Chapter 15
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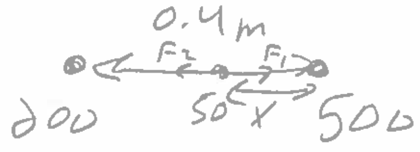
Q1) $\vec{F}_1 = \vec{F}_2$



$$\frac{\cancel{G}(5\cancel{0})(500)}{x^2} = \frac{\cancel{G}(5\cancel{0})(200)}{(0.4-x)^2}$$

$$(5)(0.4-x)^2 = 2x^2$$

$$(10-25x)^2 = 2x^2 \Rightarrow x = 0.245\,m$$

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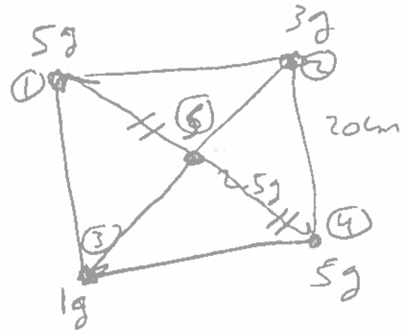
2) $\vec{F}_{net} = \vec{F}_{5g} + \vec{F}_{53}$



$$= \frac{G F_5\, 1}{\left(\frac{\sqrt{2}}{10}\right)^2}\left[m_2 - m_3\right]$$

$$= 1.67 \times 10^{-14}\,N$$

$\frac{20}{2} = 10cm$

$\sin\theta = \frac{10}{x}$

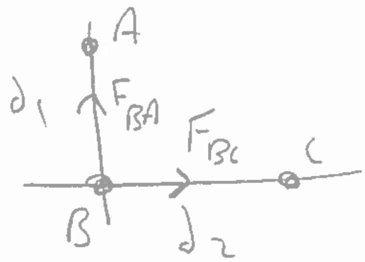
$\Rightarrow \left(x = \frac{\sqrt{2}}{10}\right) m$

$$\therefore F_{net} = 1.67 \times 10^{-14}(\cos 45\, i + \sin 45\, j)$$

Q3) $m = 5\,kg$

$d_1 = 0.3\,m$

$d_2 = 0.4\,m$



$$F_{BC} = \frac{G m_B m_C}{d_2^2} = 1.04 \times 10^{-8}\,N$$

$$F_{BA} = \frac{G m_B m_A}{d_1^2} = 1.85 \times 10^{-8}\,N$$

$$F_{net} = \sqrt{F_{BC}^2 + F_{BA}^2} = 2.16 \times 10^{-8}\,N \qquad \theta = \tan^{-1}\left(\frac{1.85}{1.04}\right) = 60.6^\circ$$

(4) $M_X = M_E$

$R_X = 0.5 R_E$

$g_X = ??$

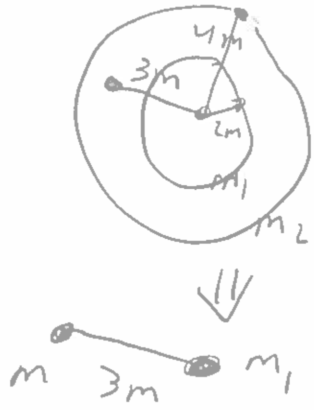
$$F_g = mg = \frac{G m M_E}{R_E^2} \quad (1)$$

$$F_{g_X} = m g_X = \frac{G m M_X}{R_X^2} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{g}{g_X} = \frac{1}{R_E^2} * \frac{R_X^2}{1} = \frac{R_X^2}{R_E^2}$$

$$\therefore \boxed{g_X = g\frac{R_E^2}{R_X^2} = g\frac{\cancel{R_E^2}}{0.5^2 \cancel{R_E^2}} = \frac{g}{\frac{1}{4}} = 4g}$$

(Q5) No force from shell (2) on (m)



$\boxed{F_{2m} = 0}$

$$F_{1m} = \frac{G m m_1}{3^2} = \frac{G m m_1}{9}$$

(Q6) $W = -\Delta U = -(U_f - U_i)$

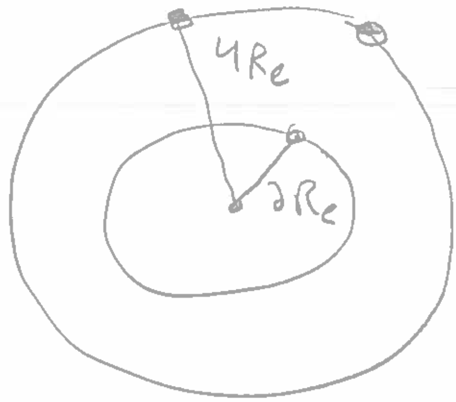
$$= -\left(-\frac{GmM}{R} - \left(-\frac{GmM}{\cancel{\infty}}\right)\right)$$

$$= +\frac{GmM}{R} = \frac{(6.67\times10^{-11})(955)(7.36\times10^{22})}{1.74\times10^{6}}$$

$$\boxed{W = 2.8\times10^{9}\ \text{J}}$$

⑦ $m = 1000\,kg$

$\Delta E = E_f - E_i$



$$= -\frac{GmM}{2R_f} - \left(-\frac{GmM}{2R_i}\right)$$

$$= +\frac{GmM}{2}\left(\frac{-1}{R_f} + \frac{1}{R_i}\right) = \frac{GmM}{2R_e}\left(\frac{-1}{4} + \frac{1}{2}\right)$$

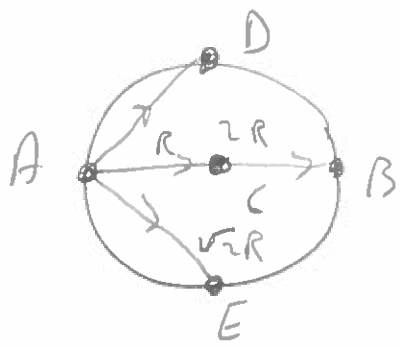
$$= \frac{GmM}{2R_e}\left(\frac{1}{4}\right) = \frac{GmM}{8R_E}$$

$$= \frac{(6.67\times10^{-11})(1000)(5.98\times10^{24})}{8(6.37\times10^{6})}$$

⑧

$$= 7.8\times10^{9}\,J$$

$F_{net,A} = F_{AB} + F_{AC} + F_{AD} + F_{AE}$



$u_{net,A} = -(u_{AB} + u_{AC} + u_{AD} + u_{AE})$

$$= -Gmm\left[\frac{1}{R} + \frac{1}{2R} + \frac{1}{\sqrt{2}R} + \frac{1}{\sqrt{2}R}\right]$$

but $(U_{net,A} + KE)_i = 0$ $(K, U)_\infty = 0$

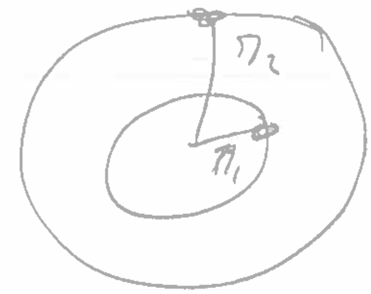
$\therefore KE = u_{net,A}$

$$\frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m}m}{R}\left[1 + \frac{1}{2} + \frac{2}{\sqrt{2}}\right]$$

$$v = \left[(3+2\sqrt{2})\frac{Gm}{R}\right]^{1/2}$$

(9) $\Delta E = E_f - E_i$

$$= \frac{-GmM}{2r_f} + \frac{GmM}{2r_i}$$

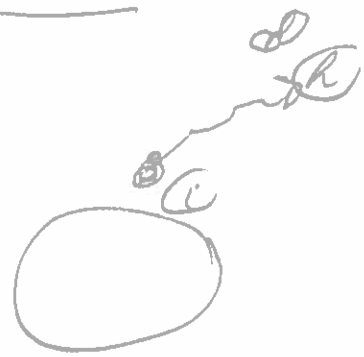


$$= \frac{GmM}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \frac{(6.67\times10^{-11})(1300)(5.98\times10^{24})}{2}\left(\frac{1}{0.665\times10^{7}} - \frac{1}{4.23\times10^{7}}\right)$$

$$= 3.29\times10^{10}\ \text{J}$$

(10)



$E_i = E_f$

$$\frac{1}{2} m v_i^2 - \frac{GMm}{R_e} = \overset{(0)}{KE_f} + U_f$$

$$\frac{1}{2} \cancel{m} v_i^2 - \frac{G\cancel{m}M}{R_e} = 0 - \frac{GmM}{(R+h)}$$

$$\cancel{\tfrac{1}{2}} v_i^2 = 2GM\left(\frac{1}{R_E} - \frac{1}{(R_E+h)}\right)$$

$$\left(\frac{1}{2} v_{es}\right)^2 = 2GM\left(\frac{1}{R_E} - \frac{1}{r}\right)$$

$$\frac{1}{4}\left(\frac{\cancel{2}\cancel{G}\cancel{M}}{R}\right) = \cancel{2}\cancel{G}\cancel{M}\left(\frac{1}{R_E} - \frac{1}{r}\right)$$

$$\frac{1}{R_E} = \frac{4}{R_E} - \frac{4}{r} \Rightarrow \frac{1}{R_E} - \frac{4}{R_E} = -\frac{4}{r}$$

$$\Rightarrow 4R_E = 3r = 3(R_E+h)$$

$$\therefore \boxed{h = \frac{1}{3} R_E}$$

Escape speed:

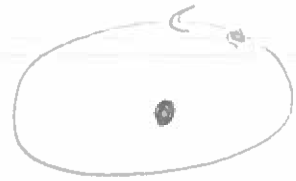
$E_i = E_f = 0$

$$\frac{1}{2} m v_{es}^2 - \frac{GMm}{R} = 0$$

$$\boxed{v_{es} = \sqrt{\frac{2GM}{R}}}$$

(11) $R = 9.4 \times 10^6 \, m$

$T = 2754 \times 10^4 \, s$



$$\frac{T}{r^3} = \frac{4\pi^2}{Gm}$$

$\therefore m = 6.5 \times 10^{23} \, kg$

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(12) $L_A = L_B$

$\cancel{m} v r)_A = \cancel{m} v r_B$

$$\frac{v_A}{v_B} = = \frac{r_B}{r_A}$$

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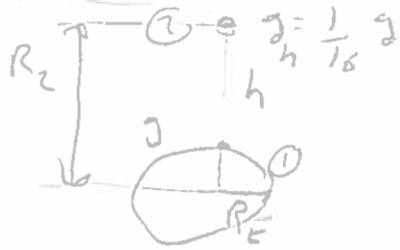
(13) $F_g = ma = \dfrac{GmM}{R_E^2}$



$$\frac{F_{g_1}}{F_{g_2}} = \frac{\cancel{m} g)_1}{\cancel{m} g)_2} = \frac{\cancel{G} \cancel{m} \cancel{M_E}}{R_E^2} \times \frac{R_2^2}{\cancel{G} \cancel{m} \cancel{M_E}}$$

$$\frac{g}{\frac{g}{16}} = 16 = \frac{R_2^2}{R_E^2} \Rightarrow 16 R_E^2 = R_2^2 \Rightarrow$$

$4R_E = R_2 = R_E + h$

$\boxed{\therefore h = 3R_E}$

(14) $a_{N_p} = 10.7\ m/s^2$ (a at the pole)

$R_N = 2.5\times10^7\ m$

$T = 16\ h = 57600\ s$

$= 16(3600)\ s$

$a_{Nq} = ?$ (a at equator)

$$w = \frac{2\pi}{T} = \frac{2\pi}{57600} = 1.091\times10^{4}\ rad/s$$

$$a_{Nq} \Rightarrow g = a_g - w^2 R = 10.4\ m/s$$

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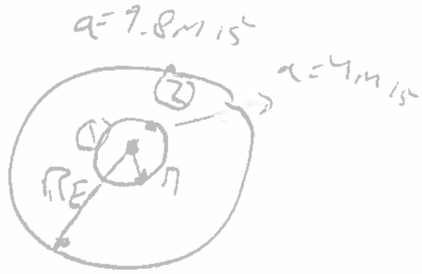
(15) $F_1 = m_1 a_1 = \frac{G m_1 M_E}{R_E^2}$



$F_2 = m_2 a_2 = \frac{G m_2 M_{ins}}{r^2}$

$$\frac{F_1}{F_2} \Rightarrow \frac{a_1}{a_2} = \frac{M_E}{R_E^2} * \frac{r^2}{M_{ins}}$$

$$\rho = \frac{M_E}{V_E} = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

$$\frac{9.8}{4} = \frac{M_E}{M_E \frac{r^3}{R_E^3}} * \frac{r^2}{R_E^2}$$

$$m_{ins} = \rho V_{ins} = \frac{M_E}{\frac{4}{3}\pi R_E^3}\ \frac{4}{3}\pi r^3$$

$$\boxed{m_{ins} = M_E \frac{r^3}{R_E^3}}$$

$$\boxed{R_E = 6370\ km}$$

$$\frac{9.8}{4} = \frac{R_E^3}{r^3} * \frac{r^2}{R_E^2}$$

$$\frac{9.8}{4} = \frac{R_E}{r}$$

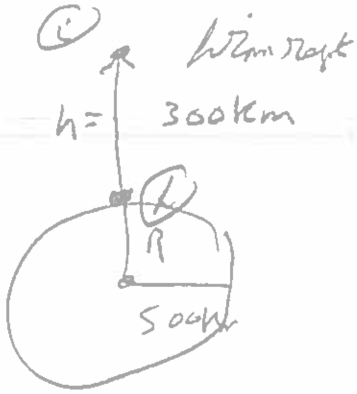
$$\boxed{\therefore r = \frac{4}{9.8} R E} = 2600\ km$$

(16) $a = 3\ m/s^2$ $\boxed{u_i = ??}$

$F_{net} = ma = \frac{GmM}{R^2}$

$\boxed{a = \frac{GM}{R^2}}$

$\therefore GM = aR^2$



$E_f = E_i$

$K_f + U_f = K_i + U_i$

$K_f = 0 + U_i - U_f$

$$\frac{1}{2} m u_f^2 = -\frac{GmM}{R+h} + \frac{GmM}{R}$$

$$u^2 = 2GM\left[\frac{1}{R} - \frac{1}{R+h}\right] = 2aR^2\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= 2(3)(2.5\times10^{11})\left[\frac{1}{5\times10^5} - \frac{1}{8\times10^5}\right] = 1125000$$

$$u = 1.06\ km/s$$

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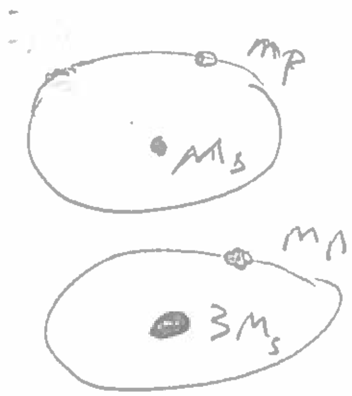
(17)

$$T_1^2 = \frac{4\pi^2}{GM_s} R^3 \quad (1)$$

$$T_2^2 = \frac{4\pi^2}{G\,3M_s} R^3 \quad (2)$$



$\frac{(1)}{(2)} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{3M_s}{M_s}$

$$\boxed{\therefore T_2 = T_1/\sqrt{3}}$$

(18) $T = 1.77\,days$

$R = 4.22 \times 10^5\,km$

$M_J = ??$

$$T^2 = \frac{4\pi^2}{G M_J} R^3$$

$$\therefore M_J = \frac{4\pi^2 R^3}{G T^2} = 1.9 \times 10^{27}\,kg$$