylindrical Symmetry

major2-992-Q10 24-07

Question 32

Consider an infinitely long line of charge density 2.0 micro-C/m lying along the x-axis as shown in Figure 4. What is the ratio of electric field strength at point A to that at point B?




(a) 0.25
(b) 1.00
(c) 2.00
(d)@ 0.50
(e) 4.00

---

final-011-Q10 24-07

Question 33 0.48-42%

An infinite line of charge produces an electric field of 6.0×104 N/C at a perpendicular distance of 2.5 m from its axis. Calculate the linear charge density.

(a) 6.3×10(-6) C/m.
(b) 7.0×10(-6) C/m.
(c)@ 8.3×10(-6) C/m.
(d) 9.5×10(-6) C/m.
(e) 5.0×10)-6) C/m.

---

major2-021-Q11 24-07

Question 34 0.53-54%

Two long, charged, concentric cylindrical shells have radii 3.0 and 6.0 cm. The charge per unit length is -2.00×10(-6) C/m on the inner cylinder and +5.00×10(-6) C/m on the outer cylinder. Find the electric field at r = 4.0 cm, where r is the radial distance from the common central axis.

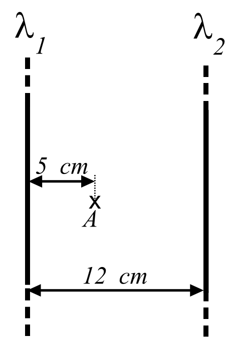
(a) 9.00×105 N/C radially outward
(b)@ 9.00×105 N/C radially inward
(c) 13.5×105 N/C radially inward
(d) 22.5×105 N/C radially inward
(e) 13.5×105 N/C radially outward

---

major2-022-Q07 24-07

Question 35 0.47-45%

Figure 3 shows two infinitely long rods carrying uniform linear charge densities $\lambda_1$ and $\lambda_2$. If the net electric field at point A is zero, then the ratio $\lambda_2 / \lambda_1$ is:




(a)@ 1.4
(b) 1.7
(c) 0.71
(d) 2.7
(e) 2.3

The two electric fields must be opposite at A, then

$$E_A = 2k\frac{\lambda_1}{r_1} - 2k\frac{\lambda_2}{r_2} = 0 \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{r_2}{r_1} = \frac{7}{5} = \underline{1.4}$$

---

major2-033-Q09 24-07

Question 36 0.35-51%

Figure 1 shows two infinitely long lines of charge with uniform linear charges densities: lambda1 = -9.00 nano-Coulomb/m and lambda2 = +3.00 nano-Coulomb/m. The separation between the two lines of charge is d = 1.00 m. What is the net electric field at point P?

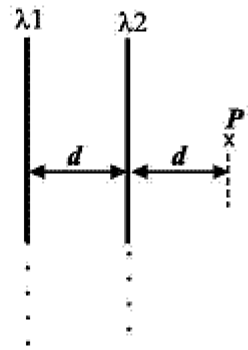



FIGURE 1

(a)@ 27 N/C to the left
(b) 72 N/C to the left
(c) 27 N/C to the right
(d) 135 N/C to the left
(e) 135 N/C to the right

---

major2-041-Q07 24-07

Question 37 0.52-35%

In figure 2, the magnitude of the electric field at point A, due to an infinite line charge density of $9.0\times10^{-6}$ C/m, is $7.2\times10^{4}$ N/C. If the point A is at a distance R from the line charge, what is R?

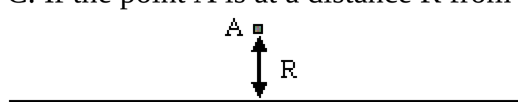



Figure (2)

(a) 1.2 m.

$$E_A = 2k\frac{\lambda}{R} \Rightarrow R = 2k\frac{\lambda}{E_A} = 2\times9.0\times10^{9}\left(\frac{9.0\times10^{-6}}{7.2\times10^{4}}\right)$$

$$= \underline{2.3\ \text{m}}$$

(b) 0.3 m.
(c) 3.4 m.
(d) 25 m.
(e)@ 2.3 m.

major2-042-Q06 24-07

Question 38 0.56-61%

A very long uniform line of charge having a linear charge density of 6.8 micro-C/m lies along x-axis. A second line of charge has a linear charge density of -3.40 micro-C/m and is parallel to x-axis at y = 0.5 m. What is the net electric field at point where y= 0.25 m on y-axis?

(a) 7.3×102 N/C along -y-axis.
(b)@ 7.3×105 N/C along +y-axis.
(c) 4.8×104 N/C along -y-axis.
(d) 4.8×106 N/C along +y-axis.
(e) 3.4×106 N/C along +y-axis.

## 24-8 Applying Gauss' Law: Planar Symmetry

final-992-Q12 24-08

Question 39

Two infinite non-conducting parallel surfaces carry uniform charge densities of 0.20 nano-C/m××2 and -0.60 nano-C/m××2. What is the magnitude of the electric field at a point between the two surfaces?

(a) 17 N/C
(b) 34 N/C
(c) 90 N/C
(d)@ 45 N/C
(e) 23 N/C

major2-002-Q11 24-08

Question 40

Fig. 7 shows two parallel plates, infinite and non-conducting, with surface charge densities of 8.9×10(-4)C/m××2 and 8.9×10(-4)C/m××2. B, a ball with negligible mass, carries a positive charge of 6.0×10(-8) C and is attached to point A with a non-conducting string of length 10 cm. At equilibrium, the tension in the string is:

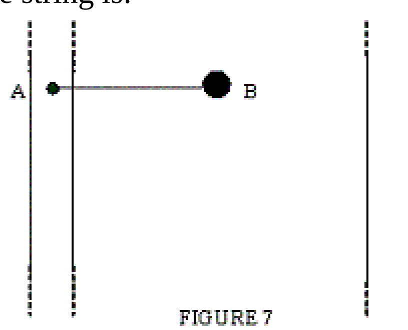



FIGURE 7

(a) 3.0 N.
(b) Zero.
(c)@ 6.0 N.
(d) 1.5 N.
(e) 0.3 N.

major2-011-Q10 24-08

Question 41 0.41-70%

As shown in figure (3), a small, non-conducting ball of mass m = 1.0×10(-6) kg and charge q = 2.0×10(-8) C, distributed uniformly through its volume, hangs from an insulating thread that makes an angle theta = 20 degrees with a vertical, uniformly charged non-conducting sheet (shown in cross section). Considering the weight of the ball and assuming that the sheet extends far vertically and into and out of the page, calculate the surface charge density of the sheet.

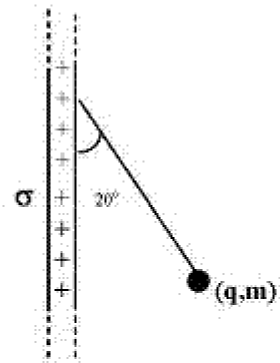



Figure 3

(a) 4.0×10(-9) C/m××2.
(b) 8.7×10(-9) C/m××2.
(c) 5.0×10(-9) C/m××2.
(d) 2.5×10(-9) C/m××2.
(e)@ 3.2×10(-9) C/m××2.

---

major2-021-Q10 24-08

Question 42 0.43-57%

Figure 4 shows cross-sections through two large, parallel non-conducting sheets with identical distributions of negative charge. The surface charge density for each sheet is 7.00×10(-15) C/m××2. What is the electric field at point A ?

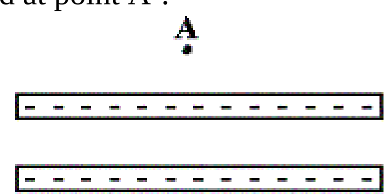



Figure 4

(a) 7.91×10(-4) N/C upward
(b) 3.96×10(-4) N/C downward
(c) 3.96×10(-4) N/C upward
(d)@ 7.91×10(-4) N/C downward
(e) 0

---

final-021-Q09 24-08

Question 43 0.36-36%

A small insulating sphere of mass m = 20.0×10(-9) kg and charge q = + 1.00 nano-Coulomb is hanging at equilibrium above a charged insulating sheet. What is the surface charge density of the sheet ?

(a) - 3.47 nano-Coulomb/m××2
(b) + 1.73 nano-Coulomb/m××2
(c)@ + 3.47 nano-Coulomb/m××2

(d) + 2.50 nano-Coulomb/m××2
(e) - 1.73 nano-Coulomb/m××2

---

major2-022-Q08 24-08

Question 44 0.42-35%

Figure 4 shows two large, parallel, non-conducting sheets, each with fixed uniform charge density: $\sigma_1 = +2.2\times10^{-6}$ C/m$^2$, $\sigma_2 = -4.3\times10^{-6}$ C/m$^2$. The ratio of the magnitude of the electric field at point A to that at point B is:

$$\vec{E}_A = \vec{E}_1 + \vec{E}_2 = \frac{1}{2\varepsilon_o}(|\sigma_1| + |\sigma_2|)\hat{i}$$

$$\vec{E}_B = \vec{E}_1 + \vec{E}_2 = \frac{1}{2\varepsilon_o}(|\sigma_2| - |\sigma_1|)\hat{i}$$

$$\frac{E_A}{E_B} = \frac{|\sigma_1| + |\sigma_2|}{|\sigma_2| - |\sigma_1|} = \frac{2.2 + 4.3}{4.3 - 2.2} = 3.1$$

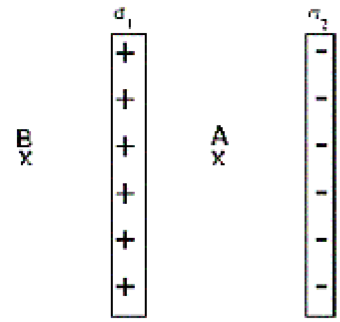


Figure 4

(a) 1.5
(b) 0.6
(c) 4.4
(d) 2.2
(e)@ 3.1

---

major2-031-Q10 24-08

Question 45 0.34-65%

For the two infinite dielectric sheets, see figure (5), find the magnitude of the electric field at a point P. Consider that each sheet has a positive surface charge density of 10××2 C/m××2.

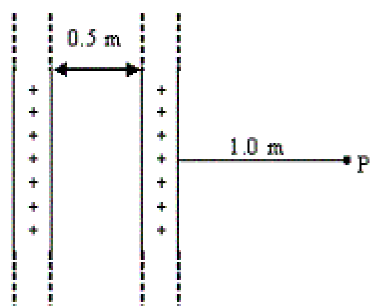


Figure (5)

(a) 2.2×1013 N/C.
(b) 0.5×1013 N/C.
(c)@ 1.1×1013 N/C.
(d) Zero.
(e) 1.7×1013 N/C.

---

major2-032-Q12 24-08

Question 46 0.41-62%

Figure 7 shows portions of two large, parallel, non-conducting sheets, A and B. The surface charge densities are: $\sigma_1$ = -4.5 μC/m$^2$ and $\sigma_2$ = -6.5 μC /m$^2$. Find the electric field at any point between the two sheets.

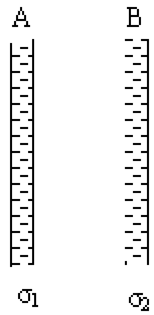


Figure 7

$$E = \frac{\sigma_1}{2\varepsilon_o} + \frac{\sigma_2}{2\varepsilon_o}, \text{ to the right because } \sigma_2 > \sigma_1$$

$$= \frac{(6.5-4.5)\times10^{-6}}{2\times9.0\times10^{-12}} = \underline{1.1\times10^{5}\ \text{N/C}} \text{ towards B}$$

(a) $1.4\times10^5$ N/C towards A.
(b)@ $1.1\times10^5$ N/C towards B.
(c) zero.
(d) $1.1\times10^5$ N/C towards A.
(e) $1.4\times10^5$ N/C towards B.

major2-032-Q15 24-08

Question 47 0.33-61%

A 40 N/C uniform electric field points perpendicularly toward a large neutral **conducting** sheet, as shown in figure 8. The surface charge densities (in C/m$^2$) on the right, sigma-R and left, sigma-L, respectively are:

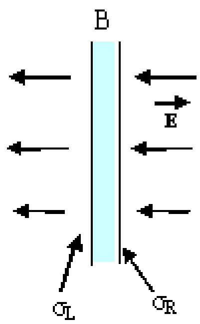


Figure 8

$$E = \frac{\sigma}{\varepsilon_o}$$

$$\Rightarrow \quad \sigma = \varepsilon_o E = 8.85\times10^{-12}\times40 = 3.5\times10^{-10}\ \text{C/m}^2,$$

So, the charges will be positive on the left side and negative on the right side

(a) $+7.0\times10^{-10}$ ; $-7.0\times10^{-10}$.
(b) $-7.0\times10^{-10}$ ; $+7.0\times10^{-10}$.
(c) $+3.5\times10^{-10}$ ; $-3.5\times10^{-10}$.
(d) zero ; zero.
(e)@ $-3.5\times10^{-10}$ ; $+3.5\times10^{-10}$.

final-033-Q10 24-08

Question 48 0.30-66%

Two large flat non-conducting sheets have equal but opposite surface charge densities. The distance between them is 2.0 cm. An electron released from rest from the negative plate strikes the positive plate after 15 nano-seconds. What is the magnitude of the surface charge density on each sheet?

(a) 3.0 nano-Coulomb/m××2
(b) 18 nano-Coulomb/m××2
(c) 7.5 nano-Coulomb/m××2
(d) 4.5 nano-Coulomb/m××2

(e)@ 9.0 nano-Coulomb/m××2

major2-042-Q10 24-08

Question 49 0.55-56%

A charged, isolated, large non-conducting plate is placed on the XY-plane. At 1.5 m from the plate, on Z-axis, the electric field measured was 10××4 N/C and directed into the plate. What is the charge density on the plate?

(a) 1.8×10(-7) C/m××2.
(b) 3.2×10(-7) C/m××2.
(c) -3.2×10(-7) C/m××2.
(d)@ -1.8×10(-7) C/m××2.
(e) zero.

24-9 Applying Gauss' Law: Spherical Symmetry

major2-991-Q10 24-09

Question 50 0.49-54%

Which one of the graphs shown in Figure 2 represents the variation of the magnitude of the electric field with the distance from the center of a solid charged conducting sphere of radius R in electrostatic equilibrium?

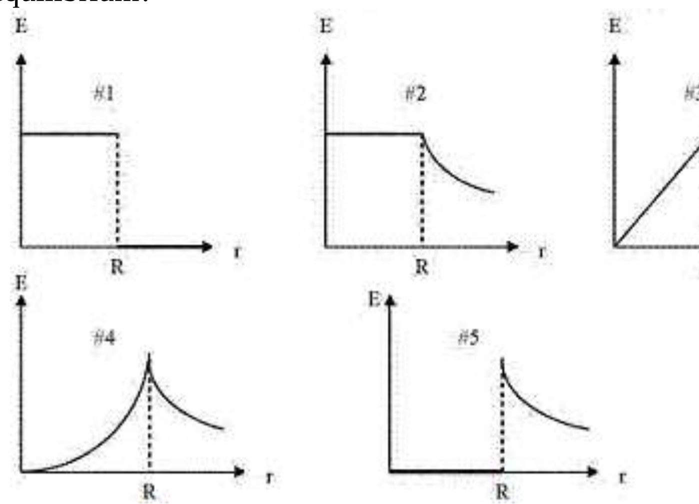



Figure 2

(a)@ #5
(b) #2
(c) #3
(d) #1
(e) #4

major2-993-Q11 24-09

Question 51 0.21-39%

Two conducting spheres are far apart. The smaller sphere carries a total charge of 4 micro-C, and the larger sphere carries a total charge of 2 micro-C. The larger sphere has a radius that is twice that of the smaller sphere. After the two spheres are connected by a thin conducting wire, the charges on the smaller and larger spheres, respectively, are:

(a) 3 micro-C and 3 micro-C
(b)@ 2 micro-C and 4 micro-C
(c) -2 micro-C and 8 micro-C
(d) 0 micro-C and 6 micro-C

(e) -4 micro-C and 10 micro-C

---

major2-001-Q08 24-09

Question 52

A solid insulating sphere has a charge of 20 micro-C uniformly distributed throughout its volume. The magnitude of the electric fields inside the sphere at r = 2 cm and outside the sphere at r = 10 cm, measured from the center of the sphere, are equal. Find the volume charge density of the sphere.

(a) 48 milli-C/m××3.
(b)@ 24 milli-C/m××3.
(c) 12 milli-C/m××3.
(d) 54 milli-C/m××3.
(e) 20 milli-C/m××3.

---

major2-031-Q08 24-09

Question 53 0.11-75%

A long noncoducting cylinder (radius 12.0 cm) has a charge of uniform density 5.0 nano-C/m××3 distributed through its column. Determin the magnitude of the electric field 5.0 cm from the axis of the cylinder. [See figure (3)].

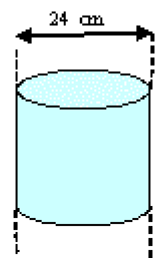



Figure (3)

(a)@ 14 N/C.
(b) 4 N/C.
(c) 22 N/C.
(d) 34 N/C.
(e) 31 N/C.

$$\Phi = \oint_{surface} \boldsymbol{E.dA} = \frac{q_{in}}{\varepsilon_o}, \quad q_{in} = \rho V_i = \rho\left(\pi r^2 h\right)$$

$$E\,(2\pi r h) = \frac{\rho\left(\pi r^2 h\right)}{\varepsilon_o} \Rightarrow E = \frac{\rho r}{2\varepsilon_o} = \frac{5\times10^{-9}\times5\times10^{-2}}{2\times9\times10^{-12}} \approx \underline{14\ \text{N/C}}.$$

---

final-031-Q12 24-09

Question 54 0.37-60%

A noncoducting shell has a uniform negative charge of magnitude 5.0×10(-5) C. Its inner and outer radii are 5.0 cm and 6.0 cm, respectively. The electric field at r = 3.0 cm, from the center, is:

(a) $4.5\times10^9$ N/C, inward.
(b) $1.5\times10^9$ N/C, outward.
(c) $4.5\times10^9$ N/C, inward.
(d) $1.5\times10^9$ N/C, inward.
(e)@ zero.

---

major2-032-Q13 24-09

Question 55 0.33-49%

A hollow metallic sphere, of radius 2.0 cm, is filled with a non-conducting material which carries a charge of $5.0\times10^{-12}$ C distributed uniformly throughout its volume. What is the magnitude of the electric field 1.5 cm from the center of the sphere?

(a) Zero.
(b) 17 N/C.
(c) 90 N/C.
(d)@ 84 N/C.
(e) 68 N/C.

$$E = k\,\frac{q}{R^3}\,r = 9.0\times10^9\left(\frac{5.0\times10^{-12}}{2.0\times10^{-2}}\right)1.5\times10^{-2}$$

$$= \underline{84\ \text{N/C}}$$

---

major2-041-Q08 24-09

Question 56 0.40-65%

A non conducting sphere, of radius 4.0 m, has a charge density of 2.0 micro-C/m$^3$. What is the electric field at a distance 1.7 m from the center?

(a) 4.8×103 N/C.
(b) 6.2×103 N/C.
(c)@ 1.3×105 N/C.
(d) 1.9×105 N/C.
(e) 2.5×105 N/C.

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \Rightarrow \frac{Q}{R^3} = \frac{4}{3}\pi\rho = \frac{4}{3}\pi \times 2.0\times10^{-6} = 8.4\times10^{-6}$$

$$E = k\left(\frac{q}{R^3}\right)r = 9.0\times10^9\left(8.4\times10^{-6}\right)1.7$$

$$= \underline{1.3\times10^5 \text{ N/C}}$$

major2-991-Q11 24-10

Question 57 0.40-58%

A solid insulating sphere has a charge of 20 micro-C uniformly distributed throughout its volume. The magnitude of the electric fields inside the sphere at r = 2 cm and outside the sphere at r = 10 cm, measured from the center of the sphere, are equal. Find the volume charge density of the sphere.

(a) 12 milli-C/m××3
(b)@ 24 milli-C/m××3
(c) 20 milli-C/m××3
(d) 54 milli-C/m××3
(e) 48 milli-C/m××3