Chapter "15" oscillation

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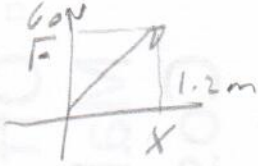
(1) $m = 5\,kg$

$X = A\cos(\omega t)$

$F = kx$

(-) restoring force

$k = \frac{F}{x}$



$k = \frac{60}{1.2} = 50\,N/m$

a) $T = \frac{2\pi}{\omega}$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{50}{5}} = 10\,\frac{rad}{s}$

b) $f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = 1.6\,Hz$

c) $A = 1.2\,m$

d) $X = A\cos(\omega t)$

$X_{(2)} = 1.2\cos(10 \times 2) = 0.5\,m$

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(2) $m = 0.2\,kg$

$k = 25\,N/m$

$x_m = 0.10\,m$

$f = ?$

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.02}} = 5.6\,Hz$

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(3) $v_{max} = 6\,m/s$

$m = 3\,kg$

$a_{max} = 5\,m/s^2$

$T = ?$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

$v_{max}\,\omega = a_{max}$

$6\omega = 5$

$\therefore \omega = \frac{5}{6} = \frac{2\pi}{T}$

$\therefore T = \frac{2\pi}{5}6$

$T = 7.54\,s$

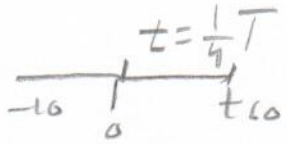
(4) $m = 0.5\,kg$ $\quad t = \frac{1}{4}T$

$K = 8\,N/m$ $\quad T = 2\pi\sqrt{\frac{m}{k}}$

$A = 0.10\,m$ $\quad = 2\pi\sqrt{\frac{0.5}{8}} = 1.57\,s$

$t = ?$ $\quad t = \frac{1}{4}T = 0.3935$

$x = 0 \to x = 10\,cm$



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(5) $A = 0.1\,m$ $\quad E_t = \frac{1}{2}kX_m^2$ $\quad U = \frac{1}{2}kx^2$

$E = 16\,J$ $\quad 16 = \frac{1}{2}k(0.1^2)$ $\quad = \frac{1}{2}(3200)(0.05)^2$

$k = ?$ at $x = 0.05\,m$ $\quad \therefore k = 3200\,N/m$ $\quad = 4\,J$

$E_t = K + 4 \Rightarrow 16 = 4 + k \Rightarrow \boxed{K = 12\,J}$

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(6) $m = 2\,kg$ $\quad \omega = \sqrt{\frac{k}{m}}$

$x = \mp 20\,cm$ $\quad T = \frac{2\pi}{\omega}$

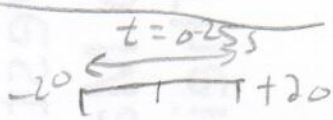
$\therefore A = 0.2\,m$ $\quad \therefore T = 2\pi\sqrt{\frac{m}{k}}$

$t = 0.25\,s$ $\quad \therefore T^2 = 4\pi^2\frac{m}{k}$

$E_t = ?$ $\quad \therefore k = 4\pi^2\frac{m}{T^2}$

$= 4\pi^2\frac{2}{0.5^2}$

$k = 315.8\,N/m$



$T = 2t = 0.5\,s$

$E_t = \frac{1}{2}kX_m^2$

$= \frac{1}{2}(316)(0.2)^2$

$\boxed{E = 6.3\,J}$

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(7) $m = 2\,kg$ $\quad E_t = \frac{1}{2}kX_m^2$ (1)

$k = 8\,N/m$ $\quad E_t = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ (2)

$A = 0.05\,m$

$v = ?$ if $x = 0.03\,m$ $\quad \therefore \frac{1}{2}mv^2 = \frac{1}{2}k(X_m^2 - x^2)$

$\therefore v = \sqrt{\frac{k}{m}(X_m^2 - x^2)} = \sqrt{\frac{8}{2}(0.05^2 - 0.03^2)}$

$\boxed{v = 0.08\,m/s}$

(8) $x = x_m \cos(\omega t + \frac{\pi}{6})$

at $t=0$ $\frac{u}{E_t} = ?$

$u = \frac{1}{2} k x^2 = \frac{1}{2} k \left[x_m^2 \cos^2(\omega t + \frac{\pi}{6}) \right]$ ①

$E_t = \frac{1}{2} k x_m^2$ ②

$\frac{①}{②} \Rightarrow \frac{u}{E_t} = \frac{\frac{1}{2} k x_m^2 \cos^2(\omega t + \pi/6)}{\frac{1}{2} k x_m^2}$

$= \cos^2(\omega t + \frac{\pi}{6})$

$= \cos^2(\pi/6) = 0.75$

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(9) $m = 6kg$ $\quad B = 0.08 \cos(4.43t + \pi)$

$L = ??$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$ $\quad T = \frac{2\pi}{\omega}$

$\therefore \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$

$\therefore \frac{1}{\omega^2} = \frac{L}{g}$

$\therefore L = \frac{g}{\omega^2}$

$L = \frac{g}{4.43^2}$

$L = 0.5m$

(10) $m = 3kg$

$X = 2\cos(50t)$

$E = ?$

$E_t = K_{max} = U_{max}$

$E_t = K_m = \frac{1}{2} m v_{max}^2$

$v_{max} = A\omega$

$\therefore E_t = \frac{1}{2} m A^2 \omega^2$

$= \frac{1}{2}(3)(2)^2(50)^2$

$\boxed{E_t = 1.5 \times 10^4 J}$

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(11) $f = 5Hz$ $\quad v(0) = 0$ ①

$X(t=0.02s) = ?$ $\quad X(0) = 0.5$ ②

$X_0 = 0.5m$

$T = \frac{2\pi}{\omega}$

$\therefore f = \frac{\omega}{2\pi}$

$\therefore \omega = 2\pi(5)$

① $\Rightarrow v = -\omega X_m \sin(\omega t + \phi)$

$0 = -\omega X_m \sin(0 + \phi)$

$0 = \sin(\phi) \Rightarrow \boxed{\phi = 0}$

② $\Rightarrow X = X_m \cos(\omega t + \phi)$

$0.5 = X_m \cos(0 + \phi)$

$\boxed{\therefore 0.5 = X_m}$

$\therefore X = X_m \cos(\omega t + \phi)$

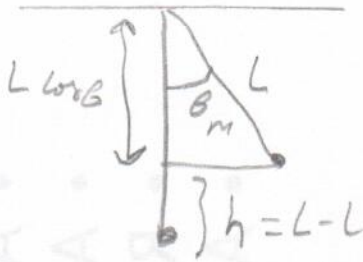
$X_{\underset{t=0.02}{=}} 0.5 \cos[(2\pi(5))(0.02)]$

$\boxed{= 0.4\,m}$

(12) $M = 0.3\,kg$

$L = 2\,?$

$E_t = K_{max} = U_{max}$



$K_{max} = mgh = mgL(1-\cos\theta_m)$ (1)

$\theta_m = 100 \times 10^{-3}\,rad \dfrac{360^\circ}{2\pi\,rad} = 5.73^\circ$ (2)

$K_{max} = 30 \times 10^{-3}\,J$ (3)

(2) (3) in (1) $\Rightarrow$

$30 \times 10^{-3} = 0.3\,g\,L(1-\cos 5.73)$

$\Rightarrow L = 2.04\,m$

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(13) $K_{max} = \frac{1}{2} m v_m^2 = \frac{1}{2} m\omega^2 X_m^2 = U_{max} = \frac{1}{2} k X_m^2$

$K_{max} \propto \omega^2 \propto \dfrac{1}{T^2}$

$\therefore$ C, B, A

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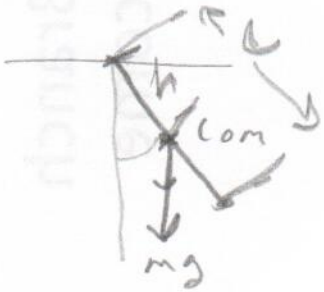
(14) $L = 1\,m$

$T = 2\pi\sqrt{\dfrac{I}{mgh}}$ $\quad I = \frac{1}{3}mL^2$ $\quad h = \frac{L}{2}$



$\therefore T = 2\pi\sqrt{\dfrac{\frac{1}{3}mL^2}{mg\frac{L}{2}}}$

$= 2\pi\sqrt{\frac{2}{3}\frac{L}{g}} \Rightarrow \boxed{T = 1.64\,s}$

(15) $R = 2.35\,cm$ $\quad T = 2\pi\sqrt{\frac{I}{mgh}}$ (3)

$= 0.0235\,m$

$d = 0.0175\,m$ $\quad \boxed{h = d}$ (1)

$I_{cm} = \frac{1}{2}mR^2$

$I_o = I_{cm} + Md^2$

$= \frac{1}{2}mR^2 + Md^2$ (2)

(1) and (2) in (3) $\Rightarrow T = 2\pi\sqrt{\frac{\frac{1}{2}mR^2 + Md^2}{mgd}}$

$T = 0.3665$

(16) $T = 2\pi\sqrt{\frac{I}{mgh}}$ (1)

$\boxed{h = R}$ (2)

$I_o = I_{com} + mR^2 = \frac{1}{2}mR^2 + MR^2 = \frac{3}{2}MR^2$ (3)

(3) and (2) in (1) $\Rightarrow$

$T = 2\pi\sqrt{\frac{\frac{3}{2}MR^2}{MgR}} = \sqrt{\frac{3R}{2g}} = 0.37\,s$