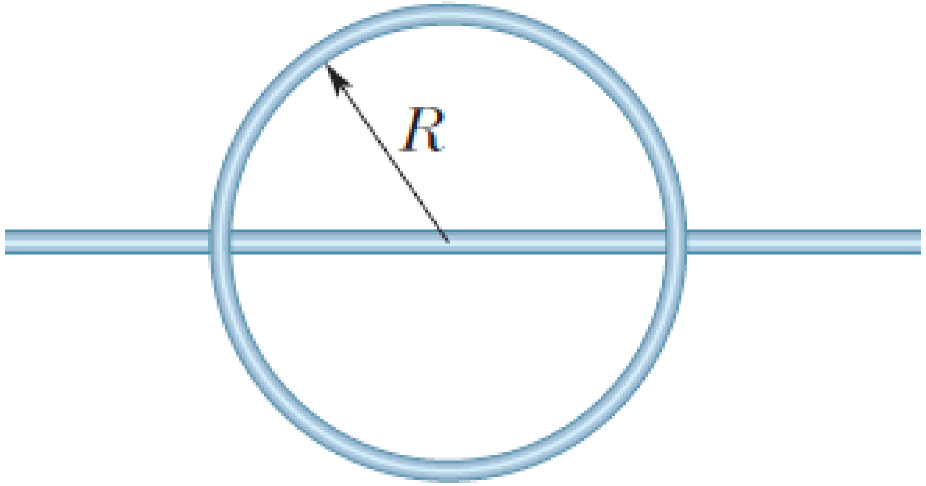
Recitation 14

•5 In Fig. 30-36, a wire forms a closed circular loop, of radius $R = 2.0$ m and resistance 4.0 Ω. The circle is centered on a long straight wire; at time $t = 0$, the current in the long straight wire is 5.0 A rightward. Thereafter, the current changes according to $i = 5.0\text{ A} - (2.0\text{ A/s}^2)t^2$. (The straight wire is insulated; so there is no electrical contact between it and the wire of the loop.) What is the magnitude of the current induced in the loop at times $t > 0$?



The field of the wire is parallel to the plane of the loop. Thus, the flux is zero at any time and $\mathcal{E}_{\text{ind}} = 0$!

••15 GO A square wire loop with 2.00 m sides is perpendicular to a uniform magnetic field, with half the area of the loop in the field as shown in Fig. 30-43. The loop contains an ideal battery with emf $\mathscr{E} = 20.0$ V. If the magnitude of the field varies with time according to $B = 0.0420 - 0.870t$, with B in teslas and t in seconds, what are (a) the net emf in the circuit and (b) the direction of the (net) current around the loop?

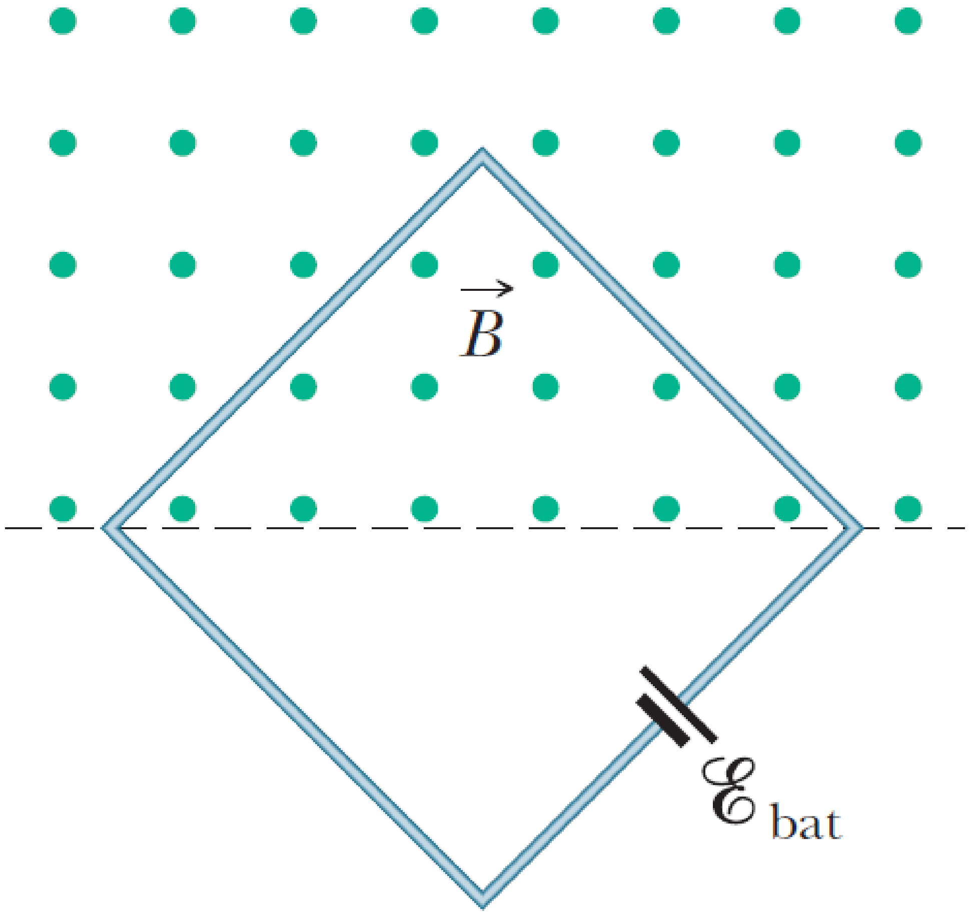


Figure 30-43 Problem 15.

a)

$$\Phi_B = AB = \frac{L^2}{2}(0.0420 - 0.870t)$$

$$\frac{d\Phi_\mathrm{B}}{dt} = -0.870\left(\frac{L^2}{2}\right) = -1.74.$$

$$\mathcal{E}_\mathrm{ind} = -\frac{d\Phi_\mathrm{B}}{dt} = 1.74\ \mathrm{V}.$$

The direction of the induced current is counterclockwise, like that generated by the battery. Thus,

$$\mathcal{E}_{net} = \mathcal{E}_{bat} + \mathcal{E}_\mathrm{ind} = 21.7\ \mathrm{V}.$$

b)

Counterclockwise.

••19 ILW An electric generator contains a coil of 100 turns of wire, each forming a rectangular loop 50.0 cm by 30.0 cm. The coil is placed entirely in a uniform magnetic field with magnitude $B = 3.50$ T and with $\vec{B}$ initially perpendicular to the coil's plane. What is the maximum value of the emf produced when the coil is spun at 1000 rev/min about an axis perpendicular to $\vec{B}$?

$$\Phi_B = BA\cos\theta(t)$$

$$\theta(t) = \omega t$$

$$\omega = 1000\,\frac{\text{rev}}{\text{min}}\,\frac{2\pi}{1\text{ rev}}\,\frac{1\text{ min}}{60\text{ s}} = 105\,\frac{\text{rad}}{\text{s}}.$$

$$\mathcal{E}_{\text{ind}} = -N\frac{d\Phi_{\text{B}}}{dt} = -N\frac{d\Phi_{\text{B}}}{dt}(BA\cos\omega t) = \omega NBA\sin\omega t$$

$$\mathcal{E}_{\text{ind,max}} = \omega NBA = \left(105\,\frac{\text{rad}}{\text{s}}\right)(100)(3.50\text{ T})(0.500\text{ m}\times 0.300\text{ m})$$

$$= 5.50\text{ kV}.$$

•**32** A loop antenna of area $2.00\ cm^2$ and resistance $5.21\ \mu\Omega$ is perpendicular to a uniform magnetic field of magnitude $17.0\ \mu T$. The field magnitude drops to zero in 2.96 ms. How much thermal energy is produced in the loop by the change in field?

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{\Delta\Phi_B}{\Delta t} = -A\frac{\Delta B}{\Delta t}$$

$$= -(2.00 \times 10^{-4}\ \text{m}^2)\frac{0 - 17.0 \times 10^{-6}\ \text{T}}{2.96 \times 10^{-3}\ \text{s}} = 1.15 \times 10^{-6}\ \text{V}.$$

$$E = P\Delta t = \frac{\mathcal{E}_{\text{ind}}^2}{R}\Delta t = \frac{(1.149 \times 10^{-6}\ \text{V})^2}{5.21 \times 10^{-6}\ \Omega}(2.96 \times 10^{-3}\ \text{s})$$

$$= 7.50 \times 10^{-10}\ \text{J}.$$