Recitation 8

•1 SSM The square surface shown in Fig. 23-30 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude $E = 1800$ N/C and with field lines at an angle of $\theta = 35°$ with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.

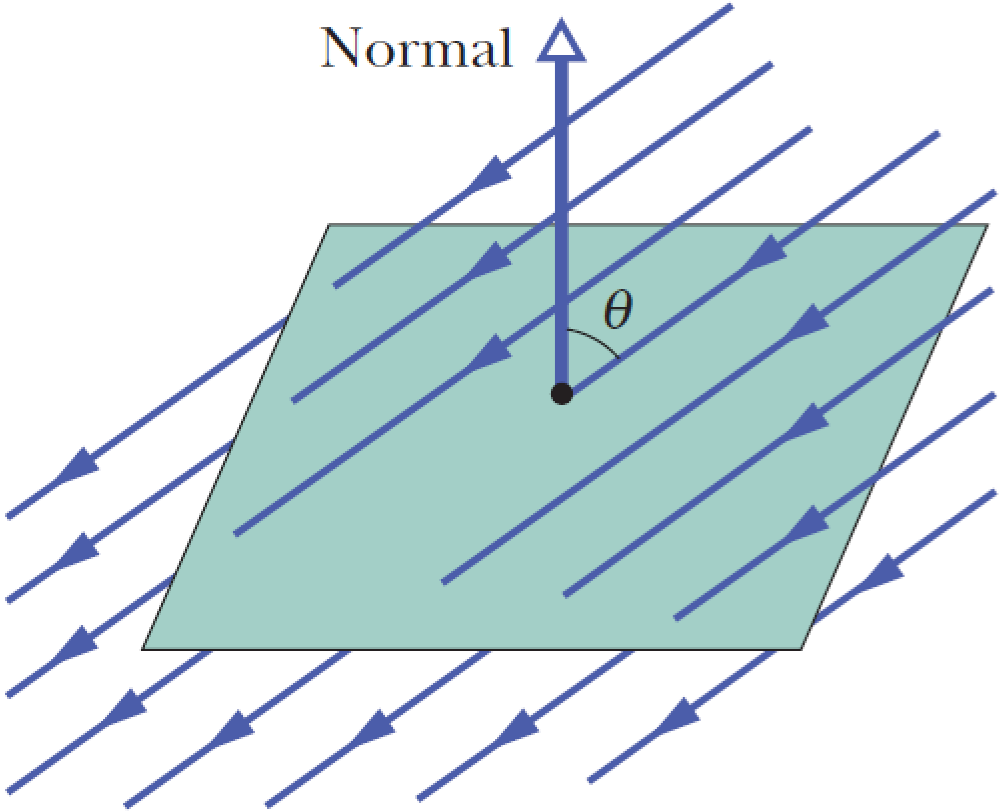
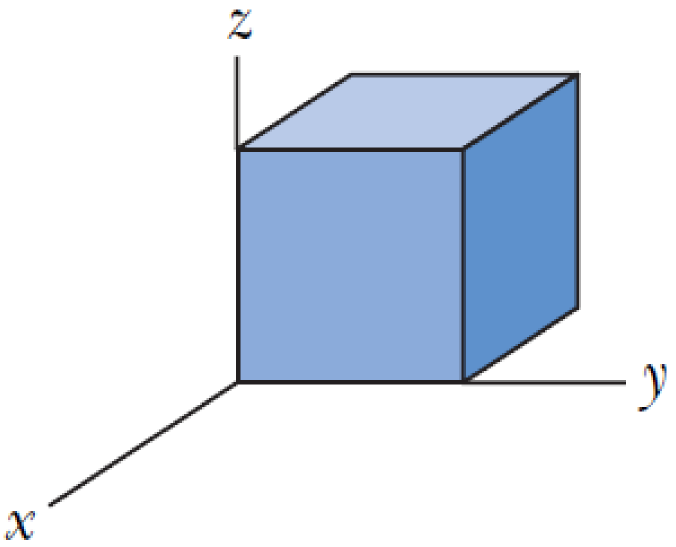


Figure 23-30 Problem 1.

$$
\begin{aligned}
\Phi = \vec{E} \cdot \vec{A} &= EA\cos\phi \\
&= (1800\ \mathrm{N/C})(3.2 \times 10^{-3}\ \mathrm{m})^2 \cos 145^\circ \\
&= -0.015\ \frac{\mathrm{N \cdot m^2}}{\mathrm{C}}.
\end{aligned}
$$

••10 Figure 23-30 shows a closed Gaussian surface in the shape of a cube of edge length 2.00 m. It lies in a region where the nonuniform electric field is given by $\vec{E} = (3.00x + 4.00)\hat{\mathrm{i}} + 6.00\hat{\mathrm{j}} + 7.00\hat{\mathrm{k}}$ N/C, with x in meters. What is the net charge contained by the cube?



Hint: We can divide the electric field into two parts, uniform and non-uniform:

$$\vec{E} = \vec{E}_{\mathrm{u}} + \vec{E}_{\mathrm{nu}} = \left(4.00\,\hat{\mathrm{i}} + 6.00\,\hat{\mathrm{j}} + 7.00\,\hat{\mathrm{k}}\right) + (3.00x\,\hat{\mathrm{i}})$$

The net flux due to the uniform part is zero!

The non-uniform field can result in non-zero flux through the front and back faces of the cube only.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \Phi_{\mathrm{F}} + \Phi_{\mathrm{B}}$$

$$\Phi_F = \vec{E}_{\mathrm{nu}} \cdot \vec{A}_{\mathrm{F}} = (3.00x\ \hat{\imath}) \cdot (4.00\ \hat{\imath}) = (3.00)(0)(4.00) = 0$$

$$\Phi_B = \vec{E}_{\mathrm{nu}} \cdot \vec{A}_{\mathrm{B}} = (3.00x\ \hat{\imath}) \cdot (-4.00\ \hat{\imath})$$

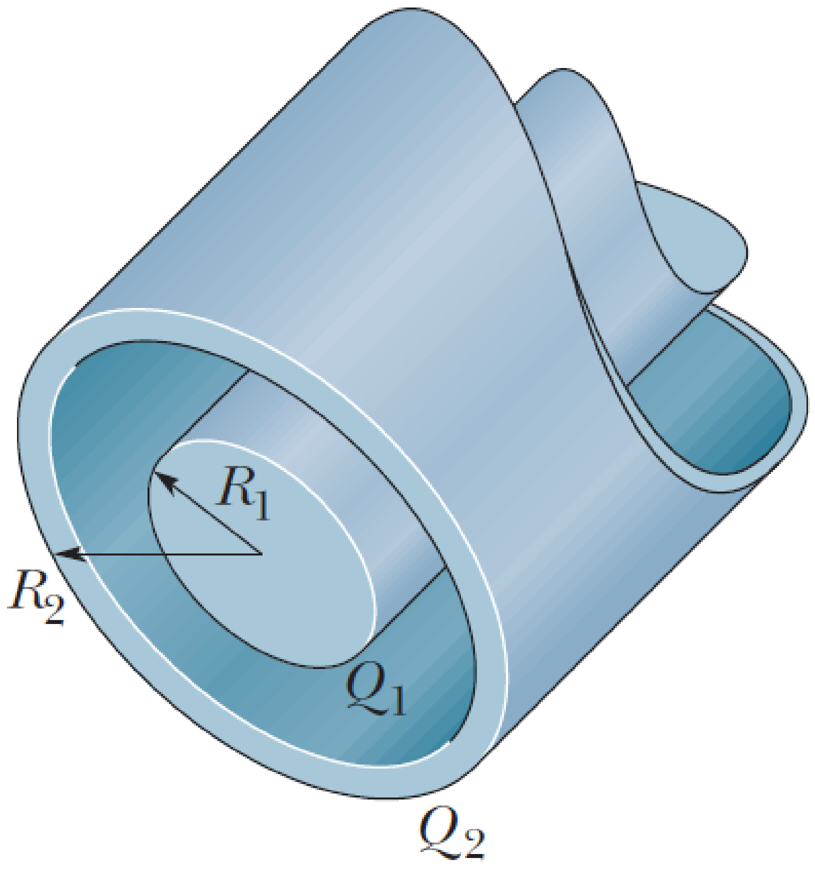
$$= (3.00)(-2.00)(-4.00) = 24\,\frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}}.$$

$$\Phi = 24.0\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}.$$

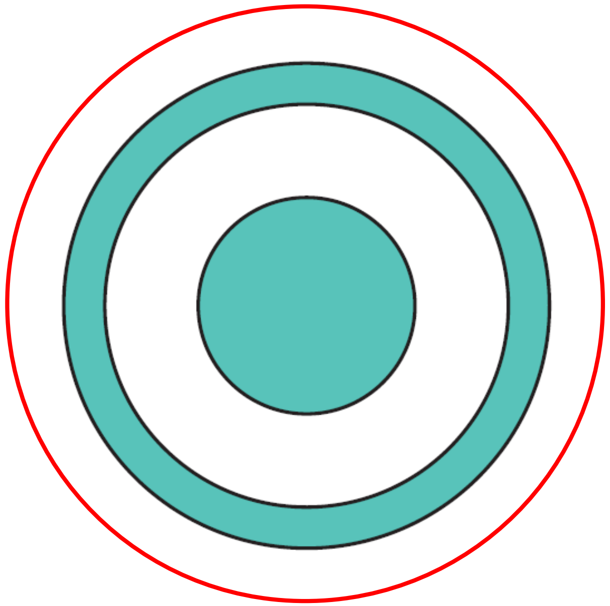
$$q_{\mathrm{enc}} = \varepsilon_0 \Phi = (8.85\ \times 10^{-12} \mathrm{N} \cdot \mathrm{C}^2/\mathrm{m}^2)(24.0\ \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C})$$

$$= 2.12 \times 10^{-10}\ \mathrm{C} = 21.1\ \mathrm{nC}.$$

••29 SSM WWW Figure 23-38 is a section of a conducting rod of radius $R_1 = 1.30$ mm and length $L = 11.00$ m inside a thin-walled coaxial conducting cylindrical shell of radius $R_2 = 10.0R_1$ and the (same) length L. The net charge on the rod is $Q_1 = +3.40 \times 10^{-12}$ C; that on the shell is $Q_2 = -2.00Q_1$. What are the (a) magnitude E and (b) direction (radially inward or outward) of the electric field at radial distance $r = 2.00R_2$? What are (c) E and (d) the direction at $r = 5.00R_1$? What is the charge on the (e) interior and (f) exterior surface of the shell?



a) We draw a cylindrical Gaussians surface of radius r and length h. Assuming $\vec{E}$ is outward,



$$\Phi = \vec{E} \cdot \vec{A} = EA = E(2\pi rh)$$

$$q_{enc} = (\lambda_1 + \lambda_2)h = \left(\frac{Q_1}{L} + \frac{Q_2}{L}\right)h$$

$$E(2\pi rh) = \frac{1}{\varepsilon_0}\left(\frac{Q_1}{L} + \frac{Q_2}{L}\right)h$$

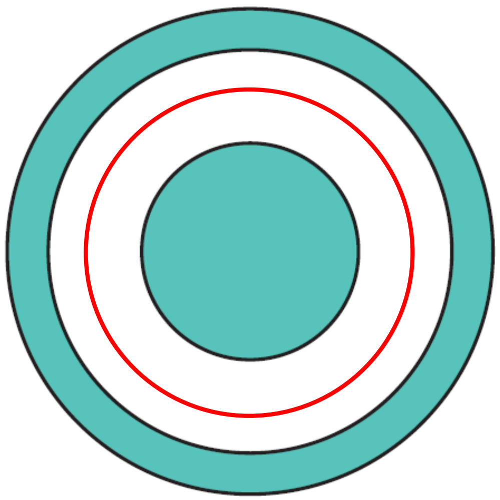
$$E = \frac{Q_1 + Q_2}{2\pi\varepsilon_0 rL} = \frac{Q_1 - 2.00Q_1}{2\pi\varepsilon_0(2.00R_2)L} = \frac{-Q_1}{40.0\pi\varepsilon_0 R_1 L}$$

$$= \frac{-3.40 \times 10^{-12}\text{C}}{40.0\pi\varepsilon_0(1.30 \times 10^{-3}\text{m})(11.0\text{ m})} = -0.214\text{ N/C}.$$

The magnitude of the electric field must be 0.214 N/C.

b) $\vec{E}$ must be radially inward, since we got the wrong sign for E.

c) We draw a cylindrical Gaussians surface of radius r and length h. Assuming $\vec{E}$ is outward,



$$\Phi = \vec{E}\cdot\vec{A} = EA = E(2\pi r h)$$

$$q_{enc} = \lambda_1 h = Q_1 h/L$$

$$E(2\pi r h) = \frac{1}{\varepsilon_0}(Q_1 h/L)$$

$$E = \frac{Q_1}{2\pi\varepsilon_0 r L} = \frac{Q_1}{2\pi\varepsilon_0(5.00R_1)L} = \frac{Q_1}{10.0\pi\varepsilon_0 R_1 L}$$

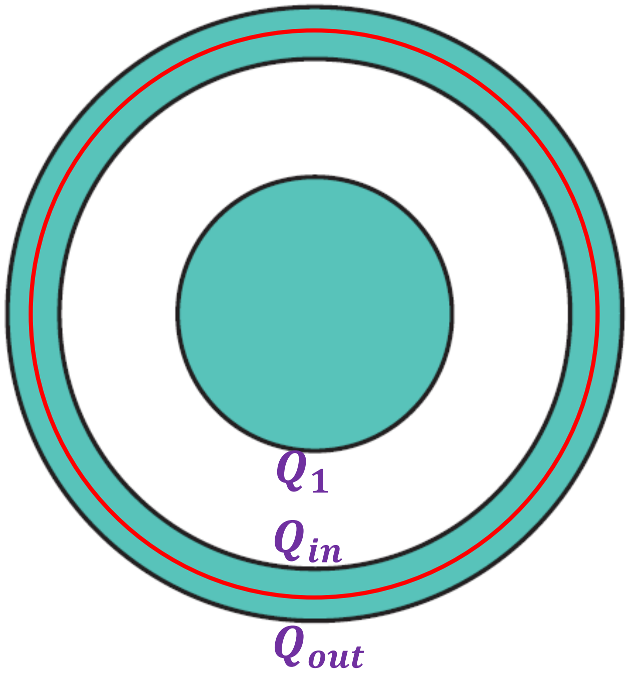
$$= \frac{3.40\times10^{-12}\mathrm{C}}{10.0\pi\varepsilon_0(1.30\times10^{-3}\mathrm{m})(11.0\ \mathrm{m})} = 0.885\ \mathrm{N/C}.$$

d) $\vec{E}$ is radially outward.

e) We draw a cylindrical Gaussians surface of radius r inside the shell, where the electric field is zero.

$$q_{\text{enc}} = Q_1 + Q_{in} = \varepsilon_0 \Phi = 0.$$

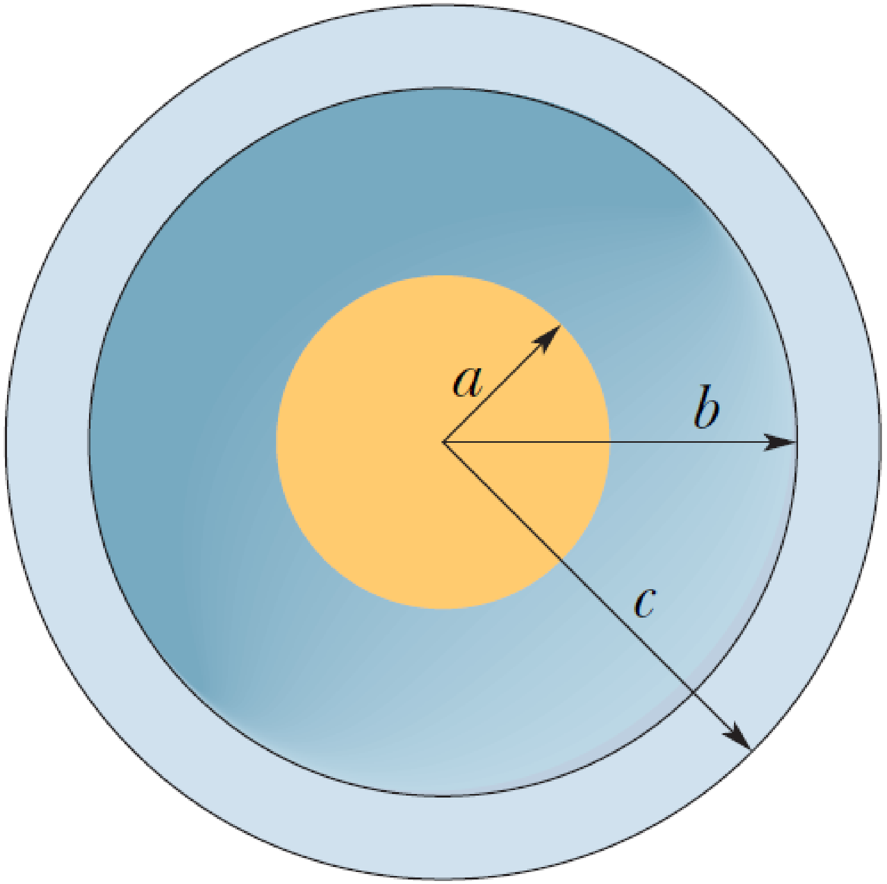
$$Q_{in} = -Q_1 = -3.40 \times 10^{-12}\ \text{C}.$$



f)

$$Q_{out} = Q_2 - Q_{in} = -2.00Q_1 - (-Q_1) = -Q_1$$
$$= -3.40 \times 10^{-12}\ \text{C}$$

••49 In Fig. 23-50, a solid sphere of radius $a = 2.00$ cm is concentric with a spherical conducting shell of inner radius $b = 2.00a$ and outer radius $c = 2.40a$. The sphere has a net uniform charge $q_1 = +5.00$ fC; the shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances (a) $r = 0$, (b) $r = a/2.00$, (c) $r = a$, (d) $r = 1.50a$, (e) $r = 2.30a$, and (f) $r = 3.50a$? What is the net charge on the (g) inner and (h) outer surface of the shell?



a)

$$E = \frac{q_1}{4\pi\varepsilon_0}\frac{r}{a^3} = \frac{q_1}{4\pi\varepsilon_0}\frac{0}{a^3} = 0.$$

b)

$$E = \frac{q_1}{4\pi\varepsilon_0}\frac{r}{a^3} = \frac{q_1}{4\pi\varepsilon_0}\frac{a/2}{a^3} = \frac{q_1}{8\pi\varepsilon_0 a^2} = \frac{5.00\times10^{-9}\text{C}}{8\pi\varepsilon_0(2.00\times10^{-2}\text{ m})^2}$$

$$= 0.0562\text{ N/C}.$$

c)

$$E = \frac{q_1}{4\pi\varepsilon_0}\frac{a}{a^3} = \frac{q_1}{4\pi\varepsilon_0 a^2} = 0.112\text{ N/C}.$$

d)

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{q_1}{4\pi\varepsilon_0(3a/2)^2} = \frac{q_1}{9\pi\varepsilon_0 a^2} = 0.0500\text{ N/C}.$$

e) At $r = 2.3a$, $E = 0$ because this radius is inside the conducting shell.

f)

$$E = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1 - q_1}{4\pi\varepsilon_0 (7a/2)^2} = 0.$$

g) To have zero electric field inside the shell, we must have the charge on the inner surface of the shell to be

$$q_{in} = -q_1 = -5.00 \times 10^{-9}\ \text{C}.$$

h)

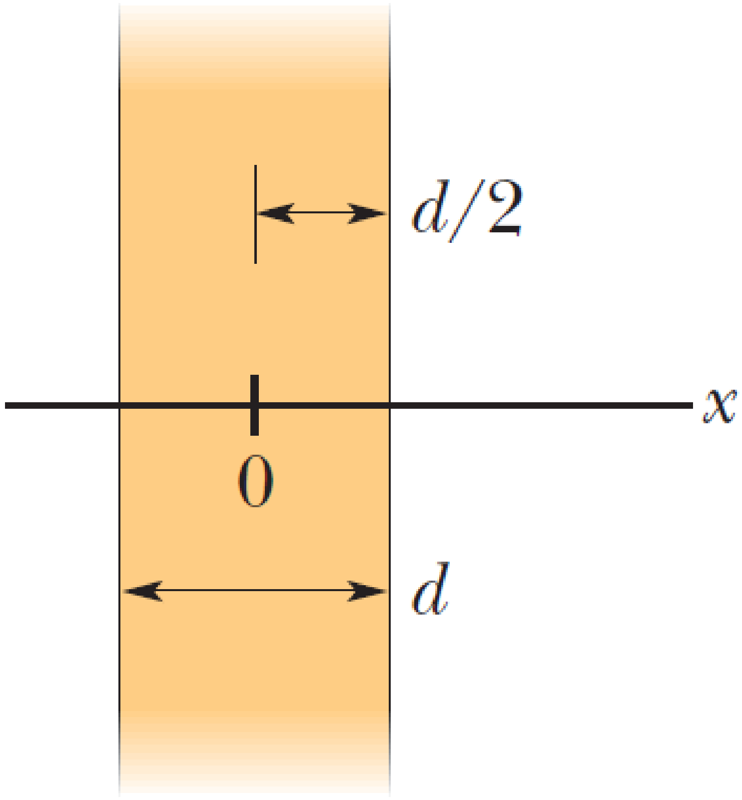
$$q_{out} = q_2 - q_{in} = -q_1 - (-q_1) = 0$$

Optional

••**15** A particle of charge $+q$ is placed at one corner of a Gaussian cube. What multiple of q/ε_0 gives the flux through (a) each cube face forming that corner and (b) each of the other cube faces?

Optional

•••43 Figure 23-47 shows a cross section through a very large nonconducting slab of thickness $d = 9.40$ mm and uniform volume charge density $\rho = 5.80\ \text{fC/m}^3$. The origin of an x axis is at the slab's center. What is the magnitude of the slab's electric field at an x coordinate of (a) 0, (b) 2.00 mm, (c) 4.70 mm, and (d) 26.0 mm?



Optional

••51 SSM WWW In Fig. 23-52, a nonconducting spherical shell of inner radius $a = 2.00$ cm and outer radius $b = 2.40$ cm has (within its thickness) a positive volume charge density $\rho = A/r$, where A is a constant and r is the distance from the center of the shell. In addition, a small ball of charge $q = 45.0$ fC is located at that center. What value should A have if the electric field in the shell ($a \leq r \leq b$) is to be uniform?