Aymen Ghannam
Ch "11"

Q1 $M = 4.0\ kg$

$I_{com} = 1.6 \times 10^{-2}\ kg \cdot m^2$

$R = 0.10\ m$

+X-direction

(No slipping)

$v_{com} = 4.0\ m/s$

$E = ??$

$KE = KE_T + KE_{Rot}$

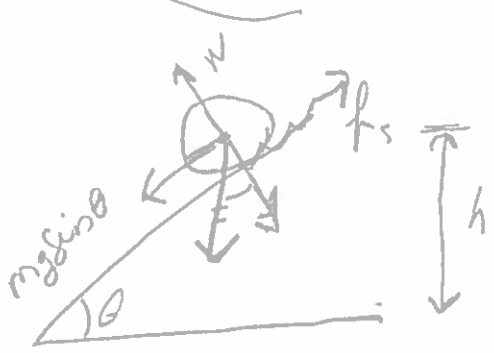
$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$= \frac{1}{2}(4)(4)^2 + \frac{1}{2}(1.6 \times 10^{-2})\left(\frac{4}{0.1}\right)^2$

$= 32 + 12.8$

$= 44.8\ J$

Q3 and **Q2** :-



$F_{net} = ma$

$mg\sin\theta - f_s = ma$ ①

$\tau_{net} = I\alpha$

$f_s R = I \frac{a}{R}$

$f_s = I \frac{a}{R^2}$ ②

② in ① ⇒

$mg\sin\theta - I\frac{a}{R^2} = ma$

$mg\sin\theta = (m + \frac{I}{R^2})a$

$a = \frac{mg\sin\theta}{m + (I/R^2)}$

$$\boxed{a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}}$$

$I_{cy} = \frac{1}{2} M R^2$

$v^2 = v_0^2 + 2aL$ ($v_0 = 0$)

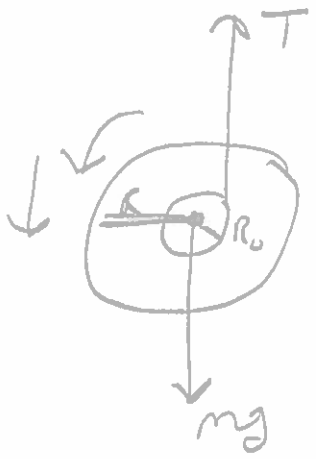
$$\boxed{v = \sqrt{\frac{2g\sin\theta L}{1 + \frac{I}{mR^2}}}}$$

$v_{cy} = \left(\frac{2g(h/L)L}{1 + \frac{1}{2}\frac{MR^2}{MR^2}}\right)^{1/2}$

$= \left(\frac{2gh}{\frac{3}{2}}\right)^{1/2} = \sqrt{\frac{4}{3}gh}$

$\sin\theta = h/L$

Q4 The Yo-Yo



$mg - T = ma$ (1)

$\tau_{net} = I\alpha$

$T R_0 = I \frac{a}{R_0}$ (2)

$\therefore T = I \frac{a}{R_0^2}$ (3)

(3) in (1) $\Rightarrow$

$mg - I \frac{a}{R_0^2} = ma$

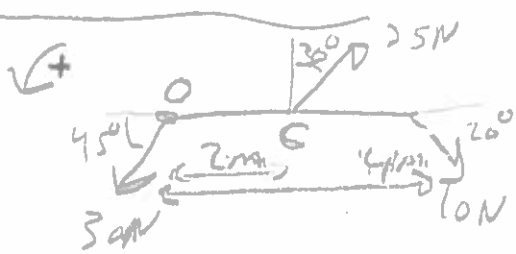
$mg = ma + I \frac{a}{R_0^2}$

$a\left(m + \frac{I}{R_0^2}\right) = mg$

$a = \frac{mg}{m + \frac{I}{R_0^2}}$

$a = \frac{g}{1 + \frac{I}{MR_0^2}}$

Q5



a) about O-axis:

$\vec{\tau}_{net} = \vec{\tau}_{10} + \vec{\tau}_{25}$

$= (0.4)(10\sin 20) + (0.4)(25\cos 30)$

$= -13.7 + 43.3 = +29.6\ N\cdot m$ Counterclockwise

b) about C-axis:

$\vec{\tau}_{net} = \vec{\tau}_{30} + \vec{\tau}_{10}$

$= (2)(30\sin 45) - (2)(10\sin 20)$

$= 42.4 - 6.8 = +35.6\ N\cdot m$ Counterclockwise

Q6 $\vec{F} = (2i + 3j)$ N

$P = (4, 5, 0)$ m

$\tau = ??$

$\vec{r} = (P - \text{origin})$

$\vec{r} = (4i + 5j + 0k)$

$\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} i & j & k \\ 4 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix} = 0i - 0j + (12 - 10)k$$

$$= +2k$$

Q7 $M = 2.0$ kg

$\vec{r} = (9i + 15j)$ m

$\vec{a} = (-3i + 3j)$ m/s^2

$\vec{\tau}_{net} = ?$

$\vec{r}_0 = (9i)$ m

$\vec{R} = \vec{r} - \vec{r}_0$

$= 9i + 15j - 9i$

$\vec{R} = +15j$

$\vec{\tau} = \vec{R} \times \vec{F}$

$= (+15j) \times (-3i + 3j)(2)$

$= (+15j) \times (-6i + 6j)$

$= -(-90k)$

$= +90k$

(Q8) $\omega = 3\ \text{rev/s} = 3 * 2\pi \frac{\text{rad}}{s} = 6\pi\ \text{rad/s}$

$R = 0.75\ m$

$M = 0.15\ kg$

$L = ?$

$L = I.\omega = (mR^2)(\omega)$

$= (0.15)(0.75)^2(6\pi) = 1.6\ kg \cdot m^2/s$

(Q9) $m_1 = 4\ kg$

$m_2 = 3\ kg$

$d = 1.00\ m$

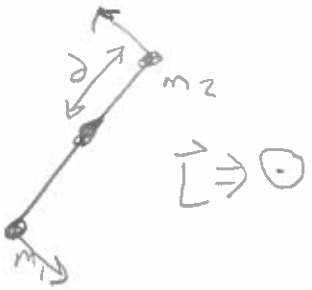
$v = 2\ m/s$

$\vec{L} = \vec{L}_1 + \vec{L}_2$

$L_1 = m_1 \vec{r}_1 \times \vec{v}_1 = +4\left(\frac{1}{2}\right)(2)\,\hat{k} = +4\hat{k}\ kg\,m^2/s$

$L_2 = m_2 \vec{r}_2 \times \vec{v}_2 = +3\left(\frac{1}{2}\right)(2)\,\hat{k} = +3\hat{k}\ kg\,m^2/s$

$L = +7\,\hat{k}\ kg.m^2/s$



(Q6) $m = 2.94\ kg$

$r = 0.200\ m$

$\omega = 6.02\ rad/s$

$I_{com} = \frac{1}{2}mr^2$

a) $\vec{L} = ?$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$= \left(\frac{1}{2}Mr^2\right) \times \vec{\omega}$

$= \frac{1}{2}(2.94)(0.2)^2(6.02)$

$= 0.345\ kg\text{-}m^2/s$ down

b) $h = r/2 = \frac{0.2}{2}$

$= 0.1\ m$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$I = I_{com} + MR^2$

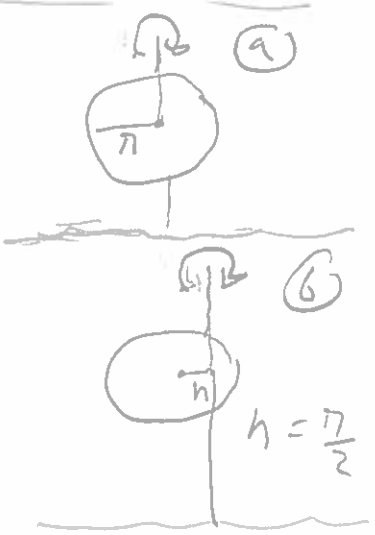
$= \frac{1}{2}Mr^2 + M\left(\frac{r}{2}\right)^2$

$= Mr^2\left(\frac{1}{2} + \frac{1}{4}\right)$

$I = (2.94)(0.2)^2(0.75)$

$L = I\omega = 0.0882 \times 6.02$

$= 0.531\ kg\ m^2/s$ down



---

c) $I_o = I_{com} + MR^2$

$= \frac{1}{2}Mr^2 + Mr^2$

$= Mr^2\left(1 + \frac{1}{2}\right)$

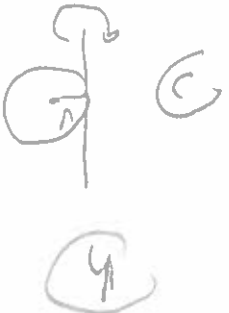
$= (2.94)(0.2)^2\left(\frac{3}{2}\right)$

$I = 0.1764\ kg.m^2$

$\vec{L} = I\vec{\omega} = (0.177)(6.02)$

$= 1.06\ kg\,m^2/s$

down



(11) $\vec{r}(t) = 4t^2\hat{i} - (2t+6t^2)\hat{j}$

$m = 2\,kg$

$\vec{\tau} = ?\quad \vec{\tau} = \frac{d\vec{L}}{dt}$

$\vec{L} = m\,\vec{r} \times \vec{v}$

$\therefore \vec{v} = \frac{d\vec{r}}{dt} = 8t\,\hat{i} - (2+12t)\,\hat{j}$

$\Rightarrow \vec{L} = (2)\,\vec{r} \times \vec{v} = 16t^2\,\hat{k}$

$\therefore \tau = \frac{d\vec{L}}{dt} = 32t\,\hat{k}\ (N.m)$

Note: $\vec{r} \times \vec{v} = (4t^2)(-2-12t) - (8t)(-2t-6t^2)\,\hat{k}$

---

(12) $m = 4.4\,kg$

$R = 9cm = 0.09m$

$M = 2.5\,kg$

a) $\tau = ?$

$\tau = \vec{R} \times \vec{F}$

$= (0.07) \times (4.4)(9.8)\,\hat{k} = 3.88\,\hat{k}$

$v_m = v$

$v_M = \frac{v}{R}$

b) $\vec{L} = \vec{L}_1 + \vec{L}_2$

$= (R \times \vec{p})_m + (I\omega)_M$

$= Rmv + MR^2\frac{v}{R}$

$= Rv(m+M)$

$= 0.621\,kg\,m$

(12) $\tau = \frac{\partial L}{\partial t}$

$$mgR = (M+m)R\frac{\partial v}{\partial t} = (m+M)Ra$$

$$\therefore a = \frac{mg}{m+M} = 6.25\ m/s^2$$

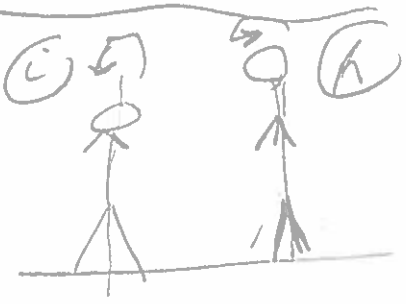
---

(13) $I_i = 45.6\ kg\cdot m^2$

$\omega_i = 32.0\ rad/s$

$I_f = 17.5\ kg\cdot m^2$

$\omega_f = ?$



$$\vec{L_i} = \vec{L_f}$$

$$I\omega)_i = I\omega)_f$$

$$(45.6)(32) = 17.5)(\omega_f)$$

$$\therefore \omega_f = 83.4\ rad/s$$

---

(14) $M_s = 1\ kg$

$R_s = 0.1\ m$

$\omega_s = 4\ rad/s$

$m_h = 0.1\ kg$

$R_h = 0.1\ m$

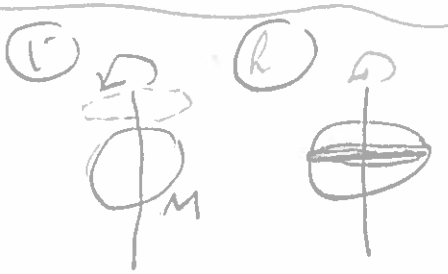
$\omega_f = ?!$

$$\vec{L_i} = \vec{L_f}$$

$$I_s\omega_s + 0 = (I_s + I_h)\omega_f$$

$$\frac{2}{5}R_s^2 M_s = \left(\frac{2}{5}M_s R_s^2 + m_h R_h^2\right)\omega_f$$

$$\therefore \omega_f = 3.2\ rad/s$$



15)

$L_i = L_f$

$I\omega_i + 0 = (I_1 + I_2)\omega_f$

$2I\omega_0 = (2I + I)\omega_f$

$\omega_f = \frac{2}{3}\omega_0$

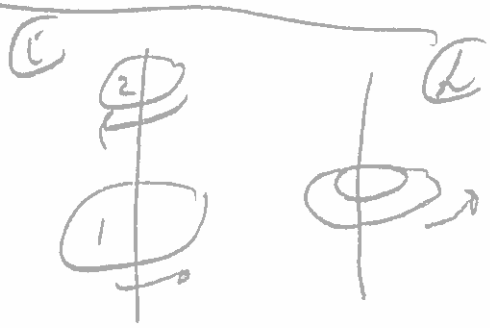
---

16) $I_1 = 4.25\ kg \cdot m^2$

$\omega_{1i} = +15.5\ \frac{rev}{s} = +31\pi\ rad/s$

$I_2 = 1.8\ kg \cdot m^2$

$\omega_{2i} = -14.2\ rev/s \Rightarrow -28.4\pi\ rad/s$



$\omega_f = ?$

$\vec{L}_i = \vec{L}_f$

$\vec{L}_1 + \vec{L}_2 = \vec{L}_f$

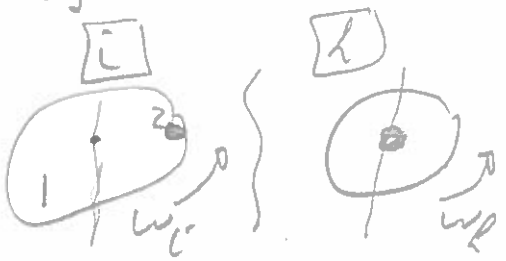
$I_1\omega_{1i} - I_2\omega_{2i} = (I_1 + I_2)\omega_f$

$\omega_f = 41.9\ rad/s$

(17) $R = 2m$

$\omega = 1 \frac{rev}{5\,sec}$

$= 0.2 \frac{rev}{2} = 0.2(2\pi) = 0.4\pi \frac{rad}{s}$



$I = 500 kg\text{-}m^2$

$m_{ch} = 25 kg$

$L_i = L_f$

$L_1 + L_{2i} = L_f$

$I_1 \omega_1 + I_{2i}\omega_2 = (I_1 + I_{2f})\omega_f$

$(500)(0.4\pi) + (MR^2)(0.4\pi) = (500 + 0)\omega_f$

$(0.4\pi)(500 + (25)(4))$

$\frac{754}{500} = \omega_f$

$\therefore \omega_f = 1.51 \frac{rad}{s}$

Q18

$\omega_R = ?$

$m_b = 0.01\,kg$

$m_D = 15\,kg$

$d_D = 1\,m$

$v_b = 400\,m/s$

$I_D = \frac{1}{3} M d^2$

(i) (R)

$L_i = L_R$

$L_i = L_{ib} + L_{iD} = m v d + 0 = (0.01)(400)(\frac{1}{2}) = 2$

$L_R = L_{R(D+b)} = (I_D + I_b)\omega_R$

$I_D = \frac{1}{3} M_D d^2 = \frac{1}{3}(15)(1)^2 = 5\,kg\cdot m^2$

$I_b = m R^2 = (0.01)(\frac{1}{2})^2 = 0.0025\,kg\cdot m^2$

$\therefore L_i = L_R \Rightarrow 2 = (0.0025 + 5)\omega_R$

$\therefore \omega_R = 0.4\,rad/s$

$K_i = \frac{1}{2} m v^2)_b = \frac{1}{2}(0.01)(400)^2 = 800\,J$

$K_R = \frac{1}{2} I_S \omega_S^2) = 0.4\,J$

Q19

$I_{wheel} = 0.25\ kg.m^2$

$R_w = 0.12m$

$m_{hom} = 0.3kg$

$L_f = L_i$

$L_{wheel} + L_{homotes} = 0$

$-I\omega)_w + m(\omega R)_h = 0$

$\therefore \omega_w = \frac{m(\omega R)_h}{I}$

$= \frac{(0.3)(0.12)(3.2)}{0.25} = 0.461\ rad/s$