Chapter (1)
Units, Dimensions, and significant figures

A. The physical quantities:

Physical quantities are the building blocks of the physical science. The laws of physics are expressed as relationships between the physical quantities.

Examples of physical quantities:

Examples of the physical quantities are mass, length, time, distance, speed, displacement, velocity, acceleration, linear momentum, force, angular velocity, angular momentum, torque, work, energy, temperature, volume, density, electric charge, electric potential, electric current, magnetic susceptibility and many others.

Types of physical quantities

1- **Fundamental quantities:** which are defined in terms of measurements or comparison with established standards. There are seven **fundamental quantities in physics**, these are **mass, length, time, electric current, temperature, amount of substance, and luminous intensity**.

length	meter	m
mass	kilogram	kg
time	second	s
temperature (absolute)	kelvin	K
amount of substance	mole	mol
electric curent	ampere	A
luminous intensity	candela	cd

2- **Derived quantities:** which are expressed in terms of the fundamental quantities. **Examples** of the derived quantities are speed, velocity, acceleration, linear momentum, force, angular velocity, angular momentum, torque, work, energy, volume, density

The fundamental quantities in mechanics

In mechanics, the three fundamental quantities are the length (L), the mass (M), and the time (T). All other physical quantities in mechanics can be expressed in terms of these three quantities.

Systems of units

Three systems of units are most commonly used in science and engineering;

Quantity	Dimension	SI Unit	cgs
Length	L	meter, m	centimeter, cm
Mass	M	kilogram, kg	gram, g
Time	T	second, s	second, s

Table 1 Units of Area, Volume, Speed, and Acceleration. Derived quantities

Quantity	Dimensions	SI Unit	cgs
Area	L^2	m^2	cm^2
Volume	L^3	m^3	cm^3
Speed	L/T	m/s	cm/s
Acceleration	L/T^2	m/s^2	cm/s^2
Force	$M\,L/T^2$	Newton, N $1\,N = 1\ kg\ m/s^2$	dyne $1 dyne = 1\ g\ cm/s^2$

Note: The following table is given for your information. No need to memorize it.

Quantity	Sympole	Defining formula	Dimensions	Units SI
Mass	m	-	M	m
Length	l	-	L	Kg
Time	t	-	T	S
Frequency	f	1/t	T^{-1}	S^{-1} (Hz)
Area	A	l.l	L^2	m^2
Volume	V	l.l.l	L^3	m^3
Density	ρ	m/V	ML^{-3}	Kgm^{-3}
Velocity	v	ds/dt	LT^{-1}	mS^{-1}
Momentum	p	mv	MLT^{-1}	$KgmS^{-1}$
Acceleration	a	dv/dt	LT^{-2}	mS^{-2}
Force	F	ma	MLT^{-2}	$KgmS^{-2} = 1newton\ (N)$
Impulse	I	$p\Delta t$	ML	Kgm
Kinetic Energy	KE	$1/2\ mv^2$	ML^2T^{-2}	$Kgm^2S^{-2} = 1joul\ (J)$
Work	W	Fr	ML^2T^{-2}	$Kgm^2S^{-2} = 1joul\ (J)$
Power	p	W/t	ML^2T^{-3}	$Kgm^2S^{-3} = J/S = Watt$
Pressure	P	F/A	$ML^{-1}T^{-2}$	$Kgm^{-1}S^{-2} = N/m^2$
Moment of inertia	I	Mr^2	ML^2	$Kgm^2 = N.m^2$
Torque	τ	F x r	ML^2	$Kgm^2 = N.m$
Surface tension	γ	F/l	MT^{-2}	$Kgm^{-2}\ N/m = J/m^2$
Viscosity	η	(F/A)/(dv/dr)	$ML^{-1}T^{-1}$	$Kgm^{-1}S^{-1} = N.S/m^2$
Elastic Modulus	Y	$(F/A)/(\Delta l/l)$	$ML^{-1}T^{-2}$	$Kgm^{-1}S^{-2} = N/m^2$

B. Changing Units

In all of our mathematical operations we must always write down the units and we always treat the unit symbols as multiplicative factors. For example, if me multiply 3.0 kg by 2.0 m/s we get

$$(3.0\ \text{kg}) \cdot (2.0\ \text{m/s}) = 6.0\ \text{kg} \cdot \text{m/s}$$

We use the same idea in changing the units in which some physical quantity is expressed. We can multiply the original quantity by a **conversion factor**, i.e. a ratio of values for which the numerator is the same thing as the denominator. The conversion factor is then equal to 1, and so we do not change the original quantity when we multiply by the conversion factor.

Examples of conversion factors are:

$$\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \qquad \left(\frac{100\ \text{cm}}{1\ \text{m}}\right) \qquad \left(\frac{1\ \text{yr}}{365.25\ \text{day}}\right) \qquad \left(\frac{1\ \text{m}}{3.28\ \text{ft}}\right)$$

Ex. The Empire State Building is 1472 ft high. Express this height in both meters and centimeters.

To do the first unit conversion (feet to meters), we can use the relation (see the Conversion Factors in the back of this book):

$$1\ \text{m} = 3.281\ \text{ft}$$

We set up the conversion factor so that "ft" cancels and leaves meters:

$$1472\ \text{ft} = (1472\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right) = 448.6\ \text{m}\ .$$

So the height can be expressed as 448.6 m. To convert this to centimeters, use:

$$1\ \text{m} = 100\ \text{cm}$$

Ex. A rectangular building lot is 100.0 ft by 150.0 ft. Determine the area of this lot in m^2.

The area of a rectangle is just the product of its length and width so the area of the lot is

$$A = (100.0\ \text{ft})(150.0\ \text{ft}) = 1.500 \times 10^4\ \text{ft}^2$$

To convert this to units of m^2 we can use the relation

$$1\ \text{m} = 3.281\ \text{ft}$$

but the conversion factor needs to be applied *twice* so as to cancel "ft^2" and get "m^2". We write:

$$1.500 \times 10^4\ \text{ft}^2 = (1.500 \times 10^4\ \text{ft}^2) \cdot \left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)^2 = 1.393 \times 10^3\ \text{m}^2$$

The area of the lot is $1.393 \times 10^3\ \text{m}^2$.

Ex. Calculate the number of kilometers in $20.0\,\text{mi}$ using only the following conversion factors: $1\,\text{mi} = 5280\,\text{ft}$, $1\,\text{ft} = 12\,\text{in}$, $1\,\text{in} = 2.54\,\text{cm}$, $1\,\text{m} = 100\,\text{cm}$, $1\,\text{km} = 1000\,\text{m}$.

Set up the "factors of 1" as follows:

$$20.0\,\text{mi} = (20.0\,\text{mi}) \cdot \left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right) \cdot \left(\frac{12\,\text{in}}{1\,\text{ft}}\right) \cdot \left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right) \cdot \left(\frac{1\,\text{m}}{100\,\text{cm}}\right) \cdot \left(\frac{1\,\text{km}}{1000\,\text{m}}\right)$$
$$= 32.2\,\text{km}$$

Setting up the "factors of 1" in this way, all of the unit symbols cancel except for km (kilometers) which we keep as the units of the answer.

The area of the lot is $1.393 \times 10^3\,\text{m}^2$.

Ex. Express the speed of light, $3.0 \times 10^8\,\frac{\text{m}}{\text{s}}$ in (a) feet per nanosecond and (b) millimeters per picosecond.

(a) For this conversion we can use the following facts:

$$1\,\text{m} = 3.28\,\text{ft} \qquad \text{and} \qquad 1\,\text{ns} = 10^{-9}\,\text{s}$$

to get:

$$3.0 \times 10^8\,\frac{\text{m}}{\text{s}} = (3.0 \times 10^8\,\frac{\text{m}}{\text{s}}) \cdot \left(\frac{3.28\,\text{ft}}{1\,\text{m}}\right) \cdot \left(\frac{10^{-9}\,\text{s}}{1\,\text{ns}}\right)$$
$$= 0.98\,\frac{\text{ft}}{\text{ns}}$$

In these new units, the speed of light is $0.98\,\frac{\text{ft}}{\text{ns}}$.

(b) For this conversion we can use:

$$1\,\text{mm} = 10^{-3}\,\text{m} \qquad \text{and} \qquad 1\,\text{ps} = 10^{-12}\,\text{s}$$

and set up the factors as follows:

$$3.0 \times 10^8\,\frac{\text{m}}{\text{s}} = (3.0 \times 10^8\,\frac{\text{m}}{\text{s}}) \cdot \left(\frac{1\,\text{mm}}{10^{-3}\,\text{m}}\right) \cdot \left(\frac{10^{-12}\,\text{s}}{1\,\text{ps}}\right)$$
$$= 3.0 \times 10^{-1}\,\frac{\text{mm}}{\text{ps}}$$

prefix	abbreviation	multiplier	--	prefix	abbreviation	multiplier
peta	P	10^{15}		deci	s	10^{-1}
tera	T	10^{12}		centi	c	10^{-2}
giga	G	10^{9}		milli	m	10^{-3}
mega	M	10^{6}		micro	µ	10^{-6}
kilo	k	10^{3}		nano	n	10^{-9}
hecto	h	10^{2}		pico	p	10^{-12}
deca	da	10		femto	f	10^{-15}

Density

A quantity which will be encountered in your study of liquids and solids is the **density** of a sample. It is usually denoted by ρ and is defined as the ratio of mass to volume:

$$\rho = m/V$$

The SI units of density are kg/m^3 but you often see it expressed in g/cm^3.

C- Dimensional Analysis

The word *dimension* has a special meaning in physics. It denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or fathoms, it is still a distance. We say its dimension is *length.*

The symbols we use in this book to specify the dimensions of length, mass, and time are L, M, and T, respectively [the *dimensions* of a quantity will be symbolized by a capitalized, non-italic letter, such as L. The *symbol* for the quantity itself will be italicized, such as *L* for the length of an object, or *t* for time]. We shall often use brackets [to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this hook is *v,* and in our notation the dimensions of speed are written $[v] = L/T$. As another example, the dimensions of area *A* are $[A] = L^2$. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

This method is used to
(1)   Derive the dimensions of a physical quantity e.g.
$$[force]=[m\ a]= MLT^{-2},$$
(2)   Derivation of physical formulas and expressions.
(3)   Check the correctness of formula or expression.

In (2) and (3) we use the fact that in equations and formulas the left hand side (LHS) must have the same dimensions as the right hand side (RHS)

Example

Use the method of dimensional analysis to show that $x = \frac{1}{2} at^2$ is dimensionally correct where x is the displacement, a acceleration, and t is the time

Solution : The dimension of LHS $[x]=L$,
           The dimension of RHS $[at^2]=LT^{-2}.T^2 =L$
The equation is dimensionally correct

In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called *dimensional analysis* can be used to assist in the derivation or to check your final expression.

dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive an equation for the position *x* of a car at a time *t* if the car starts from rest and moves with constant acceleration a. In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2} at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the

dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ **is** $L = \frac{L}{T^2}T^2 = L$. The dimensions of time cancel as shown, leaving the dimension of length on the right- hand side.

A more general procedure using dimensional analysis is to set up an expression of the form $x \propto a^n t^m$, where n and m are exponents that must be determined and the symbol $\propto$ **indicates a** proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$\left[a^n t^m\right] = L = L^1 T^0$$

Because the dimensions of acceleration are **L/T^2 and the dimension** of time is T, we have

$$(L/T^2)^n T^m = L^1 T^0 \qquad \Rightarrow \qquad (L^n T^{m-2n}) = L^1 T^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m-2n=0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x = a^n t^m$, we conclude that $x = at^2$. This result differs by a factor of ½ from the correct expression, which is $x = \frac{1}{2}at^2$.

Quick Quiz: True or False: Dimensional analysis can give you the numerical value of constant of proportionality that may appear in an algebraic expression.

Example: Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t an instant of time

Solution For the speed term, we have from Table 1 $[v] = L/T$

The same table gives us L T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \frac{L}{T^2}T = \frac{L}{T}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$ it would be dimensionally *incorrect.* Try it and see!)

Example: Analyses of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. Determine the values of n and m and write the simplest form of an equation for the acceleration.

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r and v, we see that the dimensional equation must be

$$\frac{L}{T^2} = L^n \left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

This dimensional equation is balanced under the conditions $n + m = 1$ and $m = 2$. Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2.

Questions

Q1.: The position y of a particle moving along the y axis depends on the time t according to the equation $y = At - Bt^2$. The dimensions of the quantities A and B are respectively:

Ans:

$$[y] = L; \quad [A] = \frac{[y]}{[t]} = \frac{L}{T}; \quad [B] = \frac{[y]}{[t]^2} = \frac{L}{T^2}$$

Q2: A swimming pool is filled with 16,500 ft^3 of water. What is the volume of water in m^3? (12 inch = 1 ft and 2.54 cm = 1 inch). (Ans: 467 m^3)

Ans:

$$\text{Volume} = 1.65 \times 10^4 \text{ ft}^3 = 1.65 \times 10^4 \text{ ft}^3 \times \left(\frac{12 \text{ inch}}{1 \text{ ft}}\right)^3 \times \left(\frac{2.54 \text{ cm}}{1 \text{ inch}}\right)^3 \times \left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3 = 4.67 \times 10^3 \text{ m}^3$$

Q3: The position x of a particle is given by $x = B t^2 + \frac{C}{B} t$, where x is in meters and t is in seconds. The dimension of C is:

Ans:

$$[x] = L; \quad [B] = \frac{[x]}{[t]^2} = \frac{L}{T^2}; \quad [C] = [B]\frac{[x]}{[t]} = \frac{L}{T^2} \times \frac{L}{T} = \frac{L^2}{T^3}$$

Q4: From the fact that the average density of the Earth is 5.50 g/cm^3 and its mean radius is 6.37×10^6 m, the mass of the Earth is: (Ans: 5.95×10^{24} kg)

Q5: Suppose A=B^n/C^m where A has dimensions LT, B has dimensions L^2T^{-1}, and C has dimensions LT^2. Then the exponents n and m have the values: (Ans: n = 1/5 ; m = −3/5)

Ans: For the left hand side

$$[A] = LT \tag{1}$$

For the right hand side

$$\frac{[B]^n}{[C]^m} = \frac{(L^2T^{-1})^n}{(LT^2)^m} = \frac{L^{2n}T^{-n}}{L^m T^{2m}} = L^{2n-m}T^{-n-2m} \tag{2}$$

Equate the powers in (1) and (2), one find:

$$2n - m = 1, \quad -n - 2m = 1 \implies n = \frac{1}{5}, \quad m = -\frac{3}{5}$$

Q6: An aluminum cylinder of density 2.70 g/cm^3, a radius of 2.30 cm, and a height of 1.40 m has the mass of: (Ans: 6.28 kg)

Q7: A nucleus of volume 3.4 x 10^3 fm^3 and mass of 1.0x 10^2 u has a density of: (1 fm = 10^{-15} m, 1 u = 1.7 x 10^{-27} kg) **(Ans: 5.0×10^{16} kg/m^3)**

Q8: The mass of 1.0 cm^3 of gold is 19.3 g. What is the mass of a solid cube of gold having a side of 0.70 cm ? **(Ans: 6.6×10^{-3} kg)**

Q9: Express speed of sound, 330 m/s in miles/h . (1 mile = 1609 m)(**Ans:** A1 738 miles/h)

Ans:

$$\text{Speed of sound} = 330\frac{m}{s} \times \left(\frac{12 \text{ mile}}{1609 \text{ m}}\right) \times \left(\frac{3600 \text{ s}}{1 \text{ hour}}\right) = 738 \frac{\text{miles}}{\text{hour}}$$

Q10: The position y of a particle moving along the y axis depends on the time t according to the equation $y = At - Bt^2$. The dimensions of the quantities A and B are respectively:

    A) L/T,   L/T^2
    B) L^2/T,   L/T^2
    C) L/T,   L^2/T
    D) L^3/T,   T^2/L
    E) Both are dimensionless

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Tutorial on the Use of Significant Figures

All measurements are approximations--no measuring device can give perfect measurements without experimental uncertainty. By convention, a mass measured to 13.2 g is said to have an absolute uncertainty of 0.1 g and is said to have been measured to the nearest 0.1 g. In other words, we are somewhat uncertain about that last digit —it could be a "2"; then again, it could be a "1" or a "3". A mass of 13.20 g indicates an absolute uncertainty of 0.01 g.

The objectives of this tutorial are:

—Explain the concept of significant figures.
—Define rules for deciding the number of significant figures in
  a measured quantity.
—Explain the concept of an exact number.
—Define rules for determining the number of significant figures in
  a number calculated as a result of a mathematical operation.
—Explain rules for rounding numbers.
—Present guidelines for using a calculator.
—Provide some exercises to test your skill at significant figures.

What is a "significant figure"?

The number of significant figures in a result is simply the number of figures that are known with some degree of reliability. The number 13.2 is said to have 3 significant figures. The number 13.20 is said to have 4 significant figures

Rules for deciding the number of significant figures in a measured quantity:

(1) All nonzero digits are significant:

1.234 g has 4 significant figures,
1.2 g has 2 significant figures.

(2) Zeroes between nonzero digits are significant:

1002 kg has 4 significant figures,
3.07 mL has 3 significant figures.

(3) Leading zeros to the left of the first nonzero digits are not significant; such zeroes merely indicate the position of the decimal point:

0.001 ºC has only 1 significant figure,
0.012 g has 2 significant figures.

(4) Trailing zeroes that are also to the right of a decimal point in a number are significant:

0.0230 mL has 3 significant figures,
0.20 g has 2 significant figures.

(5) When a number ends in zeroes that are not to the right of a decimal point, the zeroes are not necessarily significant:

190 miles may be 2 or 3 significant figures,
50,600 calories may be 3, 4, or 5 significant figures.

The potential ambiguity in the last rule can be avoided by the use of standard exponential, or "scientific," notation. For example, depending on whether the number of significant figures is 3, 4, or 5, we would write 50,600 calories as:

5.06×10^4 calories (3 significant figures)
5.060×10^4 calories (4 significant figures), or
5.0600×10^4 calories (5 significant figures).

What is a "exact number"?
Some numbers are exact because they are known with complete certainty.
Most exact numbers are integers: exactly 12 inches are in a foot, there might be exactly 23 students in a class. Exact numbers are often found as conversion factors or as counts of objects.
Exact numbers can be considered to have an infinite number of significant figures. Thus, the number of apparent significant figures in any exact number can be ignored as a limiting factor in determining the number of significant figures in the result of a calculation.

Rules for mathematical operations
In carrying out calculations, the general rule is that the accuracy of a calculated result is limited by the least accurate measurement involved in the calculation.
(1) In addition and subtraction, the result is rounded off to the last common digit occurring furthest to the right in all components. Another way to state this rules, it that,in addition and subtraction, the result is rounded off so that it has the same number of decimal places as the measurement having the fewest decimal places. For example,
100 (assume 3 significant figures) + 23.643 (5 significant figures) = 123.643,
which should be rounded to 124 (3 significant figures).
Addition and Subtraction with Significant Figures
When combining measurements with different degrees of accuracy and precision, *the accuracy of the final answer can be no greater than the least accurate measurement.* This principle can be translated into a simple rule for addition and subtraction: **When measurements are added or subtracted, the answer can contain no more decimal places than the least accurate measurement.**

> 150.0 g H_2O (using significant figures)
>
> + 0.507 g salt
>
> 150.5 g solution

(2) In multiplication and division, the result should be rounded off so as to have the same number of significant figures as in the component with the least number of significant figures. For example,
3.0 (2 significant figures) $\times$ 12.60 (4 significant figures) = 37.8000
which should be rounded off to 38 (2 significant figures).
Multiplication and Division with Significant Figures
The same principle governs the use of significant figures in multiplication and division: the final result can be no more accurate than the least accurate measurement. In this case, however, we count the significant figures in each measurement, not the number of decimal places: **When measurements are multiplied or divided, the answer can contain no more significant figures than the least accurate measurement.**
Example: To illustrate this rule, let's calculate the cost of the copper in an old penny that is pure copper. Let's assume that the penny has a mass of 2.531 grams, that it is

essentially pure copper, and that the price of copper is 67 cents per pound. We can start by from grams to pounds.

$$2.531 \text{ g} \times \frac{1 \text{ lb}}{453.6 \text{ g}} = 0.005580 \text{ lb}$$

We then use the price of a pound of copper to calculate the cost of the copper metal.

$$0.005580 \text{ lb} \times \frac{67¢}{1 \text{ lb}} = 0.3749 \text{ g}$$

There are four significant figures in both the mass of the penny (2.531) and the number of grams in a pound (453.6). But there are only two significant figures in the price of copper, so the final answer can only have two significant figures.

General guidelines for using calculators

When using a calculator, if you work the entirety of a long calculation without writing down any intermediate results, you may not be able to tell if an error is made. Further, even if you realize that one has occurred, you may not be able to tell where the error is.
In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately. For example,
(5.00 / 1.235) + 3.000 + (6.35 / 4.0)=4.04858... + 3.000 + 1.5875=8.630829...
The first division should result in 3 significant figures. The last division should result in 2 significant figures. The three numbers added together should result in a number that is rounded off to the last common significant digit occurring furthest to the right; in this case, the final result should be rounded with 1 digit after the decimal. Thus, the correct rounded final result should be 8.6. This final result has been limited by the accuracy in the last division.
Warning: carrying all digits through to the final result before rounding is critical for many mathematical operations in statistics. Rounding intermediate results when calculating sums of squares can seriously compromise the accuracy of the result.
Rounding Off
When the answer to a calculation contains too many significant figures, it must be rounded off.
There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves *underestimating* the answer for five of these digits (0, 1, 2, 3, and 4) and *overestimating* the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.
If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.
If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.

Last Note: Significant figures are critical when reporting scientific data because they give the reader an idea of how well you could actually measure/report your data. Before looking at a few examples, let's summarize the rules for significant figures.

      1) ALL non-zero numbers (1,2,3,4,5,6,7,8,9) are ALWAYS significant.

      2) ALL zeroes between non-zero numbers are ALWAYS significant.

      3) ALL zeroes which are SIMULTANEOUSLY to the right of the decimal point AND at the end of the number are ALWAYS significant.

      4) ALL zeroes which are to the left of a written decimal point and are in a number >= 10 are ALWAYS significant.

A helpful way to check rules 3 and 4 is to write the number in scientific notation. If you can/must get rid of the zeroes, then they are NOT significant.

Examples: How many significant figures are present in the following numbers?

Number	# Significant Figures	Rule(s)
48,923	5	1
3.967	4	1
900.06	5	1,2,4
0.0004 (= 4 E-4)	1	1,4
8.1000	5	1,3
501.040	6	1,2,3,4
3,000,000 (= 3 E+6)	1	1
10.0 (= 1.00 E+1)	3	1,3,4

This gives you some idea of how to determine the number of significant figures in a single number. To find out how to use this information when performing mathematical operations (addition, subtraction, multiplication, division), see "Significant Figures: Mathematical Operations."

Sample problems on significant figures

<u>Instructions</u>: print a copy of this page and work the problems. When you are ready to check your answers, go to the next page.
1. 37.76 + 3.907 + 226.4 =
2. 319.15 - 32.614 =
3. 104.630 + 27.08362 + 0.61 =
4. 125 - 0.23 + 4.109 =
5. $2.02 \times 2.5 =$
6. 600.0 / 5.2302 =
7. $0.0032 \times 273 =$
8. $(5.5)^3 =$
9. $0.556 \times (40 - 32.5) =$
10. $45 \times 3.00 =$
11. $3.00 \times 10^5 - 1.5 \times 10^2 =$ (Give the exact numerical result, then express it the correct number of significant figures).
12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?

Answer key to sample problems on significant figures

1. 37.76 + 3.907 + 226.4 = 268.1
2. 319.15 - 32.614 = 286.54
3. 104.630 + 27.08362 + 0.61 = 132.32
4. 125 - 0.23 + 4.109 = 129 (assuming that 125 has 3 significant figures).
5. $2.02 \times 2.5 = 5.0$
6. 600.0 / 5.2302 = 114.7
7. $0.0032 \times 273 = 0.87$
8. $(5.5)^3 = 1.7 \times 10^2$
9. $0.556 \times (40 - 32.5) = 4$
10. $45 \times 3.00 = 1.4 \times 10^2$ (assuming that 45 has two significant figures)
11. $3.00 \times 10^5 - 1.5 \times 10^2 = 299850 = 3.00 \times 10^5$
12. What is the average of 0.1707, 0.1713, 0.1720, 0.1704, and 0.1715?
Answer = 0.1712

Links to other resources on the use of significant figures

Uncertainty in Measurements: A tutorial by Professor Frederick A. Sensese at Frostburg State University that shows how uncertainty arises from length, temperature, and volume measurements. How to count significant figures for a single measurement and for a series of measurements. How to round measurements to the correct number of significant figures. http://antoine.frostburg.edu/cgi-bin/senese/tutorials/sigfig/index.cgi
Significant figures quiz: A JavaScript tutorial by Professor Frederick A. Sensese at Frostburg State University that lets you test your knowledge of the use of significant figures. http://antoine.frostburg.edu/chem/senese/101/measurement/sigfig-quiz.shtml
Frequently asked questions about measurements, including FAQs on significant figures (such as:
"Why should the rules for propagating significant digits not be applied to averages?
"Why does 1101 cm - 1091 cm = 10 cm with 2 significant figures?"
"Are there simpler rules for counting significant digits?"
http://antoine.frostburg.edu/chem/senese/101/measurement/faq.shtml
Determining the number of significant figures: a drill by Scott Van Bramer of Widener University involving significant figures that presents you with successive number displays and grades you on your answer to the question, "How many significant figures are there?" http://science.widener.edu/svb/tutorial/sigfigures.html
Significant Figures and Rounding Rules: a discussion of issues relating to the proper teaching of significant figures and rounding rules. This site, authored by Christopher Mulliss, includes links to other pages and to interactive tutorials. http://www.angelfire.com/oh/cmulliss

Extra Notes:
Measuring and significant figures
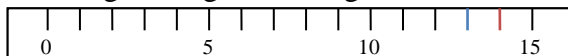
Ruler with only centimeter

The mark "x" is measured to be at 13.2 cm with the above ruler which has only centimeter marks as shown above. We have estimated the tenth of the centimeter; the digit "2" here is the estimated uncertain digit. The uncertainty of the measurement is approximately ±0.5 cm and the measurement has three significant figures (s.f.)
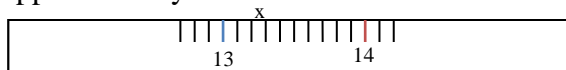
Ruler with millimeter marks

The mark "x" is now measured to be at 13.24 cm with the estimated digit at one tenth of a millimeter. The uncertainty of the measurement is approximately ±0.05 cm and the measurement has four significant figures.

Rules for counting significant figures in a number:

- The leftmost non-zero digit is the first significant figure.
- If there is no decimal point, the rightmost non-zero digit is the last significant figure.
- If there is a decimal point, the rightmost digit is significant, zero or not. Number "200" has one significant figure and "200." has three significant figures, or you write it as 2 x 10^3, 2.0 x 10^3, and 2.00 x 10^3 to indicate 1, 2, or 3 significant figures respectively.
- Any digits between significant figures are also significant.

Rounding the result of addition/subtraction:

The number that results from **addition or subtraction should be rounded at the lowest decimal place that the original numbers have in common**. If we add the numbers 2.2 + 41.066 + 19.11 we get the result 62.376. This implies that we measured to the thousandths place, but one of the numbers was only measured to the tenths place. The sum should be rounded to 62.4 at the tenths place, which is the lowest decimal place that all three numbers share.

Rounding the result of multiplication/division:

The number that results from **multiplication or division should be rounded so that it has the same number of significant figures as the original number with the fewest number of significant figures**. For example, if we multiply the numbers 22 x 30.1 x 1.4 we get the result 927.08. The lowest digit is the one with uncertainty, so this implies that we made measurements that had only one in ten thousand parts uncertainty! We need to round the result to 930, which has two significant figures, since the original numbers had two and three significant figures.

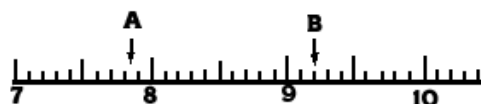## Significant Figures in Measurement and Calculations

A successful chemistry student habitually labels all numbers, because the unit is important. Also of great importance is the number itself. Any number used in a calculation should contain only figures that are considered reliable; otherwise, time and effort are wasted. Figures that are considered reliable are called *significant figures*. Chemical calculations involve numbers representing actual measurements. In a measurement, significant figures in a number consist of:

Figures (digits) definitely known + One estimated figure (digit)

In class you will hear this expressed as "all of the digits known for certain plus one that is a guess."

Recording Measurements

When one reads an instrument (ruler, thermometer, graduate, buret, barometer, balance), he expresses the reading as one which is reasonably reliable. For example, in the accompanying illustration, note the reading marked *A*. This reading is definitely beyond the 7 cm mark and also beyond the 0.8 cm mark. We read the 7.8 with certainty. We further *estimate* that the reading is five-tenths the distance from the 7.8 mark to the 7.9 mark. So, we estimate the length as 0.05 cm more than 7.8 cm. All of these have meaning and are therefore significant. We express the reading as 7.85 cm, accurate to three significant figures. All of these figures, *7.85*, can be used in calculations. In reading B we see that 9.2 cm is definitely known. We can include one estimated digit in our reading, and we estimate the next digit to be zero. Our reading is reported as 9.20 cm. It is accurate to three significant figures.

Rules for Zeros

If a zero represents a measured quantity, it is a significant figure. If it merely locates the decimal point, it is not a significant figure.

Zero Within a Number. In reading the measurement 9.04 cm, the zero represents a measured quantity, just as 9 and 4, and is, therefore, a significant number. A zero between any of the other digits in a number is a significant figure.

Zero at the Front of a Number. In reading the measurement 0.46 cm, the zero does not represent a measured quantity, but merely locates the decimal point. It is not a significant figure. Also, in the measurement 0.07 kg, the zeros are used merely to locate the decimal point and are, therefore, not significant. Zeros at the first (left) of a number are not significant figures.

Zero at the End of a Number. In reading the measurement 11.30 cm, the zero is an estimate and represents a measured quantity. It is therefore significant. Another way to look at this: The zero is not needed as a placeholder, and yet it was included by the person recording the measurement. It must have been recorded as a part of the measurement, making it significant. Zeros to the right of the decimal point, and at the end of the number, are significant figures.

Zeros at the End of a Whole Number. Zeros at the end of a whole number may or may not be significant. If a distance is reported as 1600 feet, one assumes two sig figs. Reporting measurements in scientific notation removes all doubt, since all numbers written in scientific notation are considered significant.

1 600 feet	1.6×10^3 feet	Two significant figures
1 600 feet	1.60×10^3 feet	Three significant figures
1 600 feet	1.600×10^3 feet	Four significant figures

Sample Problem #1: Underline the significant figures in the following numbers.

(a) 0.0420 cm	answer = 0.0420 cm	(e) 2 403 ft.	answer = 2 403 ft.
(b) 5.320 in.	answer = 5.320 in.	(f) 80.5300 m	answer = 80.5300 m
(c) 10 lb.	answer = 10 lb.	(g) 200. g	answer = 200 g
(d) 0.020 ml	answer = 0.020 ml	(h) 2.4×10^3 kg	answer = 2.4×10^3 kg

Rounding Off Numbers

In reporting a numerical answer, one needs to know how to "round off" a number to include the correct number of significant figures. Even in a series of operations leading to the final answer, one must "round off" numbers. The rules are well accepted rules.

1. If the figure to be dropped is less than 5, simply eliminate it.
2. If the figure to be dropped is greater than 5, eliminate it and raise the preceding figure by 1.
3. If the figure is 5, followed by nonzero digits, raise the preceding figure by 1
4. If the figure is 5, not followed by nonzero digit(s), and preceded by an odd digit, raise the preceding digit by one
5. If the figure is 5, not followed by nonzero digit(s), and the preceding significant digit is even, the preceding digit remains unchanged

Sample Problem #2: Round off the following to three significant figures.

(a) 3.478 m   answer = 3.48 m   (c) 5.333 g   answer = 5.33 g
(b) 4.8055 cm   answer = 4.81 cm   (d) 7.999 in.   answer = 8.00 in.

Multiplication

In multiplying two numbers, when you wish to determine the number of significant figures you should have in your answer (the product), you should inspect the numbers multiplied and find which has the least number of significant figures. This is the number of significant figures you should have in your answer (the product). Thus the answer to 0.024 x 1244 would be rounded off to contain two significant figures since the factor with the lesser number of significant figures (0.024) has only *two* such figures.

Sample Problem #3: Find the area of a rectangle 2.1 cm by 3.24 cm.

Solution: Area = 2.1 cm x 3.24 cm = 6.804 cm^2

We note that 2.1 contains two significant figures, while 3.24 contains three significant figures. Our product should contain no more than *two* significant figures. Therefore, our answer would be recorded as 6.8 cm^2

Sample Problem #4: Find the volume of a rectangular solid 10.2 cm x 8.24 cm x 1.8 cm

Solution: Volume = 10.2 cm x 8.24 cm x 1.8 cm = 151.2864 cm^3

We observe that the factor having the least number of significant figures is 1.8 cm. It contains two significant figures. Therefore, the answer is rounded off to 150 cm^3.

Division

In dividing two numbers, the answer (quotient) should contain the same number of significant figures as are contained in the number (divisor or dividend) with the least number of significant figures. Thus the answer to 528 ÷ 0.14 would be rounded off to contain *two* significant figures. The answer to 0.340 ÷ 3242 would be rounded off to contain three significant figures.

Sample Problem #5: Calculate 20.45 ÷ 2.4

Solution: 20.45 ÷ 2.4 = 8.52083

We note that the 2.4 has fewer significant figures than the 20.45. It has only *two* significant figures. Therefore, our answer should have no more than two significant figures and should be reported as 8.5.

Addition and Subtraction

In adding (or subtracting), set down the numbers, being sure to keep like decimal places under each other, and add (or subtract). Next, note which column contains the first estimated figure. This column determines the last decimal place of the answer. After the answer is obtained, it should be rounded off in this column. In other words, round to the least number of decimal places in you data.

Sample Problem #6: Add 42.56 g + 39.460 g + 4.1g

Solution:

                     42.56 g
                     39.460 g
                     4.1 g
Sum =   86.120 g

Since the number 4.1 only extends to the first decimal place, the answer must be rounded to the first decimal place, yielding the answer 86.1 g.

Average Readings

The average of a number of successive readings will have the same number of decimal places that are in their sum.

Sample Problem #7: A graduated cylinder was weighed three times and the recorded weighings were 12.523 g, 12.497 g, 12.515 g. Calculate the average weight.

Solution:

                     12.523 g
                     12.497 g
                     12.515 g
                     37.535 g

In order to find the average, the sum is divided by 3 to give an answer of 12.51167. Since each number extends to three decimal places, the final answer is rounded to three decimal places, yielding a final answer of 12.512 g. Notice that the divisor of 3 does not effect the rounding of the final answer. This is because 3 is an exact number - known to an infinite number of decimal places.