Chapter 27

Current and Resistance

27-1 Moving Charging and Electric Currents

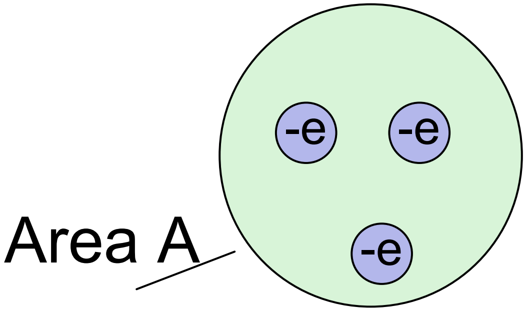
Electrostatics → Charges do not move

Electric current → Charges in motion

27-1 Moving Charging and Electric Currents

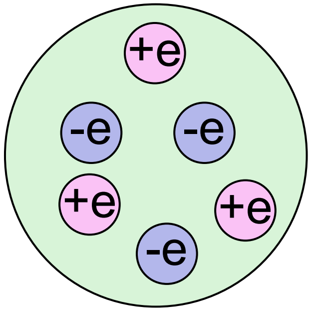
Electric current through a surface is the flow rate of net charges through the surface

3 electrons moving towards us



Current ≠ 0

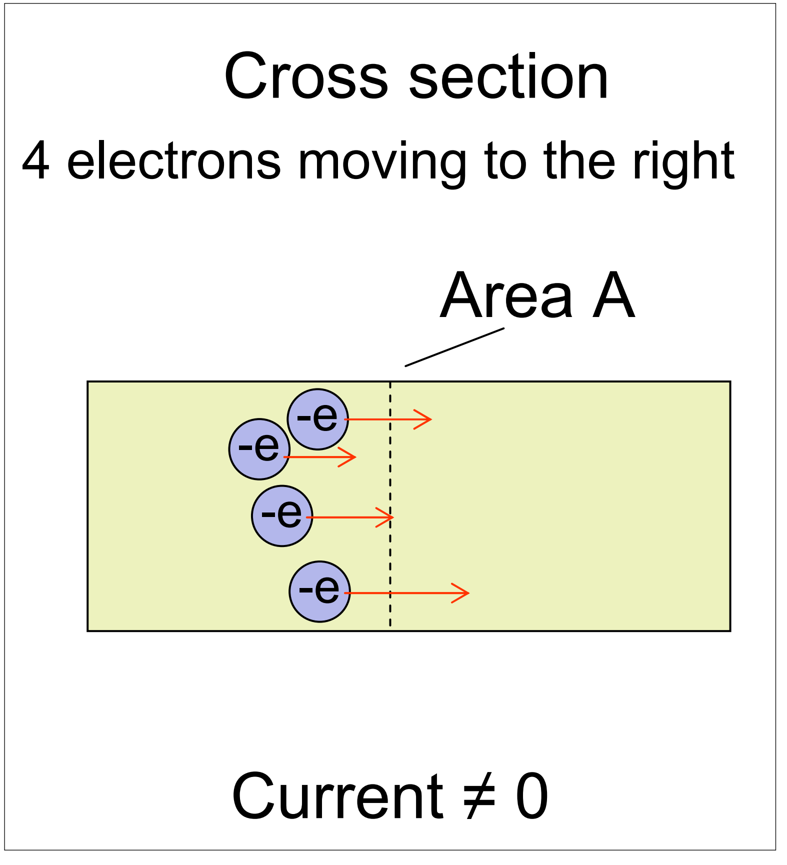
3 electrons and 3 protons moving towards us

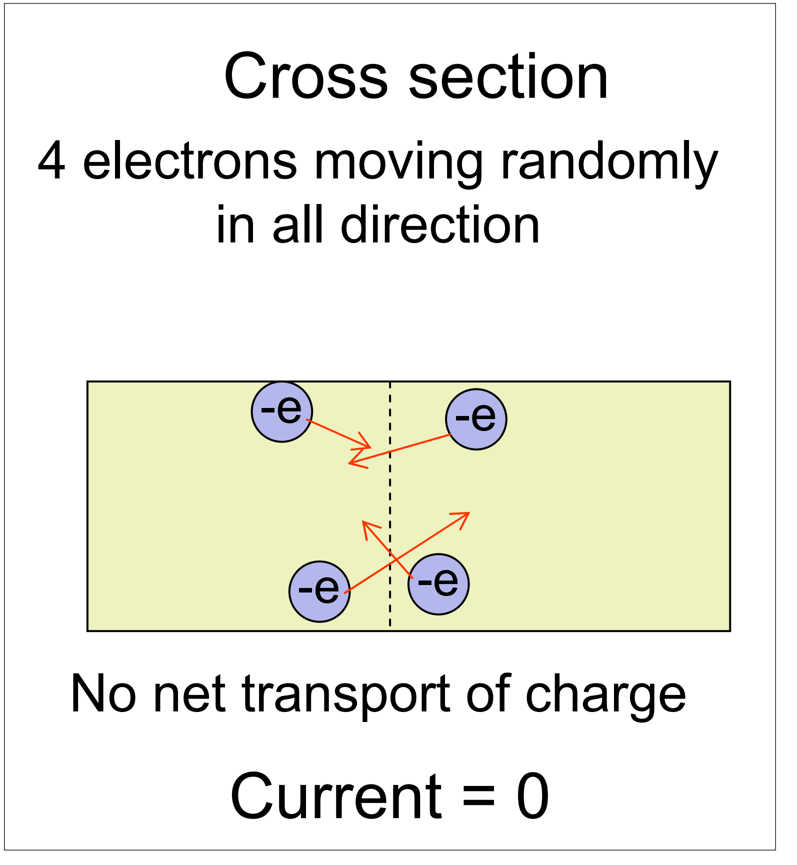


Current = 0

27-1 Moving Charging and Electric Currents

Electric current through a surface is the flow rate of net charges through the surface





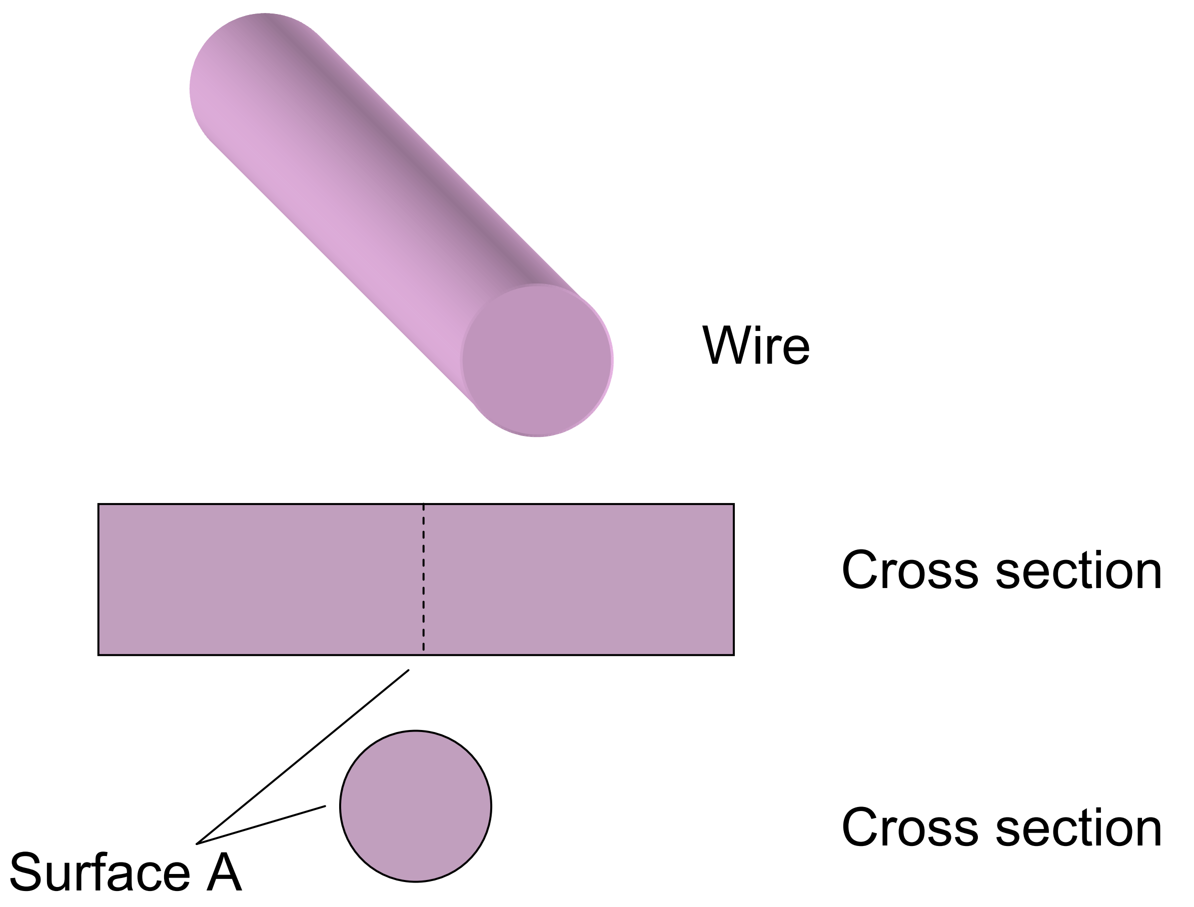
electrons in isolated conductor
(no external electric field applied)
Random motion
Speed ≈ 10^6 m/s

27-2 Electric Currents

Electric current through a surface A is the flow rate of net charges through the surface

Charge dq passes though surface A in time dt

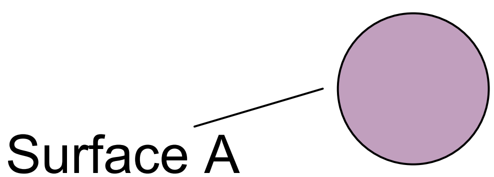
$$i = \frac{dq}{dt}$$



27-2 Electric Currents

$$i = \frac{dq}{dt}$$

Total charge q passes through surface A in time t

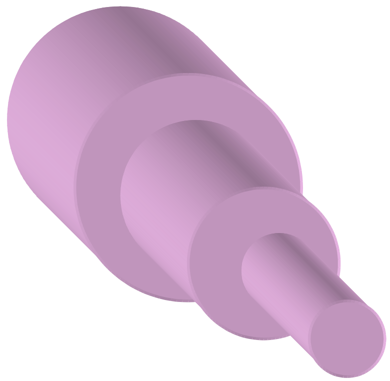


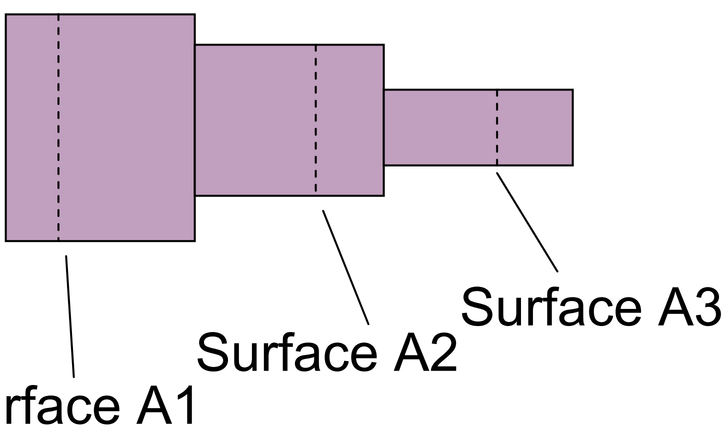
$$q = \int_0^t i\,dt$$

In a steady-state condition, current does not change with time

$$q = i\,t$$

27-2 Electric Currents





In a steady-state condition
Current through A1
= current though A2
= Current though A3

Since charge is conserved,
any electron passed through A1 should pass though A2 and A3

27-2 Electric Currents

SI unit for current

Ampere

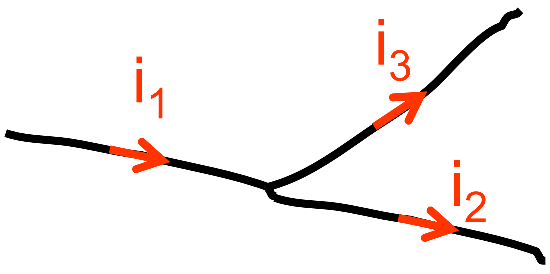
$$i = \frac{dq}{dt}$$

$$1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ Second}}$$

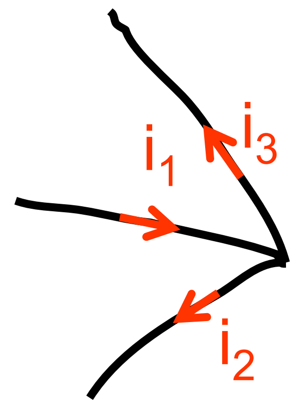
27-2 Electric Currents

Current is a scalar quantity

We use arrows to indicate directions of currents in wires.



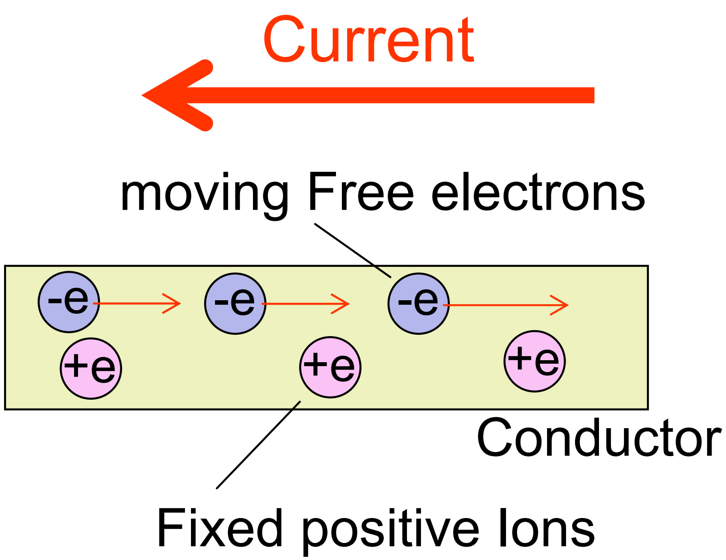
$I_1 = i_2 + i_3$



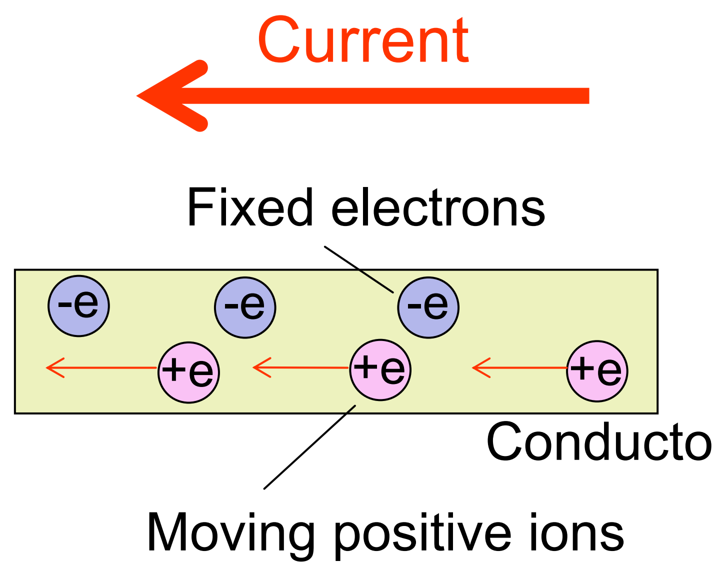
$I_1 = i_2 + i_3$

These arrows are not vectors.

27-2 Electric Currents



Real world

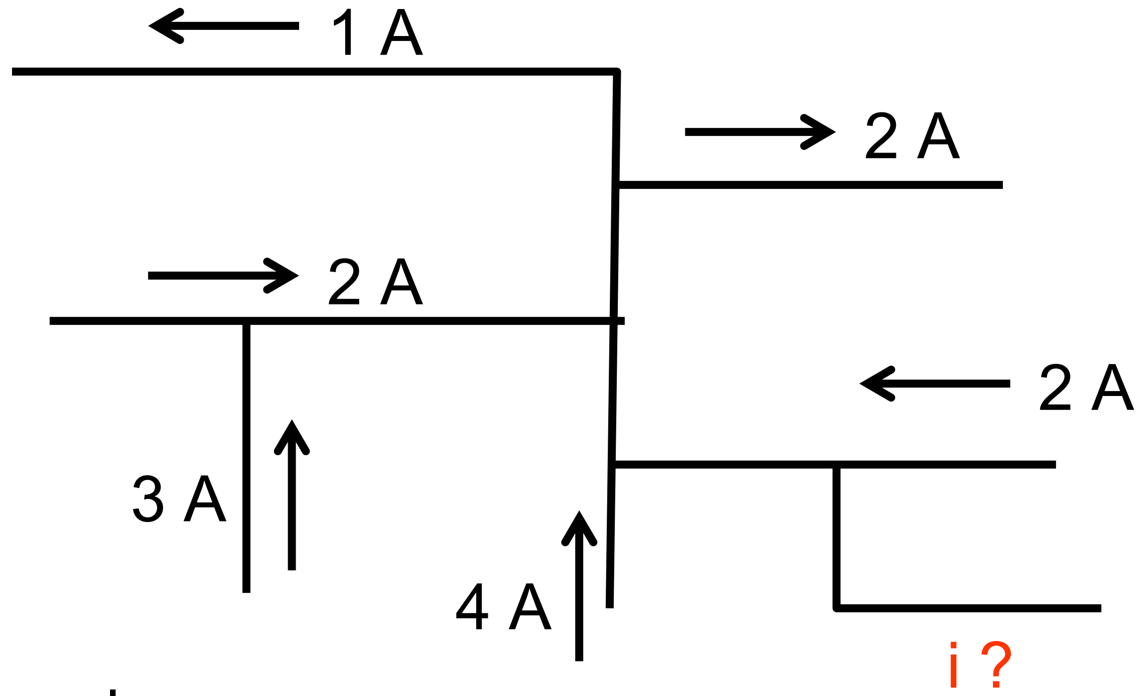


Convention (agreement)
Not physical

A current arrow is drawn the direction in which positive charge would move, even if the actual carriers (electrons) are negative and move in the opposite direction

27-2 Electric Currents

Checkpoint 1



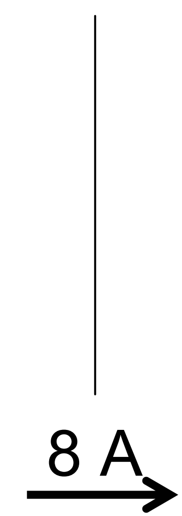
Charge is conserved

Current in = current out

Given current in= 3 + 2+ 4+ 2 = 11A

Given current out = 1 + 2 = 3 A

i = 8 A out

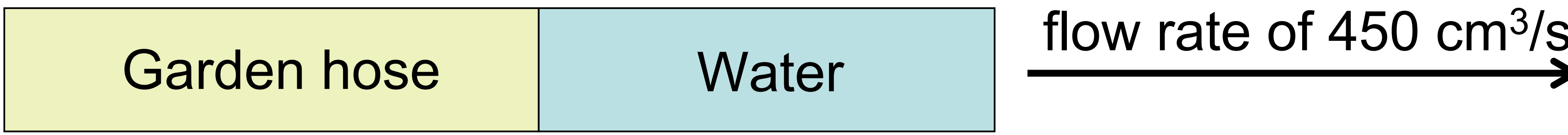


27-2 Electric Currents

Sample Problem 27-1

Water flows from a garden hose at a rate of 450 cm^3/s

What is the current of negative charge?



$$i = \frac{dq}{dt} = \frac{\text{negative charge}}{\text{time}}$$

27-2 Electric Currents

Sample Problem 27-1

$$i = \frac{\text{negative charge}}{\text{time}} = \frac{\text{negative charge}}{\text{volume}} \frac{\text{volume}}{\text{time}}$$

$\frac{\text{volume}}{\text{time}}$ = 450 cm^3/s

$$\frac{\text{negative charge}}{\text{volume}} = \frac{\text{negative charge}}{\text{mass}} \frac{\text{mass}}{\text{volume}}$$

$\frac{\text{mass}}{\text{volume}}$ = water density = 1 g/cm^3

$$\frac{\text{negative charge}}{\text{mass}} = \frac{\text{negative charge}}{\text{mole}} \frac{\text{moles}}{\text{mass}}$$

$\frac{\text{moles}}{\text{mass}}$ = 1/molar mass

= 1/(18 g/mole)

H_2O

oxygen = 8 protons

+ 8 neutorns

hydrogen = 1 proton

$$\frac{\text{negative charge}}{\text{mole}} = \frac{\text{negative charge}}{\text{molecule}} \frac{\text{molecules}}{\text{mole}}$$

$\frac{\text{molecules}}{\text{mole}}$ = Avogadro''s Number

= $6.02X10^{23}$

Molecules/mole

$$\frac{\text{negative charge}}{\text{molecule}} = \frac{\text{negatvie charge}}{\text{electron}} \frac{\text{electrons}}{\text{moleucle}}$$

$\frac{\text{negatvie charge}}{\text{electron}}$ = e = $1.6X10^{-19}$ C/electron

$\frac{\text{electrons}}{\text{moleucle}}$ = 10 electrons/molecule

= 2 from hydrogen atoms and 8 from oxygen atom

27-2 Electric Currents

Sample Problem 27-1

The current of negative charge

$$i = \left(\frac{1.6X10^{-19}\ C}{electron}\right)\left(\frac{10\ electrons}{molecule}\right)\left(\frac{6.02X10^{23}\ molecules}{mole}\right)\left(\frac{1\ mole}{18\ g}\right)\left(\frac{1\ g}{cm^3}\right)\left(\frac{450\ cm^3}{s}\right)$$

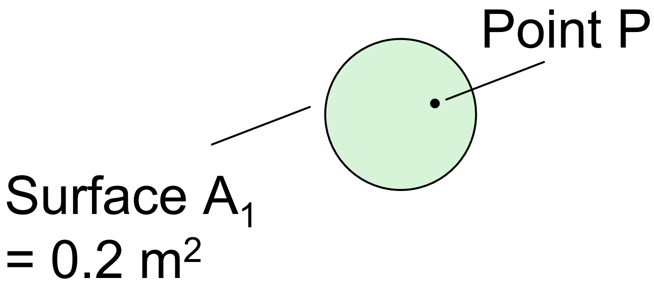
$$i = 24.1X10^{6}\ A$$

The current of positive charge

$$i = 24.1X10^{6}\ A$$

There is no net flow of charge (current = 0) through the hose

27-3 Current density

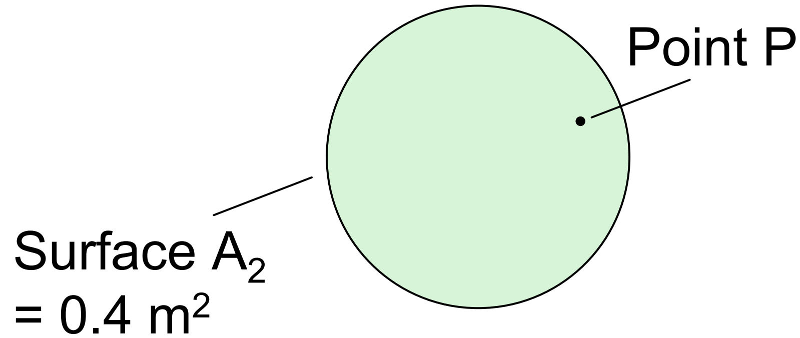


4 coulombs pass per second through surface A_1

Current through A_1 = 4 A

Current density at point P

$$= \frac{\text{current}}{\text{area}} = 20\ A/m^2$$



4 coulombs pass per second through surface A_2

Current through A_2 = 4 A

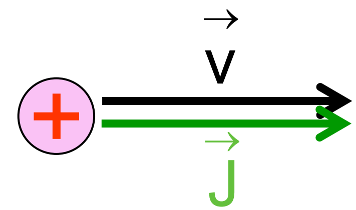
Current density at point P

$$= \frac{\text{current}}{\text{area}} = 10\ A/m^2$$

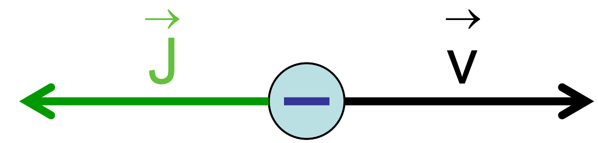
27-3 Current density

Current density is a vector quantity

Direction



Same as the direction of velocity of the moving positive charge



Opposite to the direction of velocity of the moving negative charge

Magnitude

$$J = \frac{\text{current}}{\text{area normal to the velocity}}$$

The SI unit for J is A/m^2

27-3 Current density

$$i = \int \vec{J} \bullet d\vec{A}$$

Area vector

Magnitude	Area of the surface
Direction	Normal to the surface

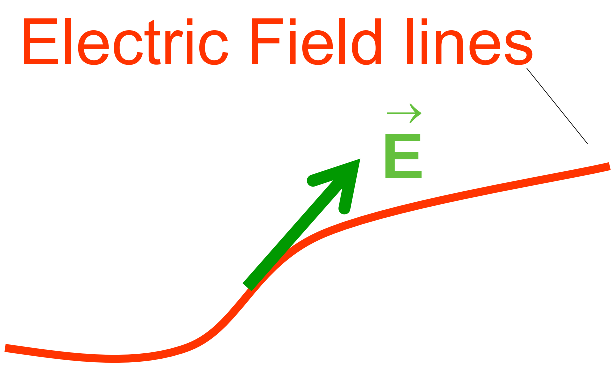
If $\vec{J}$ is uniform and parallel to $d\vec{A}$

$$i = J\,A$$

$$J = \frac{i}{A}$$

27-3 Current density

Electric Field $\vec{E}$



At any point, the tangent of the electric field lines gives the direction of the electric field

Number of lines per unit area in a plane perpendicular to the electric field lines is proportional the magnitude of the electric field

Current Density $\vec{J}$

At any point, the tangent of the streamlines gives the direction of the current density

Number of lines per unit area in a plane perpendicular to the streamlines is proportional the magnitude of the current density

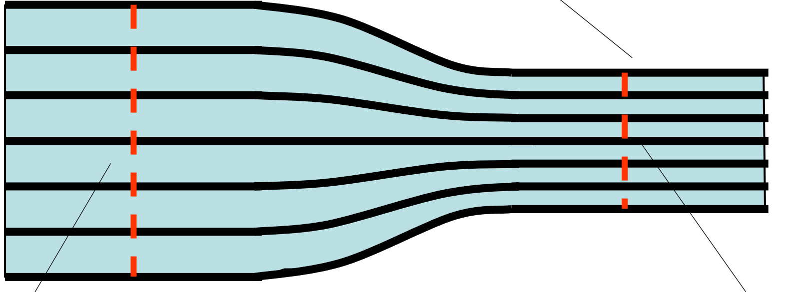
27-3 Current density

Same current

Charge is conserved

(Any charge pass though the 1st surface should pass through the 2nd surface

Streamlines representing current density in the flow of charge through a conductor

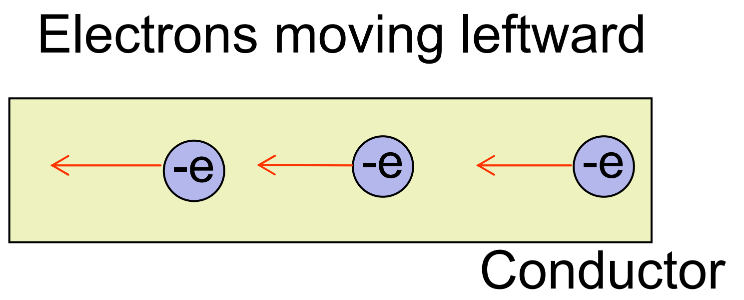


High current density

Low current density

27-3 Current density

Checkpoint 2



What is the direction of …

Current? Rightward →

Current density? Rightward →

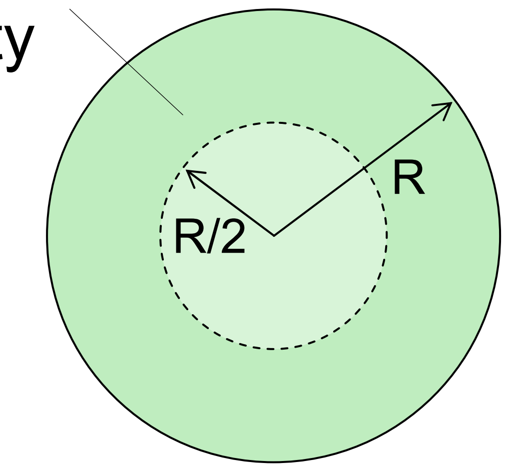
Electric Field? Rightward →

27-3 Current density

Sample Problem 27-2

R = 2.0 mm
$J = 2.0 \times 10^5 \ A/m^2$

What is the current through the outer portion of the wire between radial distances R/2 and R?



Cross section of a wire

$$i = J A$$

$$i = J\left(\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right)$$

$$i = J\left(\frac{3}{4}\pi R^2\right) = 1.9 \ A$$

27-3 Current density

Sample Problem 27-2

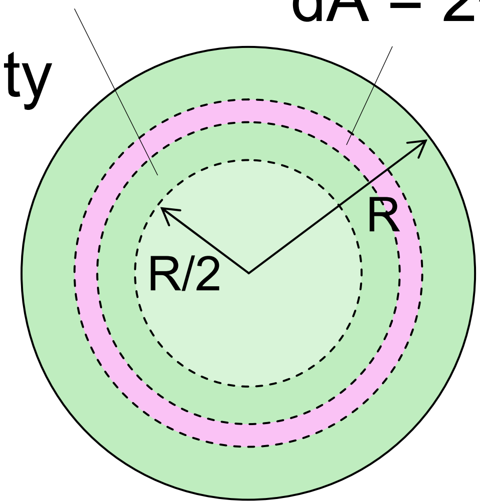
Not uniform current density

$R = 2.0$ mm

$J = a\, r^2$

$a = 3.0 \times 10^{11}$ A/m^2

r in meters



Cross section of a wire

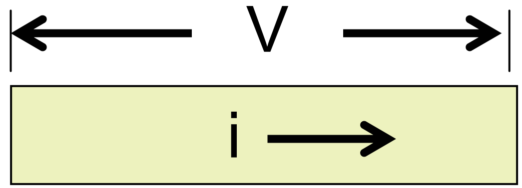
What is the current through the outer portion of the wire between radial distances R/2 and R?

$$i = \int \vec{J} \cdot d\vec{A} = \int J\, dA = \int_{R/2}^{R} J\, 2\pi r dr = \int_{R/2}^{R} a\, r^2\, 2\pi r dr$$

$$= 2\pi a \int_{R/2}^{R} r^3 dr = 2\pi a \left[\frac{r^4}{4} \right]_{R/2}^{R} = 2\pi a \left(\frac{R^4}{4} - \frac{R^4}{64} \right)$$

$$= \frac{15}{32} \pi a R^4 = 7.1 \text{ A}$$

27-4 Resistance and Resistivity



An object
Resistor

Resistor
symbol

$$V = R\,i$$

Potential difference — Resistance — Current

27-4 Resistance and Resistivity

SI unit for resistance

Ohm (Ω)

$$V = R\,i$$

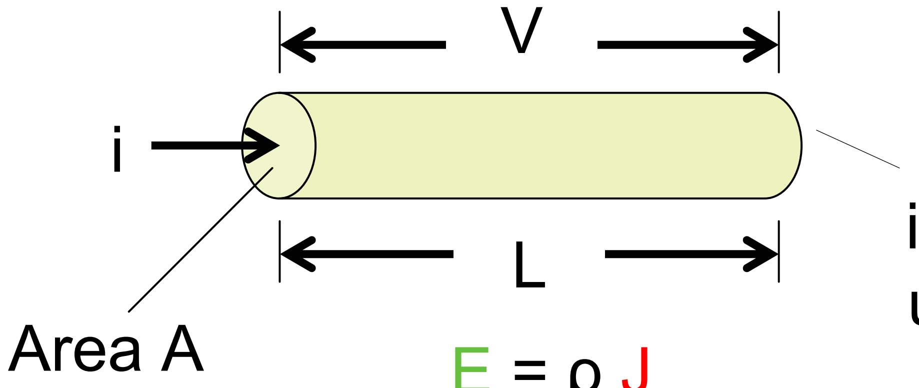
$$1\ \text{Ohm} = \frac{1\ \text{Volt}}{1\ \text{Ampere}}$$

27-4 Resistance and Resistivity

Resistor R of an object	Resistivity ρ of a material
Property of the object Depends on the shape	Property of the material Does not depend on the shape
$V = R\, i$	$\vec{E} = \rho \vec{J}$
SI unit ohm Ω	SI unit ohm•meter $\Omega \cdot m$

27-4 Resistance and Resistivity

Calculating Resistance from Resistivity



A homogenous isotropic conductor of uniform cross section A and length L

$$E = \rho J$$

$$\frac{V}{L} = \rho \frac{i}{A}$$

$$\frac{V}{i} = \rho \frac{L}{A}$$

$$\boxed{R = \rho \frac{L}{A}}$$

homogenous
property does not depend on the position

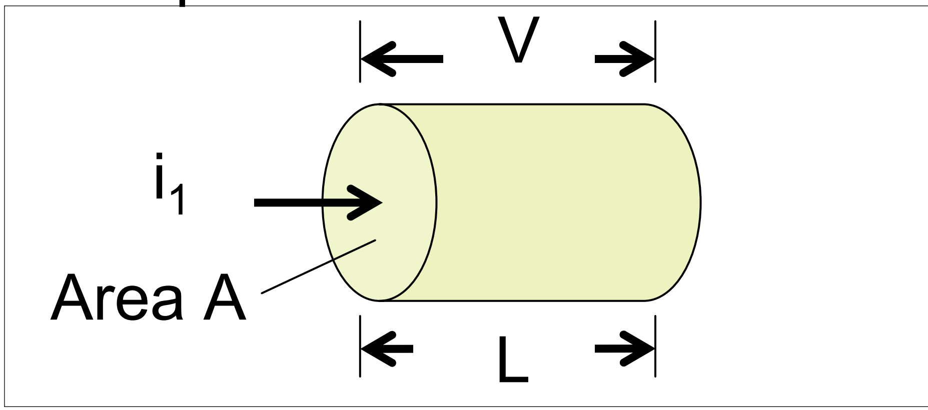
Isotropic
property does not depend on direction

27-4 Resistance and Resistivity

Checkpoint 3

All made of copper

Rank current greatest first

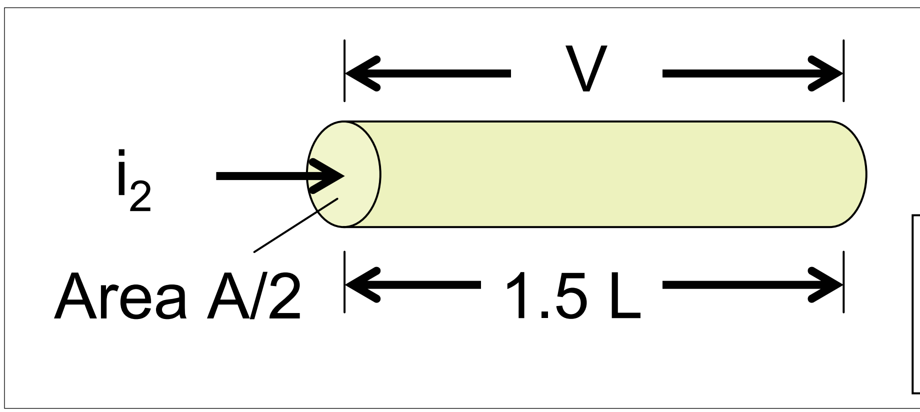


$$i_1 = \frac{V}{\rho}\frac{A}{L}$$

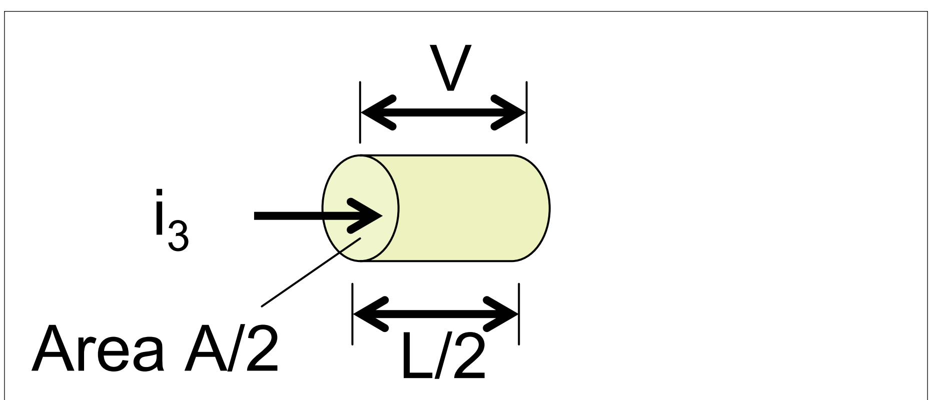
$$i = \frac{V}{R}$$

$$R = \rho\frac{L}{A}$$

$$i = \frac{V}{\rho}\frac{A}{L}$$



$$i_2 = \frac{V}{\rho}\frac{A/2}{1.5\,L} = \frac{1}{3}\frac{V}{\rho}\frac{A}{L}$$



$$i_1 = i_3 > i_2$$

$$i_2 = \frac{V}{\rho}\frac{A/2}{L/2} = \frac{V}{\rho}\frac{A}{L}$$

27-4 Resistance and Resistivity

Variation with Temperature

$$\rho - \rho_0 = \rho_0 \, \alpha \, (T - T_0)$$

Temperature coefficient of resistivity

Current density at T

Current density at T_0

Reference temperature

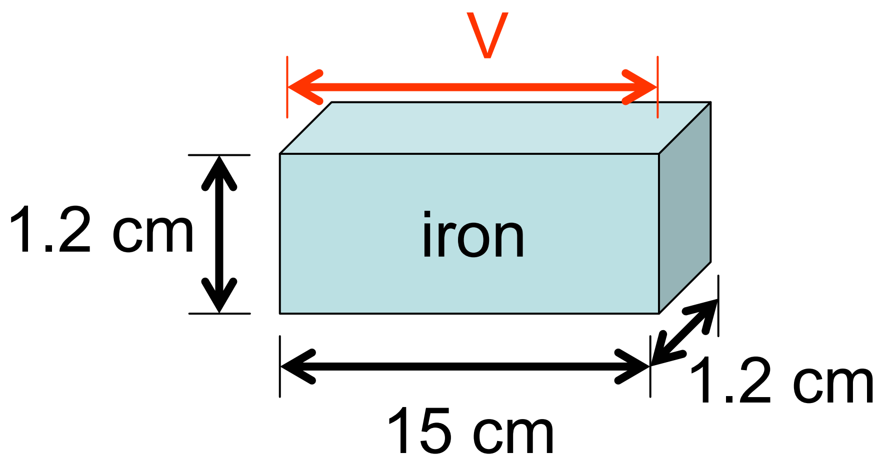
Good approximation over a wide temperature range

27-4 Resistance and Resistivity

Sample Problem 27-4

For iron $\rho = 9.68 \times 10^{-8}\ \Omega.m$

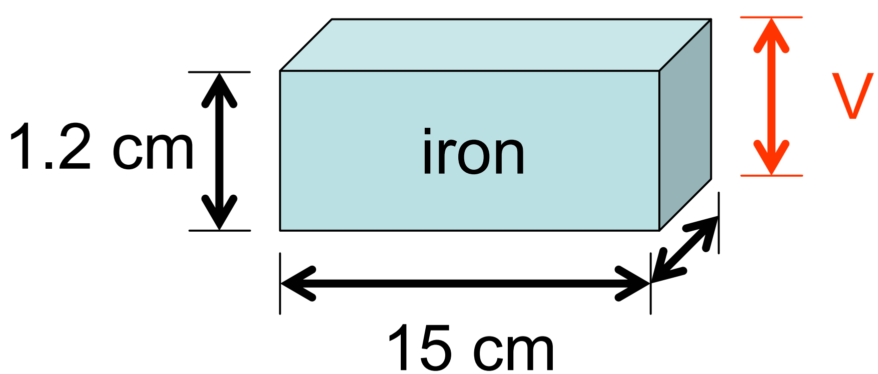
What is the resistance of the block?



$$R = \rho \frac{L}{A}$$

$$R = (9.68 \times 10^{-8}) \frac{0.15}{(.012)(0.012)}$$

$$R = 100\ \mu\Omega$$

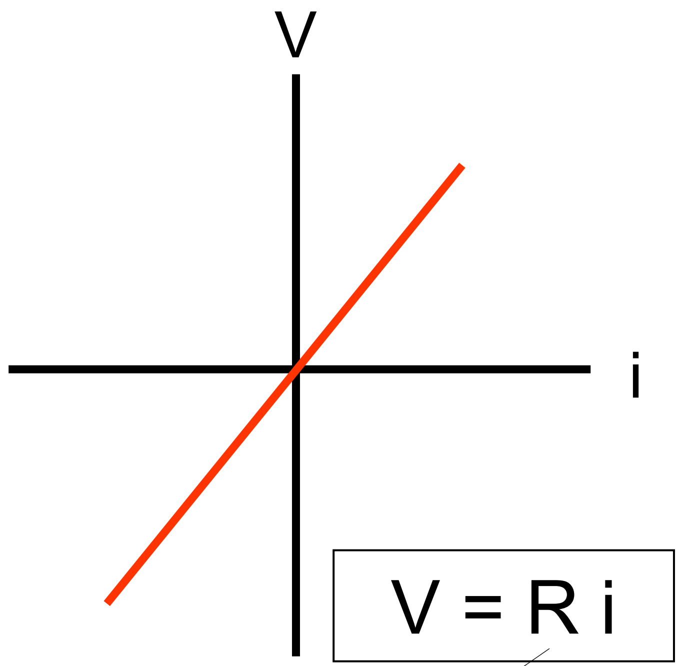


$$R = \rho \frac{L}{A}$$

$$R = (9.68 \times 10^{-8}) \frac{.012}{(0.15)(0.012)}$$

$$R = 0.65\ \mu\Omega$$

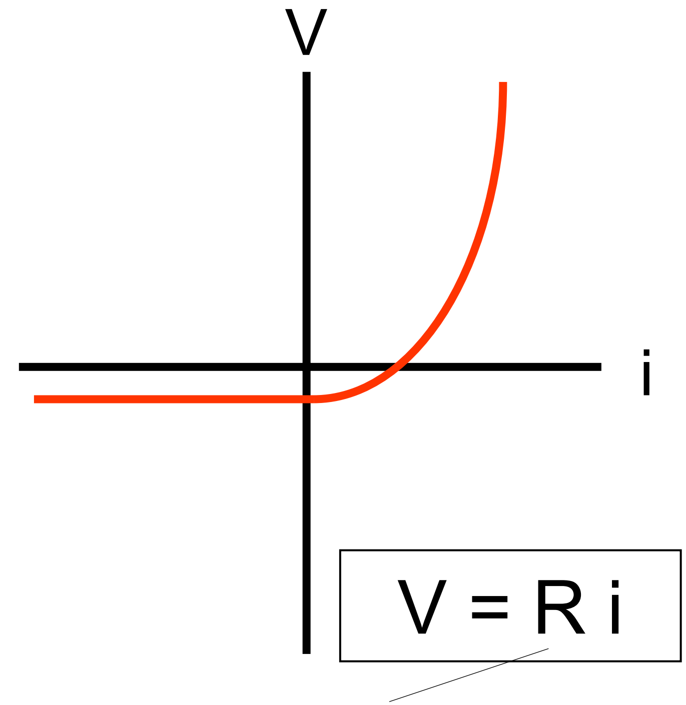
27-5 Ohm's Law



Constant

Current is directly proportional to the potential difference

Device obeys Ohm's law



Function of the potential difference

Device does not obey Ohm's law

27-5 Ohm's Law

Checkpoint 4

Which device does not obey Ohm's law?

Device 1

V	I
2.00	4.50
3.00	6.75
4.00	9.00

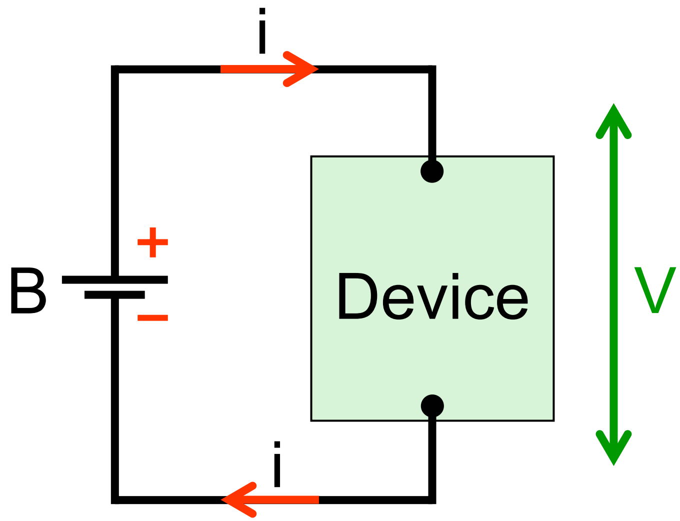
Device 1
obeys Ohm's law

Device 2

V	I
2.00	1.50
3.00	2.00
4.00	2.70

Device 2
does not obey Ohm's law

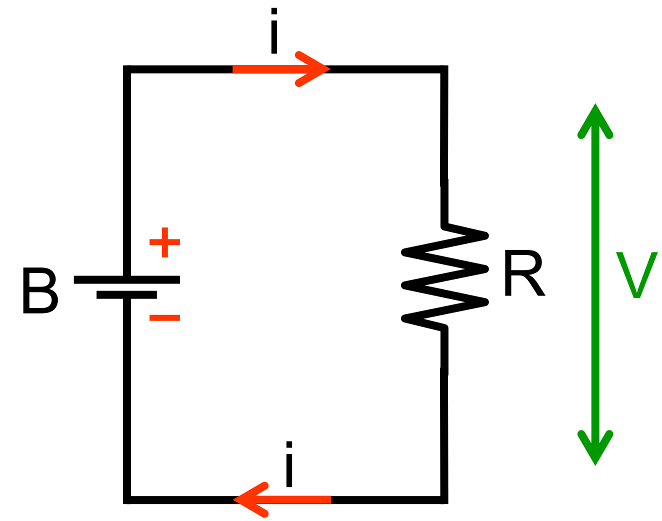
27-7 Power in Electric Circuits



Rate of electric energy transfer from the battery to the device

$$P = i\,V$$

General



Rate of electric energy transfer from the battery to the device

$$P = i\,V$$

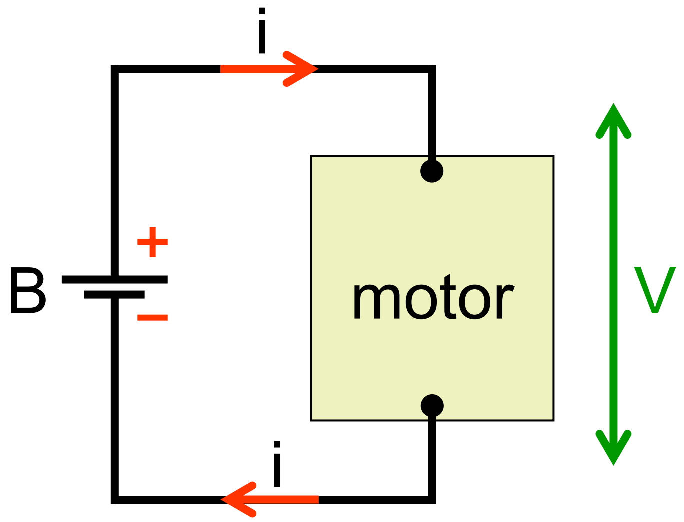
$$P = i^2 R$$

$$P = \frac{V^2}{R}$$

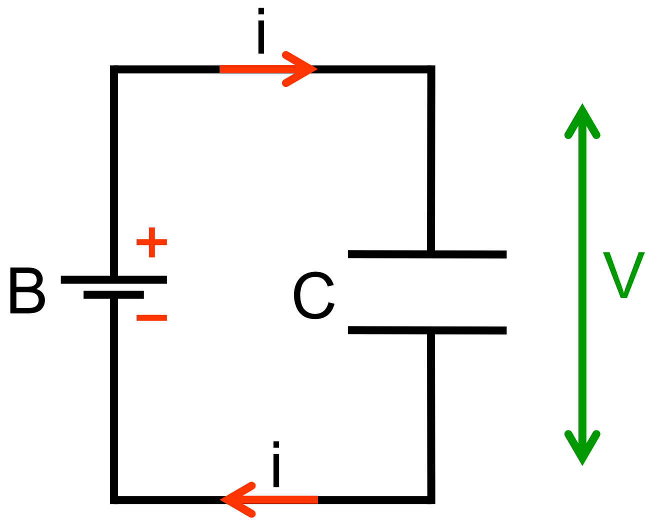
Special case for a resistor

$$V = R\,i$$

27-7 Power in Electric Circuits

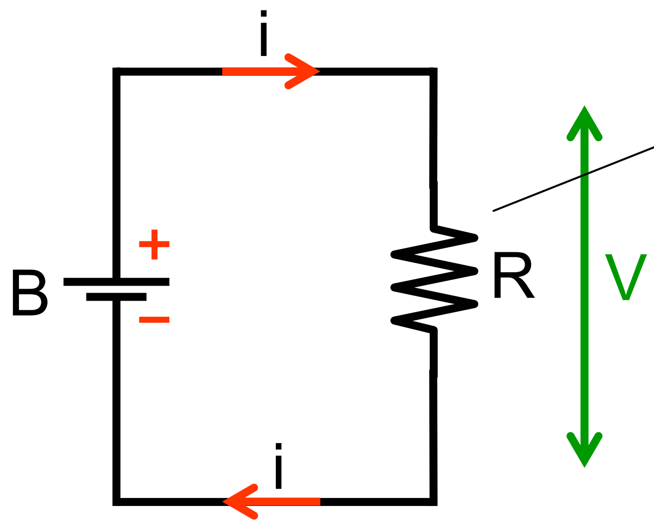


Electric energy from the battery is transferred at a rate of P = i V to rotate the motor



Electric energy from the battery is transferred at a rate of P = i V to charge the capacitor

27-7 Power in Electric Circuits

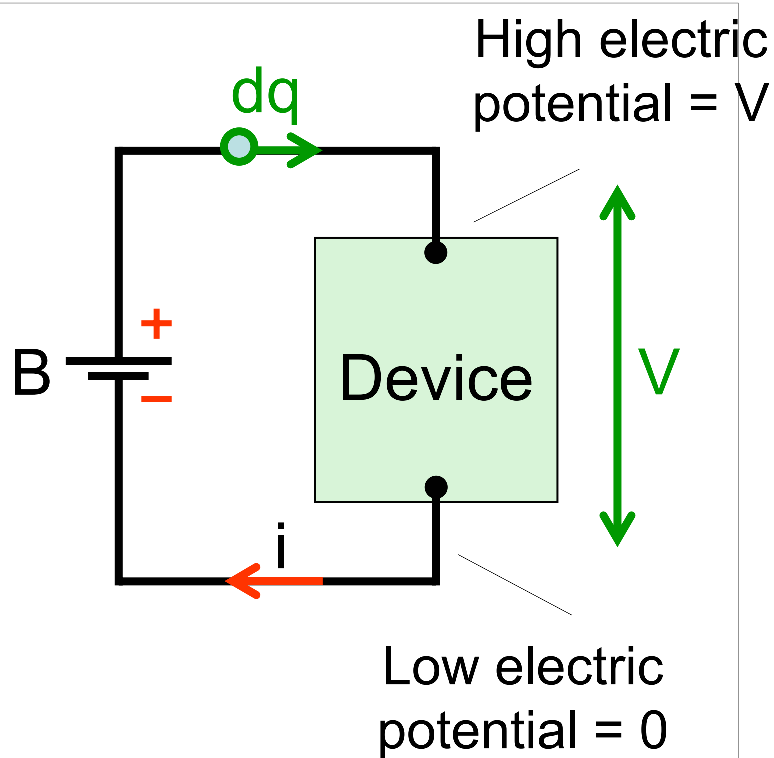


Electric energy from the battery is transferred at a rate of P = i V to heat up the resistor

When electrons drift through a resistor, they collide with the molecules of the resistor, this increases the random motion of the molecules which is equivalent to higher temperature

27-7 Power in Electric Circuits

Derivation of $\boxed{P = i\,V}$



Suppose a charge of dq moves through the device during time dt

The change in potential energy of dq is

$$dU = dq(0-V) = -\,dqV$$

The rate at which potential energy changes

$$\frac{dU}{dt} = -\frac{dq\,V}{dt} = -\frac{dq}{dt}V = -\,i\,V$$

loss

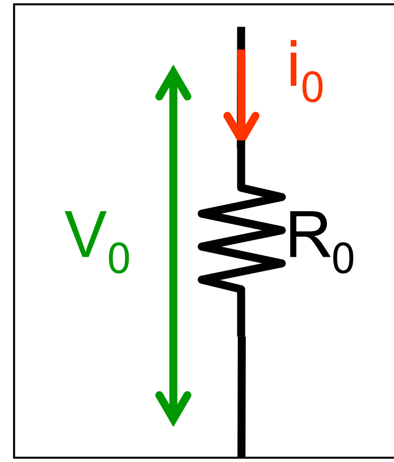
Since the energy is conserved, the rate at which potential energy is lost is the rate at which energy of other form is gained

$$P = -\frac{dU}{dt} = i\,V$$

27-7 Power in Electric Circuits

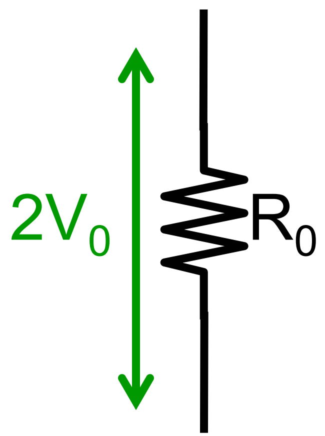
Checkpoint 5

Rank the change in the rate at which electrical energy is converted to thermal energy due to the resistance?



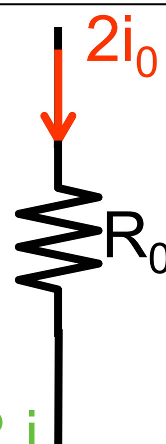
$$V_0 = R_0\, i_0$$

$$P_0 = \frac{V_0^2}{R_0}$$



$$P = \frac{(2V_0)^2}{R_0}$$

$$P = 4P_0$$

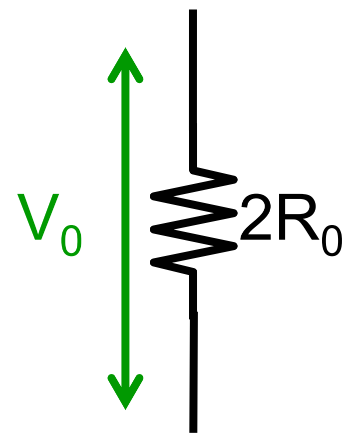


$$V = R\,i$$

$$V = R(2i_0)$$

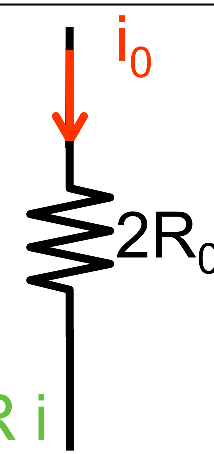
$$V = 2V_0$$

$$P = 4\,P_0$$



$$P = \frac{V_0^{\,2}}{2R_0}$$

$$P = \frac{P_0}{2}$$



$$V = R\,i$$

$$V = (2R_0)i_0$$

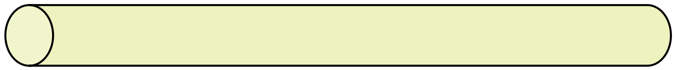
$$V = 2V_0$$

$$P = \frac{(2V_0)^2}{2R_0}$$

$$P = 2\,P_0$$

27-7 Power in Electric Circuits

Sample Problem 27-6



Nichrome heating wire
Nickel-chromium-iron alloy

$R = 72\ \Omega$

$V = 120\ V$

At what rate is energy dissipated?

$$P = \frac{V^2}{R} = 200\ W$$

If the wire is cut in half, at what rate is energy dissipated ?

For half the length $R = 36\ \Omega$

$$P = \frac{V^2}{R} = 400\ W$$