Recitation 2

•15 SSM WWW A stretched string has a mass per unit length of 5.00 g/cm and a tension of 10.0 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the negative direction of an x axis. If the wave equation is of the form $y(x, t) = y_m \sin(kx \pm \omega t)$, what are (a) y_m, (b) k, (c) ω, and (d) the correct choice of sign in front of ω?

a) $y_m = 0.12\ \text{mm}$

b)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{2\pi f}{\sqrt{\tau/\mu}} = \frac{2\pi(100\ \text{Hz})}{\sqrt{10.0\ \text{N}/(0.500\ \text{kg/m})}} = \frac{628\ \text{Hz}}{4.47\ \text{m/s}} = 140\ \frac{\text{rad}}{\text{m}}$$

c)

$$\omega = 2\pi f = 2\pi(100\ \text{Hz}) = 628\ \frac{\text{rad}}{\text{s}}.$$

d) The positive sign.

•26 A string along which waves can travel is 2.70 m long and has a mass of 260 g. The tension in the string is 36.0 N. What must be the frequency of traveling waves of amplitude 7.70 mm for the average power to be 85.0 W?

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 = \frac{1}{2}\mu v (2\pi f)^2 y_m^2 = 2\pi^2 \mu v f^2 y_m^2$$

$$\mu = \frac{M}{L} = \frac{0.260\text{ kg}}{2.70\text{ m}} = 0.0963\text{ kg/m}.$$

$$v = \sqrt{\tau/\mu} = \sqrt{36.0\text{ N}/(0.0963\text{ kg/m})} = 19.3\text{ m/s}$$

$$f = \sqrt{\frac{P_{avg}}{2\pi^2 \mu v y_m^2}} = \frac{1}{\pi y_m}\sqrt{\frac{P_{avg}}{2\mu v}} = \frac{1}{\pi(7.70\times 10^{-3}\text{ m})}\sqrt{\frac{85.0\text{ W}}{2(0.0963\text{ kg/m})(19.3\text{ m/s})}}$$

$$= 198\text{ Hz}.$$

•31 SSM Two identical traveling waves, moving in the same direction, are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

$$y'_m = 2y_m \cos\frac{1}{2}\phi = 2y_m \cos\frac{\pi}{4} = 2y_m\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}y_m$$

•43 SSM WWW What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

a)

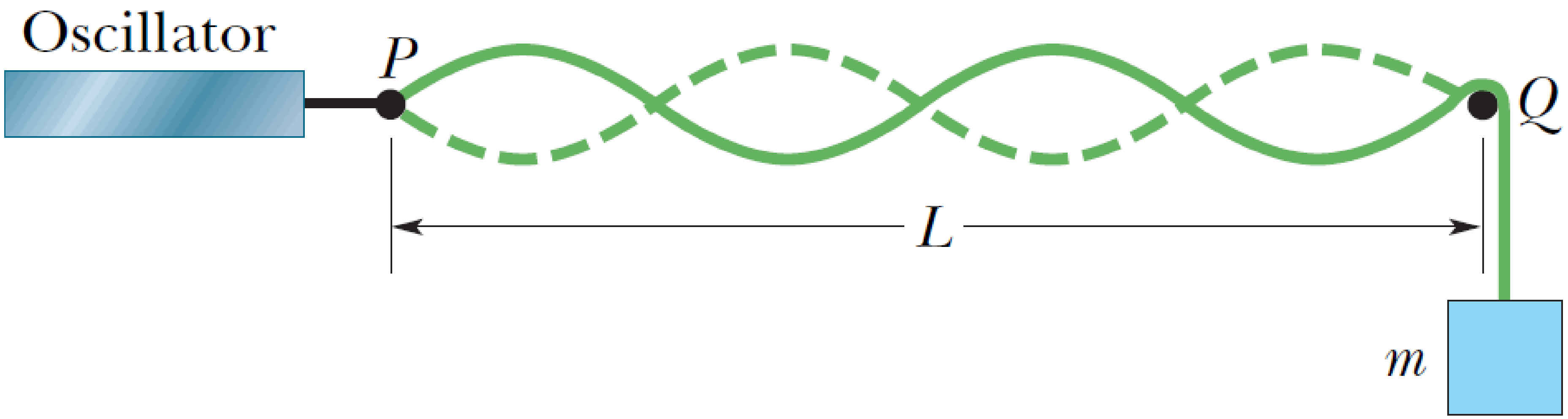
$$f_1 = \frac{v}{2L} = \frac{\sqrt{\frac{\tau}{\mu}}}{2L} = \frac{\sqrt{\frac{\tau}{M/L}}}{2L} = \frac{1}{2}\sqrt{\frac{\tau}{ML}} = \frac{1}{2}\sqrt{\frac{250\ \text{N}}{(0.100\ \text{kg})(10.0\ \text{m})}}$$

$$= 7.91\ \text{Hz}.$$

b) $f_2 = 2f_1 = 15.8\ \text{Hz}.$

c) $f_3 = 3f_1 = 23.7\ \text{Hz}.$

••58 GO In Fig. 16-41, a string, tied to a sinusoidal oscillator at P and running over a support at Q, is stretched by a block of mass m. Separation $L = 1.20$ m, linear density $\mu = 1.6$ g/m, and the oscillator frequency $f = 120$ Hz. The amplitude of the motion at P is small enough for that point to be considered a node. A node also exists at Q. (a) What mass m allows the oscillator to set up the fourth harmonic on the string? (b) What standing wave mode, if any, can be set up if $m = 1.00$ kg?



$$f_n = nf_1 = n\left(\frac{v}{2L}\right) = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}$$

a)

$$m = \frac{4\mu L^2 f_n^2}{n^2 g} = \frac{4(1.60 \times 10^{-3}\ \mathrm{kg/m})(1.20\ \mathrm{m})^2(120\ \mathrm{Hz})^2}{4^2(9.8\ \mathrm{m/s^2})} = 0.846\ \mathrm{kg}.$$

b)

$$n = 2Lf_n\sqrt{\frac{\mu}{mg}} = 2(1.20\ \mathrm{m})(120\ \mathrm{Hz})\sqrt{\frac{1.60 \times 10^{-3}\ \mathrm{kg/m}}{(1.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}} = 3.68$$

No standing wave mode can be set with $m = 1.00\ \mathrm{kg}$, since n is not an integer.