Recitation 7

•2 An ideal gas undergoes a reversible isothermal expansion at 77.0°C, increasing its volume from 1.30 L to 3.40 L. The entropy change of the gas is 22.0 J/K. How many moles of gas are present?

$$\Delta S = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i} = nR \ln \frac{V_f}{V_i} + 0$$

$$n = \frac{\Delta S}{R \ln \frac{V_f}{V_i}} = \frac{22.0 \text{ J/K}}{\left(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}\right) \ln \frac{3.40 \text{ L}}{1.30 \text{ L}}} = 2.75 \text{ mol.}$$

••9 A 10 g ice cube at −10°C is placed in a lake whose temperature is 15°C. Calculate the change in entropy of the cube–lake system as the ice cube comes to thermal equilibrium with the lake. The specific heat of ice is 2220 J/kg · K. (*Hint:* Will the ice cube affect the lake temperature?)

$$\Delta S_{sys} = \Delta S_{ice} + \Delta S_{lake}$$

$$\Delta S_{ice} = \Delta S_1 + \Delta S_2 + \Delta S_3$$

$$\Delta S_1 = \int_i^f \frac{dQ}{T} = m_{ice} c_{ice} \int_{T_1}^{T_m} \frac{dT}{T} = m_{ice} c_{ice} \ln \frac{T_m}{T_1}$$

$$= (10.0\ \text{g}) \left(2.220 \frac{\text{J}}{\text{g} \cdot \text{K}} \right) \ln \frac{273\ \text{K}}{263\ \text{K}} = 0.8285\ \frac{\text{J}}{\text{K}}.$$

$$\Delta S_2 = \int_i^f \frac{dQ}{T} = \frac{Q}{T_m} = \frac{m_{ice} L_F}{T_m} = \frac{(10.0\text{ g})(333\text{ J/g})}{273\text{ K}} = 12.20\text{ J/K}.$$

$$\Delta S_3 = \int_i^f \frac{dQ}{T} = m_{ice} c_w \int_{T_m}^{T_2} \frac{dT}{T} = m_{ice} c_w \ln \frac{T_2}{T_m}$$

$$= (10.0\text{ g})\left(4.187 \frac{\text{J}}{\text{g} \cdot \text{K}}\right) \ln \frac{288\text{ K}}{273\text{ K}} = 2.240\ \frac{\text{J}}{\text{K}}.$$

$$\Delta S_{ice} = 0.8285\ \frac{\text{J}}{\text{K}} + 12.20\ \frac{\text{J}}{\text{K}} + 2.240\ \frac{\text{J}}{\text{K}} = 15.27 \frac{\text{J}}{\text{K}}.$$

$$\Delta S_{lake} = \int_i^f \frac{dQ}{T} = \frac{Q}{T_2} = -\frac{Q_1 + Q_2 + Q_3}{T_2}$$

$$Q_1 = m_{ice} c_{ice} \Delta T = (10.0\ \mathrm{g}) \left(2.220 \frac{\mathrm{J}}{\mathrm{g} \cdot \mathrm{K}} \right) [0°\mathrm{C} - (-10°\mathrm{C})] = 222.0\ \mathrm{J}.$$

$$Q_2 = m_{ice} L_F = (10.0\ \mathrm{g})(333\ \mathrm{J/g}) = 3330\ \mathrm{J}.$$

$$Q_3 = m_{ice} c_w \Delta T = (10.0\ \mathrm{g}) \left(4.187 \frac{\mathrm{J}}{\mathrm{g} \cdot \mathrm{K}} \right) [15°\mathrm{C} - 0°\mathrm{C}] = 628.0\ \mathrm{J}.$$

$$\Delta S_{lake} = -\frac{222.0\ \mathrm{J} + 3330\ \mathrm{J} + 628.0\ \mathrm{J}}{288\ \mathrm{K}} = -14.51 \frac{\mathrm{J}}{\mathrm{K}}.$$

$$\Delta S_{sys} = \Delta S_{ice} + \Delta S_{lake} = 15.27 \frac{\mathrm{J}}{\mathrm{K}} - 14.51 \frac{\mathrm{J}}{\mathrm{K}} = 0.76 \frac{\mathrm{J}}{\mathrm{K}}.$$

••30 A 500 W Carnot engine operates between constant-temperature reservoirs at 100°C and 60.0°C. What is the rate at which energy is (a) taken in by the engine as heat and (b) exhausted by the engine as heat?

a)

$$\varepsilon_C = 1 - \frac{T_L}{T_H} = 1 - \frac{333\ \text{K}}{373\ \text{K}} = 0.1072$$

$$\dot{Q}_H = \frac{\dot{W}}{\varepsilon_C} = \frac{500\ \text{W}}{0.1072} = 4.66\ \text{kW}.$$

b)

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 4.66\ \text{kW} - 500\ \text{W} = 4.16\ \text{kW}.$$

••42 The motor in a refrigerator has a power of 200 W. If the freezing compartment is at 270 K and the outside air is at 300 K, and assuming the efficiency of a Carnot refrigerator, what is the maximum amount of energy that can be extracted as heat from the freezing compartment in 10.0 min?

$$K = \frac{T_L}{T_H - T_L} = \frac{270\ \text{K}}{300\ \text{K} - 270\ \text{K}} = 9.00.$$

$$\dot{Q}_L = K\dot{W} = (9.00)(200\ \text{W}) = 1.80\ \text{kW}.$$

$$Q_L = \dot{Q}_L t = (1.80 \times 10^3\ \text{W})(600\ \text{s}) = 1.08 \times 10^6\ \text{J} = 1.08\ \text{MJ}.$$