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Chapter 10

Quantity	Translational	Rotational	
Position	S=rθ	$\theta = \frac{s}{r}$	s = length of arc r = radius θ = angle in radians
Velocity v_t = Linear (tangential) velocity	$v_t = \frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$=v/r	ω= Angular velocity
a_t = tangential acceleration	$a_t = \frac{dv_t}{dt} = r\frac{d\omega}{dt} = r\alpha$	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$=a/r	α = angular acceleration
a_c = Radial (Centripetal) acceleration	$a_c = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$		
Force	$F = ma$	$\vec{\tau} = \vec{r} \times \vec{F} = \tau 1 + \tau 2 + \ldots$ $\tau = I\alpha$	
Mass	m	$I = \sum_i m_i r_i^2$	I = moment of inertia (rotational inertia) m_i = mass of particles i.
Kinetic energy	k	$K_{rot} = \frac{1}{2} I \omega^2$	
Parallel axes theorem		$I = I_{CM} + MD^2$	M = total mass D = distance
Linear momentum	P=mv	$mvr = I\omega$ L=RXP=Iω=l1+l2+…	
	F_{net} = dp / dt If F_{ext} = 0, p_{tot} = constant	τ_{net} = dL / dt If τ_{ext} = 0, L_{tot} = constant	
Linear impulse	J=$F\,t$	τt	
Work	w = $F\ s$	w = $\tau\ \theta$	
Power	p = $F\ v$	p = $\tau\ \omega$	

Table 11-1 More Corresponding Variables and Relations for Translational and Rotational Motion[a]

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau}\ (= \vec{r} \times \vec{F})$
Linear momentum	$\vec{p}$	Angular momentum	$\vec{\ell}\ (= \vec{r} \times \vec{p})$
Linear momentum[b]	$\vec{P}\ (= \Sigma \vec{p}_i)$	Angular momentum[b]	$\vec{L}\ (= \Sigma \vec{\ell}_i)$
Linear momentum[b]	$\vec{P} = M\vec{v}_{com}$	Angular momentum[c]	$L = I\omega$
Newton's second law[b]	$\vec{F}_{net} = \frac{d\vec{P}}{dt}$	Newton's second law[b]	$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$
Conservation law[d]	$\vec{P}$ = a constant	Conservation law[d]	$\vec{L}$ = a constant

[a]See also Table 10-3.
[b]For systems of particles, including rigid bodies.
[c]For a rigid body about a fixed axis, with L being the component along that axis.
[d]For a closed, isolated system.

Quantity	Translational motion along a fixed direction	Rotational motion about a fixed axis
Position	x (m)	θ (rad)
Velocity	v (m/s)	ω (rad/s)
Acceleration	a (m/s^2)	α (rad/s^2)
Mass	m (kg)	I (kg.m^2)
Newton's second law	$F = ma = \frac{dp}{dt}$ (N)	$\tau = I\alpha$ (m.N)
Work	$W = \int F\,dx$ (J)	$W = \int \tau\,d\theta$ (J)
Kinetic energy	$K = \frac{1}{2}mv^2$ (J)	$K = \frac{1}{2}I\omega^2$ (J)
Power	$P = Fv$ (W)	$P = \tau\omega$ (W)
Work-energy theorem	$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ (J)	$W = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$ (J)

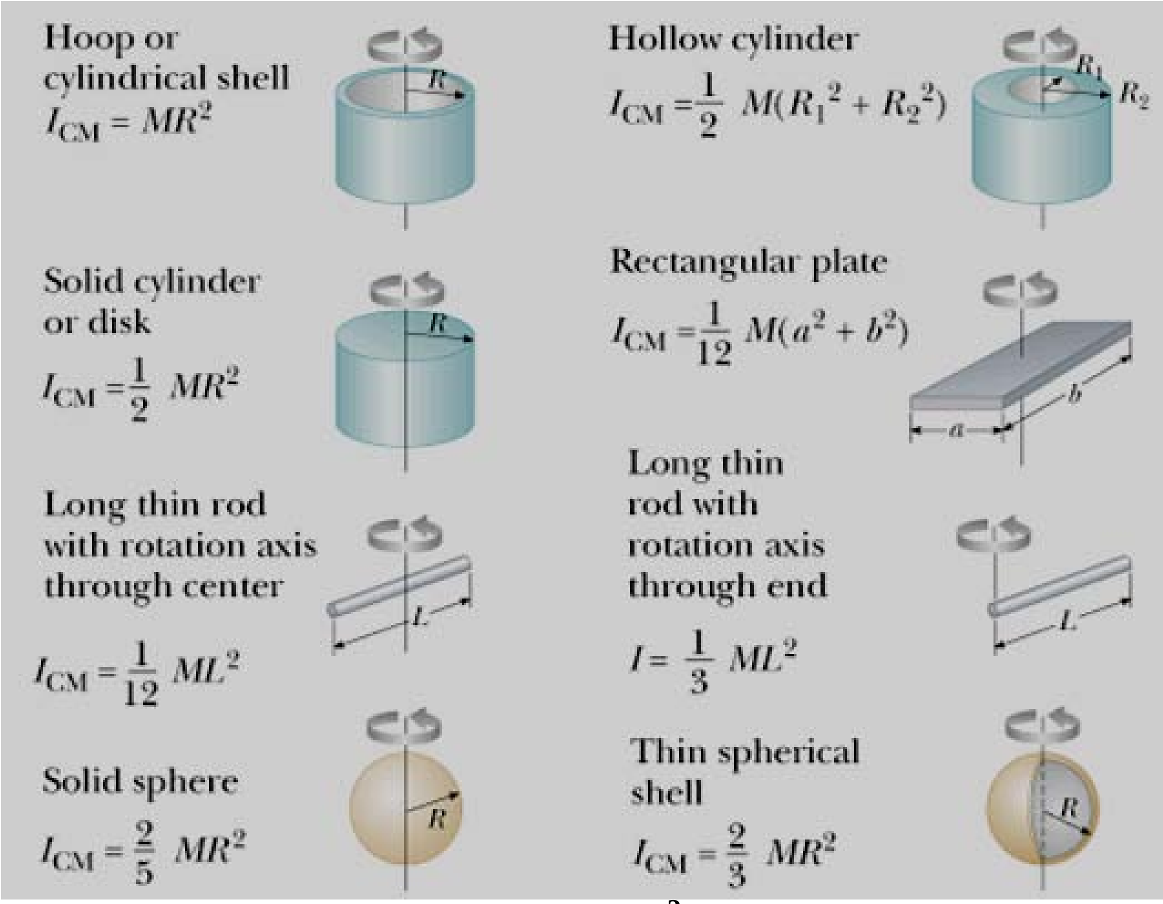
Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)

1- Torque is a vector.
2- Torque is positive when the body rotate counterclockwise (convention)
3- Torque is negative when the body rotate clockwise (convention)

SI unit of torque is **N.m (same as the work); but** Never use Joules as a unit of torque, because Joules is a unit of work.

☞ *Force* causes *linear* acceleration.
☞ *Torque* causes *angular* acceleration.



$$\mathbf{I_{hoop} = MR^2}$$