Recitation 9

•1 SSM A particular 12 V car battery can send a total charge of 84 A · h (ampere-hours) through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (*Hint:* See Eq. 21-3.) (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

a)

$$q = 84 \text{ A} \cdot \text{h} = (84 \text{ C/s})(3600 \text{ s}) = 3.0 \times 10^5 \text{ C}.$$

b)

$$\Delta U = q\Delta V = (3.0 \times 10^5 \text{ C})(12 \text{ V}) = 3.6 \times 10^6 \text{ J} = 3.6 \text{ MJ}.$$

••15 SSM ILW A spherical drop of water carrying a charge of 30 pC has a potential of 500 V at its surface (with $V = 0$ at infinity). (a) What is the radius of the drop? (b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

a)

$$r_1 = k\frac{q_1}{V_1} = \left(8.99 \times 10^9 \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{30 \times 10^{-12}\ \mathrm{C}}{500\ \mathrm{V}} = 5.4 \times 10^{-4}\ \mathrm{m}.$$

b)

$$\frac{\mathrm{Vol}_2}{\mathrm{Vol}_1} = \frac{r_2^3}{r_1^3} \Longrightarrow r_2 = r_1(\mathrm{Vol}_2/\mathrm{Vol}_1)^{1/3} = (5.4 \times 10^{-4}\mathrm{m})(2)^{1/3} = 6.8 \times 10^{-4}\mathrm{m}.$$

$$V_2 = k\frac{q_2}{r_2} = \left(8.99 \times 10^9 \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(\frac{2 \times 30 \times 10^{-12}\ \mathrm{C}}{6.80 \times 10^{-4}\ \mathrm{m.}}\right) = 790\ \mathrm{V}.$$

••37 SSM What is the magnitude of the electric field at the point $(3.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})$ m if the electric potential is given by $V = 2.00xyz^2$, where V is in volts and x, y, and z are in meters?

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2, \qquad E_y = -\frac{\partial V}{\partial y} = -2.00xz^2,$$

$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz.$$

At the point in question,

$$E_x = -2.00(-2.00)(4.00)^2 = 64.0 \text{ N/C}.$$

$$E_y = -2.00(3.00)(4.00)^2 = -96.0 \text{ N/C}.$$

$$E_z = -4.00(3.00)(-2.00)(4.00) = 96.0 \text{ N/C}.$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150 \text{ N/C}.$$

••45 ILW A particle of charge q is fixed at point P, and a second particle of mass m and the same charge q is initially held a distance r_1 from P. The second particle is then released. Determine its speed when it is a distance r_2 from P. Let $q = 3.1\ \mu C$, $m = 20$ mg, $r_1 = 0.90$ mm, and $r_2 = 2.5$ mm.

$$\Delta K = W = -\Delta U$$

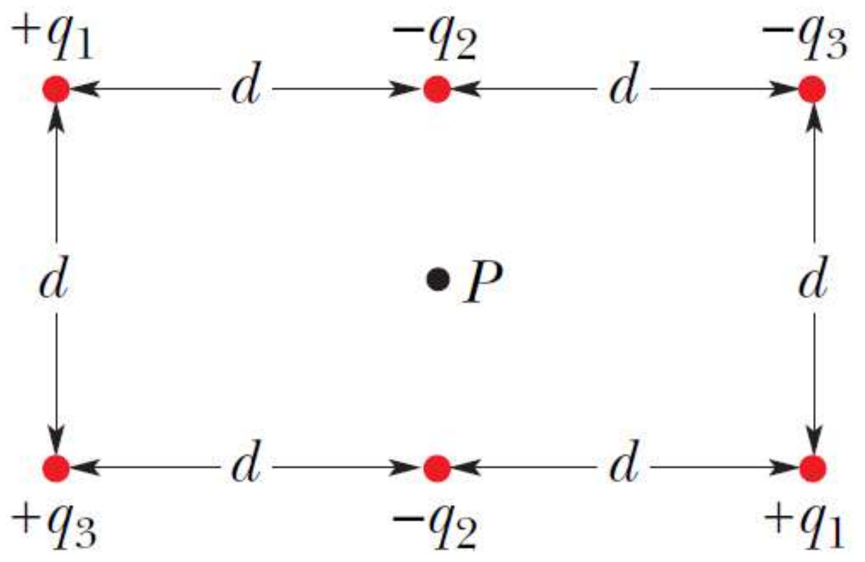
$$K_f - 0 = U_i - U_f = k\frac{q^2}{r_i} - k\frac{q^2}{r_f} = kq^2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= \left(8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)(3.1 \times 10^{-6}\ \mathrm{C})^2 \left(\frac{1}{0.90 \times 10^{-3}\ \mathrm{m}} - \frac{1}{2.5 \times 10^{-3}\ \mathrm{m}}\right)$$

$$= 61.4\ \mathrm{J}.$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(61.4\ \mathrm{J})}{20 \times 10^{-6}\ \mathrm{kg}}} = 2.5 \times 10^3\ \frac{\mathrm{m}}{\mathrm{s}}.$$

92 In Fig. 24-65, point P is at the center of the rectangle. With $V = 0$ at infinity, $q_1 = 5.00$ fC, $q_2 = 2.00$ fC, $q_3 = 3.00$ fC, and $d = 2.54$ cm, what is the net electric potential at P due to the six charged particles?



The two q_3 charges produce zero electric potential at P. The potential V at P is produced by the remaining four charges:

$$V = 2V_1 + 2V_2 = 2k\frac{q_1}{\sqrt{d^2 + (d/2)^2}} + 2k\frac{-q_2}{d/2} = 4k\frac{q_1}{\sqrt{5}d} + 4k\frac{-q_2}{d}$$

$$= \frac{4k}{d}\left(q_1/\sqrt{5} - q_2\right)$$

$$= \frac{4(8.99 \times 10^9 \mathrm{N \cdot m^2/C^2})}{2.54 \times 10^{-2}\ \mathrm{m}}\left[(5.00 \times 10^{-15}\ \mathrm{C})/\sqrt{5} - (2.00 \times 10^{-15}\ \mathrm{C})\right]$$

$$= 3.34 \times 10^{-4}\ V.$$