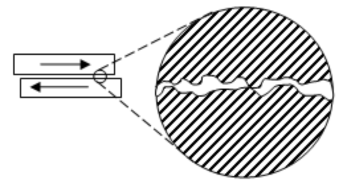
Chapter6
Force and Motion—I

6-1 FRICTION

Friction ... is very useful! We need friction to walk. Friction is not well understood. The amount of friction between two surfaces depends on difficult-to-characterize details of the surfaces, including microscopic roughness, cleanliness, and chemical composition.

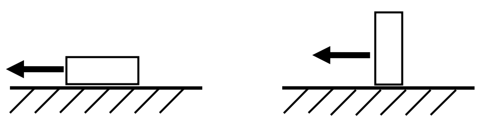


- Friction involves tearing and shearing between microscopically rough surfaces.

If two metal surfaces are <u>atomically</u> smooth and clean (almost impossible to achieve), they will <u>bond</u> on contact = "cold weld".

Empirical observations about friction:

- The magnitude of the force of friction f between 2 surfaces is proportional to the normal force N, not the area of contact, ($f \propto N$).



Pull a block of mass m along a surface. Regardless of orientation, you get the same normal force (N = mg), and you get the same frictional force f.

Why not $f \propto$ area of contact? Because more area $\Rightarrow$ less weight *per* area.

- Static friction is different than sliding friction (also called kinetic friction). The maximum magnitude of static friction force usually larger than the magnitude of kinetic friction force.

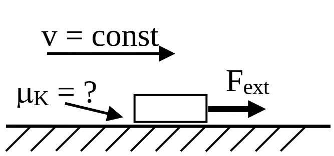
Kinetic Friction (also called sliding friction)

$$f = \mu_K N$$ (Not a law, just an empirical observation – usually, but not always, true)

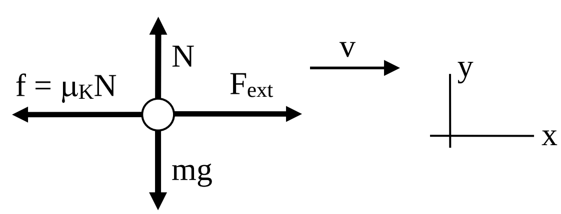
μ_K = coefficient of kinetic friction = dimensionless number $\mu_K > 0$

(μ_K can be greater than 1 but, usually, $\mu_K < 1$.)

Example: A block of mass m is being pushed along a rough horizontal table. One maintains a constant velocity v with a horizontal external force of magnitude F_{ext}. What is μ_K?



Free-body diagram: direction of frictional force *f* is always <u>opposite</u> to the motion:



velocity = constant $\Rightarrow$ $a = 0$ $\Rightarrow$ $F_{net} = 0$

$\Rightarrow$ In y-direction: $N = mg$ In x-direction: $F_{ext} = \mu_K N$ So, $F_{ext} = \mu_K mg$, or...

$$\mu_K = \frac{F_{ext}}{mg}$$

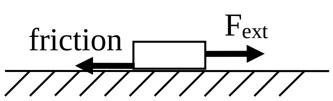
Static Friction

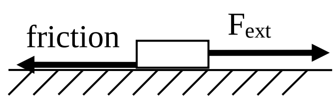
$$f_{static} < f_{static,\,max} = \mu_S N$$ (the maximum magnitude of the static friction force is $\mu_S N$)

μ_S = coefficient of static friction = dimensionless number $\mu_S > 0$

Usually, $\mu_S > \mu_K$ (maximum static friction is greater than kinetic friction)

Consider a book sitting on a table. You pull on a book with a small force F_{ext} to the right, but it doesn't move. There must be a frictional force to the left (otherwise the book would move).





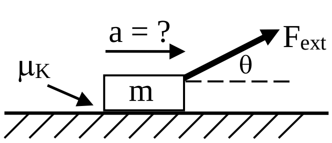
If you increase the external force and the book still does not move, the frictional force must have gotten bigger to match.

If you make the external force big enough, the book will suddenly start to move. Just before the book moved, the static friction was at its maximum value. So the magnitude of the static friction force can be anything between zero and a maximum value, given by $f_{max} = \mu_S N$. The book will remain stationary until $F_{ext} > f_{max} = \mu_S N$. Then the book will start to slide.

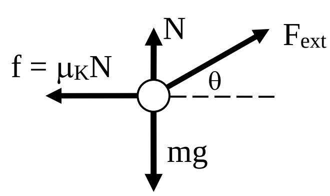
Usually, $\mu_S > \mu_K \Rightarrow$ large force is needed to start an object sliding, but then a smaller force is needed to keep it sliding. Anyone who has pushed a fridge across the kitchen floor knows this.

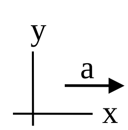
There is no good theory of friction $\Rightarrow$ μ's cannot be computed; instead, they are determined experimentally.

Example: A mass m on a flat table, with sliding friction coefficient μ_K, is pulled along the table by a force F_{ext} at angle θ. What is the (magnitude of the) acceleration *a*?



Steps 1, 2:





Step 1: Free-body diagram.

Step 2: Choose coordinate system.
(Make direction of acceleration = + direction)

Step 3:

Y-motion: $\sum F_y = m a_y$, $a_y = 0 \Rightarrow \sum F_y = 0 \Rightarrow \Sigma$ (forces up) = Σ (forces down) $\Rightarrow$

$N + F_{ext} \sin\theta = mg$, $N = mg - F_{ext} \sin\theta$ (Do you understand the sinθ here?)

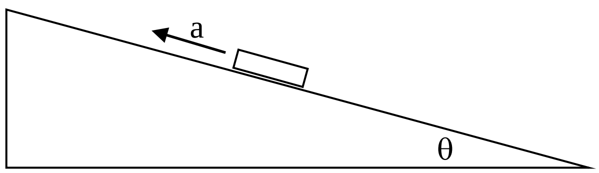
X-motion: $\sum F_x = m a_x$, $a_x = a \Rightarrow F_{ext} \cos\theta - \mu_K N = m a$

Now combine X and Y results:

$F_{ext} \cos\theta - \mu_K(mg - F_{ext} \sin\theta) = m a \Rightarrow$

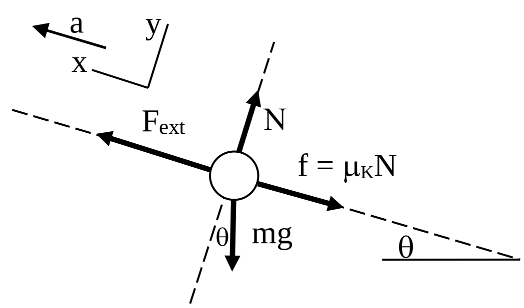
$$a = \underbrace{\frac{F_{ext}}{m}(\cos\theta + \mu_K \sin\theta) - \mu_K g}_{\text{notice that all terms have units of [a]}}$$

Example: Friction on an inclined plane



A mass m on an incline at angle θ, with kinetic friction coefficient μ_K. What size external force F_{ext} is required to maintain an acceleration of magnitude *a* **up** the incline?

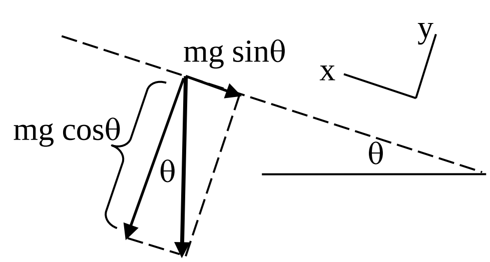
Step 1: Free-body diagram.



Step 2: Choose coordinate system.
(Make direction of acceleration = + direction)

Step 3: $\sum F_x = m a_x$, $\sum F_y = m a_y$

Notice:
x-component of weight = $-mg \sin\theta$
y-component of weight = $-mg \cos\theta$



Y: $a_y = 0 \Rightarrow +N - mg \cos\theta = 0$, $N = mg \cos\theta$

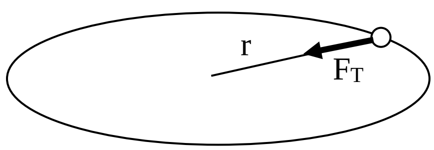
X: $a_x = a \Rightarrow +F_{ext} - \mu_K N - mg \sin\theta = m a$

Combine X and Y equations:
$+F_{ext} - \mu_K mg \cos\theta - mg \sin\theta = m a \Rightarrow$ $F_{ext} = ma + \mu_K mg \cos\theta + mg \sin\theta$

Forces and circular motion

NII: $\vec{F}_{net} = m\vec{a}$ $\Rightarrow$ To make something accelerate, we need a force in the same direction as the acceleration. $\Rightarrow$ Centripetal acceleration is always caused by centripetal force, a force toward center. "Centripetal" means "toward the center". "Centrifugal" means something totally different. Centrifugal forces don't exist! More on that below.

Example: Rock twirled on a string. (Assume no gravity)
Given: m = 0.1 kg , T (period) = 1 s , radius r = 1 m



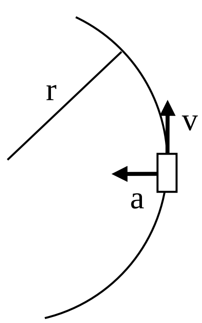
What is tension F_T in the string ? (Here, we use symbol F_T , since T already taken by period.)

F_T is the *only* force acting. (No such thing as "centrifugal force"!)

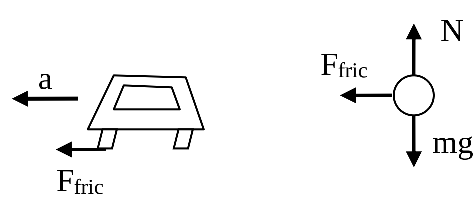
$$F_T = m\,a = m\frac{v^2}{r}, \quad v = \frac{2\pi\, r}{T}, \quad \Rightarrow$$

$$F_T = m\frac{(2\pi\, r/T)^2}{r} = 4\pi^2 m\frac{r}{T^2} = 4\pi^2(0.1)\frac{1}{1^2} = 3.9\ \text{N} \approx 1\ \text{pound}$$

Example: Rotation with friction. A car rounds a curve on a flat road (not banked). The radius of the circular curve is r = 100 m, and the speed of the car is v = 30 m/s (≅ 68 mph). How large a static friction coefficient (μ_S, not μ_K !) is needed for the car to not skid off the road?



Top view



View from rear of car

$F_{net} = F_{fric} = m\,a \quad \Rightarrow \quad \mu_S N = m v^2 / r$ (car about to skid $\Rightarrow$ $F_{fric} = \mu_S N$)

$N = mg \quad \Rightarrow \quad \mu_S\, m\, g = m\, v^2 / r$ (m's cancel) $\Rightarrow$

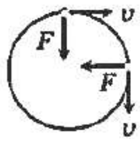
$$\mu_S = \frac{v^2}{g\,r} = \frac{(30\text{m/s})^2}{(9.8\text{m/s}^2)(100\text{m})} = 0.92 \ \text{(no units)}$$

So, need $\mu_S \geq 0.92$ or else car will skid. For rubber on dry asphalt $\mu_S \cong 1.0$, for rubber on wet asphalt $\mu_S \cong 0.7$. So car will skid if road is wet.

Chapter6
Force and Motion—II

6-3 UNIFORM CIRCULAR MOTION

In uniform circular motion only direction of velocity changes, speed remains constant. Force is always perpendicular to velocity. A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

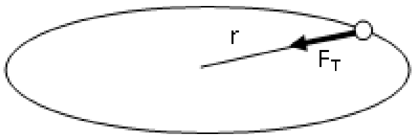


$$F = m\frac{v^2}{R} \quad \text{(magnitude of centripetal force).} \qquad (6\text{-}18)$$

Forces and circular motion

NII: $\vec{F}_{net} = m\vec{a}$ $\Rightarrow$ To make something accelerate, we need a force in the same direction as the acceleration. $\Rightarrow$ Centripetal acceleration is always caused by centripetal force, a force toward center. "Centripetal" means "toward the center".

Example: Rock twirled on a string**.** (Assume no gravity)
Given: m = 0.1 kg , T (period) = 1 s , radius r = 1 m
What is tension F_T in the string?
Answer: Here, we use symbol F_T, since T already taken by period.
F_T is the *only* force acting.

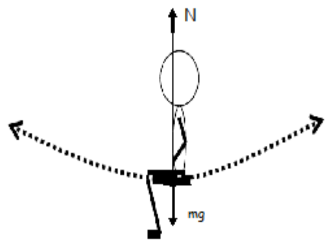


$$F_T = m\,a = m\frac{v^2}{r}, \quad v = \frac{2\pi\, r}{T}, \quad \Rightarrow$$

$$F_T = m\frac{(2\pi\, r/T)^2}{r} = 4\pi^2 m\frac{r}{T^2} = 4\pi^2(0.1)\frac{1}{1^2} = 3.9\text{ N}$$

<u>EXAMPLES OF CIRCULAR MOTION EFFECTS</u>

- **Person on a swing**

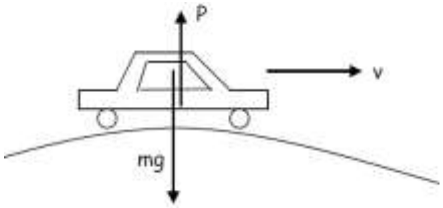


At the bottom of the swing, the forces on the person are the reaction from the seat and the weight force.

$$\frac{mv^2}{r} = N - mg \quad \Rightarrow \quad N = mg + \frac{mv^2}{r}.$$

In this case N > mg, so the person 'feels' heavier.

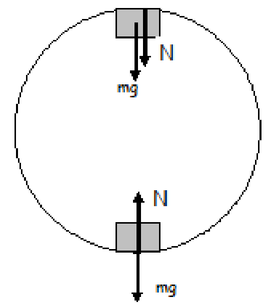
- **Person in a car going over a hump.**



$$\frac{mv^2}{r} = mg - P \qquad \therefore \qquad P = mg - \frac{mv^2}{r}$$

$\therefore$ The person feels lighter.

Example: Consider a car passing through a vertical loop. The force towards the center of the circle is given by:



<u>At the top</u>

$$\frac{mv^2}{r} = N + mg$$

Here the car speed is affected by gravity. It slows down on the upward section and speeds up on the downhill section.
Except at the top and bottom of the loop, the force of gravity means that there is a component of the car's motion.

<u>At the bottom</u>

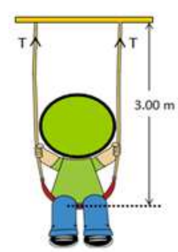
$$\frac{mv^2}{r} = N - mg$$

Example: A small remote control car with mass 1.60 kg moves in a vertical circle inside a hollow metal cylinder that has a radius of 9.80 m, as shown in the **Figure 7**. The speed at the highest point B is v = 9.80 m/s. What is the magnitude of the normal force "R" exerted on the car by the walls of the cylinder at point B (at the top of the vertical circle?)

Answer: The equation of motion at B is:

$$m\left(\frac{v^2}{r}\right) = mg + R \quad \Rightarrow \quad R = m\left(\frac{v^2}{r}\right) - mg = 0$$

Example: A 40.0 kg child swings in a swing supported by two chains, each 3.00 m long, as shown in the figure**.** The tension, T, in each chain at the lowest point of his swing is 350 N. Find the child's speeds at the lowest point of his swing.



Answer:

$$2T - mg = m\frac{v^2}{r}$$

$$700 - 392 = \frac{40}{3}v^2 \Rightarrow v = 4.8 \text{ m/s}$$

Car Negotiating a Flat Turn

Finding the maximum speed for negotiating a turn without slipping

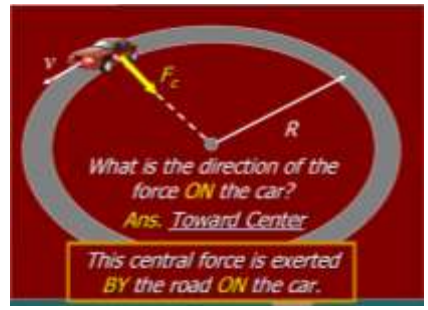
Notes:

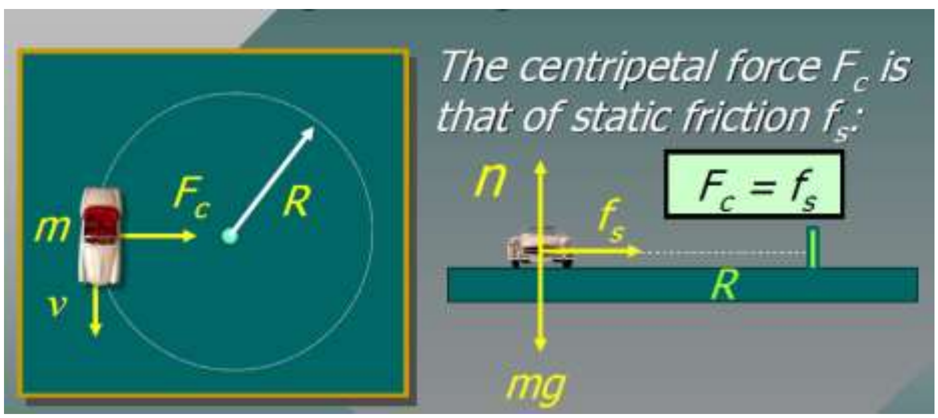
1- A centripetal force, F_c, must act on the car because the car is moving around a circular arc; that force must be directed toward the center of curvature of the arc (here, that is horizontally).

2- The only horizontal force acting on the car is a frictional force on the tires from the road. So the required centripetal force is a frictional force.

3- Because the car is not sliding, the frictional force must be a *static* frictional force (see the figure).

4- Because the car is on the verge of sliding (slipping), the magnitude $\vec{f}_s$ is equal to the maximum value $\vec{f}_{s,\max} = \mu_s F_N$, where $F_N\,(\equiv n)$ is the magnitude of the normal force acting on the car from the track.





5- The car is on the verge of slipping when F_c is equal to the maximum force of static friction f_s.

$$F_c = m\frac{v^2}{R}, \quad f_s = \mu_s mg \quad F_c = f_s \quad \Rightarrow \quad v = \sqrt{\mu_s Rg}$$

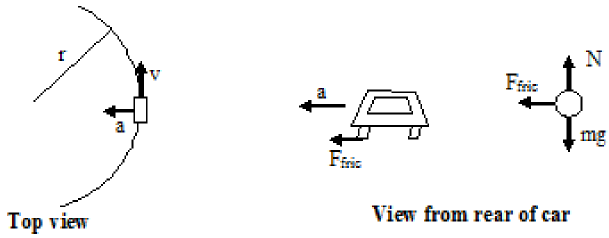
The velocity v is the maximum speed for no slipping.

Example: A car negotiates a turn of radius 70 m when the coefficient of static friction is 0.7. What is the maximum speed to avoid slipping?

Answer:

$$v = \sqrt{\mu_s Rg} = \sqrt{(0.7)(9.8)(70\text{ m})} = 21.9\text{ m/s}$$

Example: Rotation with friction. A car rounds a curve on a flat road (not banked). The radius of the circular curve is r = 100 m, and the speed of the car is v = 30 m/s. How large a static friction coefficient (μ_S, not μ_K !) is needed for the car to not skid off the road?



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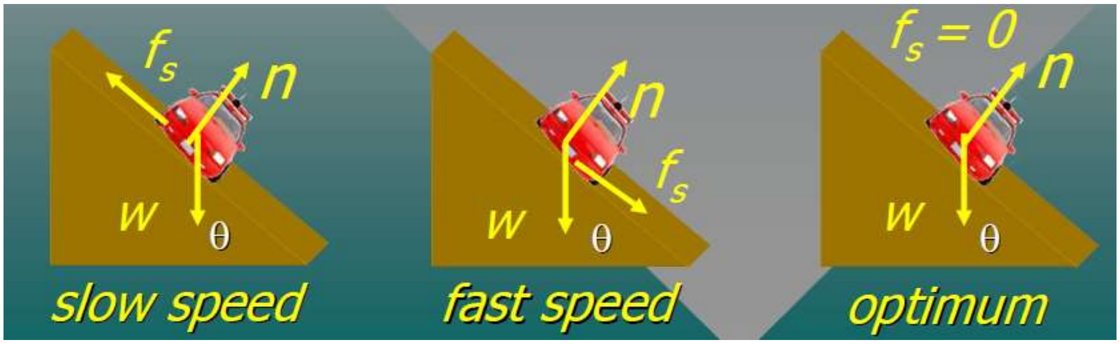
$N = mg \quad \Rightarrow \quad \mu_S\, m\, g = m v^2 / r$ (m's cancel) $\Rightarrow$

$$\mu_S = \frac{v^2}{g\,r} = \frac{(30\text{m/s})^2}{(9.8\text{m/s}^2)(100\text{m})} = 0.92 \text{ (no units)}$$

So, need $\mu_S \geq 0.92$ or else car will skid. For rubber on dry asphalt $\mu_S \cong 1.0$, for rubber on wet asphalt $\mu_S \cong 0.7$. So car will skid if road is wet.

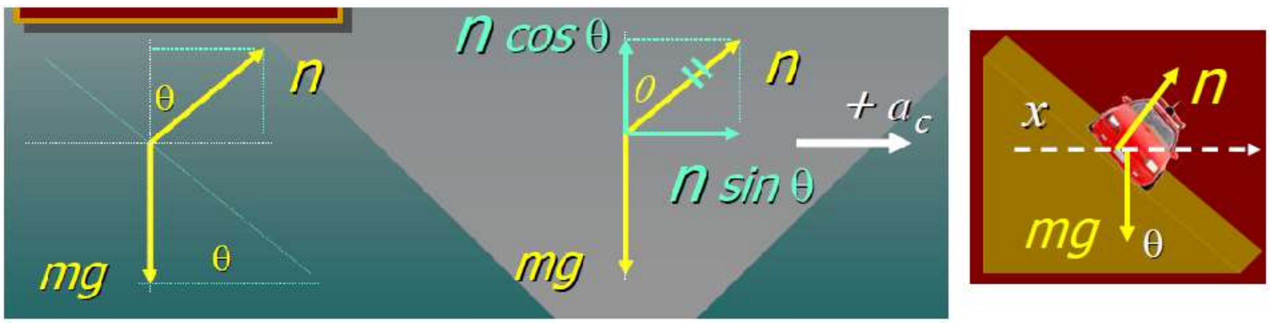
Optimum Banking (tilted) Angle

By banking a curve at the optimum angle, the normal force n can provide the necessary centripetal force without the need for a friction force.



Free-body Diagram

Acceleration a_c is toward the center. Set the x-axis along the direction of a_c, i. e., horizontal (left to right).



Apply Newton's second Law

$$\left.\begin{aligned}\sum F_x = ma_c &\Rightarrow n\sin\theta = \frac{mv^2}{R}\\ \sum F_y = 0 &\Rightarrow n\cos\theta = mg\end{aligned}\right\} \Rightarrow \quad \tan\theta = \frac{v^2}{gR}$$

Optimum banking angle θ

Example: A car negotiates a turn of radius 80 m. What is the optimum banking angle for this curve if the speed is to be equal to 12 m/s?

Answer:

$$\tan\theta = \frac{v^2}{gR} = \frac{(12\text{ m/s})^2}{(9.8\text{ m/s}^2)(80\text{ m})} = 0.184 \quad \Rightarrow \quad \theta = 10.40^\circ$$

How might you find the centripetal force on the car, knowing its mass? $F_c = \frac{mv^2}{R}$

EXAMPLE: A ball on a string (Conical pendulum)

You are whirling a ball of mass *m* in a horizontal circle at the end of a string of length *L*. The ball has a constant speed *v*, and the string makes an angle θ with the vertical.

(a) What is the tension in the string? Express your answer in terms of *m*, *g*, and θ.

(b) What is *v*? Express your answer in terms of *m*, *L*, *g*, and/or θ.

SOLUTION

Let's apply the general method for solving problems using Newton's Laws. The first step is to draw a diagram (see Figure a) showing the ball, the string, and the circular path followed by the ball. The next step is to draw a free-body diagram showing the forces acting on the ball. Although the ball is going in a horizontal circle, the string is at an angle. As shown in Figure b, only two forces act on the ball, the downward force of gravity and the force of tension that is directed away from the ball along the string.

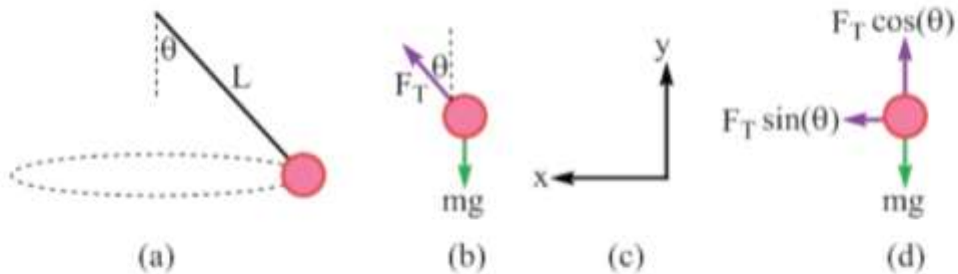


Figure: (a) A diagram and (b) free-body diagram for a ball being whirled in a horizontal circle at the end of a string. (c) An appropriate coordinate system. (d) A free-body diagram, with force components aligned with the coordinate system.

Now, choose an **appropriate coordinate system**. The key is to align the coordinate system with the acceleration. Because the ball is experiencing uniform circular motion, the acceleration is directed horizontally toward the center of the circle. We can choose a coordinate system with axes that are horizontal and vertical, as in Figure c. Finally, split the tension into components, with $F_T \cos\theta$ vertically up and $F_T \sin\theta$ toward the center of the circle, as in Figure d.

(a) To find an appropriate expression for the tension, we can apply Newton's second law in the *y*-direction. Because there is no acceleration vertically, we have:

$$\sum F_y = ma_y = 0 .$$

Looking at the free-body diagram to evaluate the left-hand side of this equation gives:

$$F_T \cos\theta = mg .$$

Solving for the tension gives: $F_T = \dfrac{mg}{\cos\theta}$

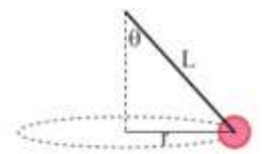
(b) To find an expression for the speed of the ball, let's apply Newton's second law in the *x*-direction. The positive *x*-direction is toward the center of the circle, in the direction of the centripetal acceleration, so we apply the special form of Newton's second law that is appropriate for use in circular motion situations. The general equation is:

$\sum F_x = \frac{mv^2}{r}$, where the acceleration is directed toward the center of the circle.

Looking at the free-body diagram in Figure d, we see that there is only one force in the *x*-direction, so:

$$F_T \sin\theta = \frac{mv^2}{r}$$

A common error in this situation is to assume that *r*, the radius of the circular path, is equal to *L*, the length of the string. Referring to Figure, however, it can be seen that $r = L\sin\theta$.



Note that *r*, the radius of the circular path, is not the same as *L*, the length of the string.

Substituting that into our equation gives:

$$F_T \sin\theta = \frac{mv^2}{L\sin\theta}, \qquad \text{so} \qquad v^2 = \frac{F_T L \sin^2\theta}{m}.$$

Using our result from part (a) to eliminate F_T gives:

$$v^2 = \frac{mgL\sin^2\theta}{m\cos\theta} = \frac{gL\sin^2\theta}{\cos\theta}.$$

Taking the square root of both sides gives: $v = \sin\theta\sqrt{\frac{gL}{\cos\theta}}$

Extra Examples

Q: A car moves with a constant speed on a flat circular track of radius 20 m. The coefficient of static friction between its tires and the track is 0.25. What is the maximum speed at which the car can travel without slipping?

Answer:

Use the equation $m\frac{v^2}{R} = \mu_s mg$ and solve for $v = \sqrt{\mu_s gR} = \sqrt{0.25 \times 9.8 \times 20} = 7.0$ m/s

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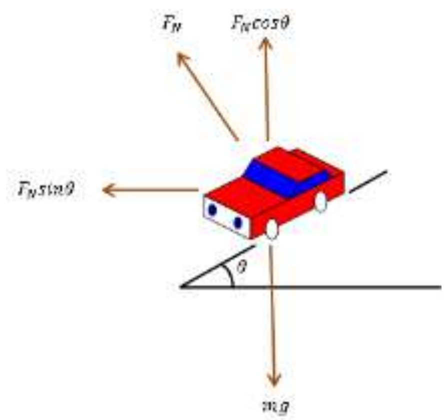
Q: A car of mass 500 kg can go around a banked circular road of radius 60 m at the maximum speed of 20 m/s without slipping. Find the normal force on the car from the banked surface (Ignore the friction force from the road).



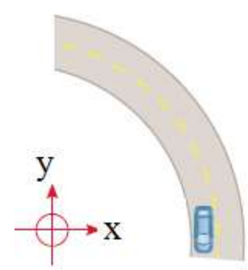
Answer:

$$F_N sin\theta = mv^2/R$$

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{20 \times 20}{500 \times 9.8}\right) = 34.23°$$

$$F_N = \frac{mg}{cos\theta} = \frac{500 \times 9.8}{\cos(34.23)} = 5926.6\ N = 5.9\ kN$$

Example: A 55.0 kg man drives his car through a flat circular track of radius 300 m with a constant speed of 80.0 km/h. What is the magnitude of the net force exerted by the seat of the car on the man at the moment shown in **Figure**?



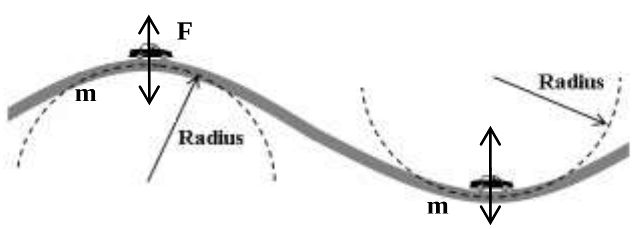
Answer:

$$N = mg = 55 \times 9.8 = 539 \text{ N}$$

$$F_c = m\frac{v^2}{R} = 55\frac{(80 \times 1000/3600)^2}{300} = 90.53 \text{ N}$$

$$F_{net} = \sqrt{90.5^2 + 539^2} = 546.547$$

Example: In the Figure, a car is driven at constant speed over a circular hill and then into a circular valley with the same radius. At the top of the hill, the normal force on the driver from the car seat is 0. The driver's mass is 70.0 kg. What is the magnitude of the normal force on the driver from the seat when the car passes through the bottom of the valley?



Solution

At the top of the hill, the situation is that the weight is greater than the normal force:

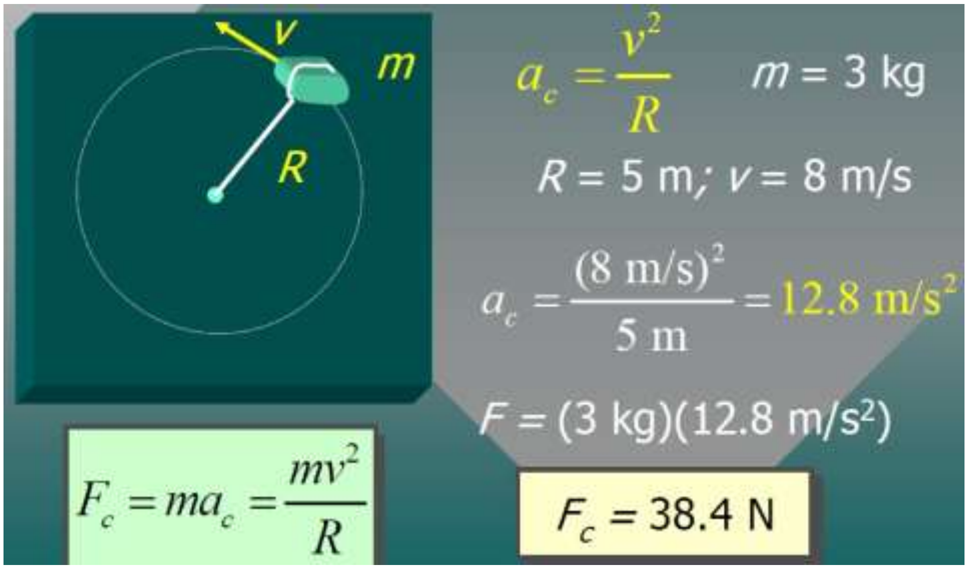
$$F_N - mg = -mv^2/R,$$

we find $F_N = m(g - v^2/R)$.

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$.

Later, at the bottom of the valley, the situation is that the normal force is greater than the weight (as shown in the Figure): $F_N - mg = mv^2/R$ accordingly.
Thus we obtain: $F_N = m(g + v^2/R) = 2mg = 1372$ N $\approx 1.37 \times 10^3$ N = 1.37 kN.

Example: A 3-kg rock swings in a circle of radius 5 m. If its constant speed is 8 m/s, what is the centripetal acceleration?
Answer:



Example: A skater moves with 15 m/s in a circle of radius 30 m. The ice exerts a central force of 450 N . What is the mass of the skater?
Answer: